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BIASED
TECHNICAL
CHANGE AND
ECONOMIC
CONSERVATION
LAWS

Ryuzo Sato

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BIASED TECHNICAL CHANGE AND ECONOMIC CONSERVATION LAWS

by

Ryuzo Sato

C.V. Starr Professor of Economics
and Director of the Center for Japan-U.S.
Business and Economic Studies
Leonard Stern School of Business
New York University



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**To my grand children:
Keimi, Yūki, Ryuka and Yūna**

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Preface

Of the two topics mentioned in the title of this book, *Biased Technical Change and Economic Conservation Laws*, the former deals with an issue that has been frequently discussed in the existing literature. The latter, on the other hand, is still relatively unfamiliar to many economists. They both have one thing in common, however: they have basically never been thoroughly analyzed before.

Although terms such as “labor saving” and “capital saving” fall under the category of biased technical change, the first of these topics, no analysis has ever been made of its basic statistical estimation methodology or its structure relative to production functions. In particular, no model exists in which biased technical change gives rise endogenously to technical progress. A special feature of this book is its thorough investigation and analysis of these issues, which go far beyond existing studies in this area.

The concept of economic conservation laws dates back to Ramsey’s classic study of 1928, but until recently no one has ever dealt with its present-day significance or its application to economics attendant upon its subsequent development in the other social sciences. This book primarily makes use of Lie groups to shed new light on the analysis of economic conservation laws. Economic conservation laws are not simply abstract concepts; this book shows that they are tools of empirical analysis that can be applied to such topics as analyses of macro performance and corporate efficiency.

In short, the principal aim of this book is twofold: to reveal the new economic significance of the old concept of biased technical change and the current application of the new concept of economic conservation laws.

Many of the chapters here are revised versions of papers that were originally published in academic journals, subsequent to the publication of Volume One and Volume Two of *The Selected Essays* of Ryuzo Sato (Edward Elgar, 1996 & 1999). This volume is, in a sense, Volume Three of my *Selected Essays*. I owe a great debt of gratitude to my co-authors of several articles in this volume, Rama Ramachandran, Chanpin Lian, Youngduk Kim and Mariko Fuji. I would also like to thank Blackwell, Cambridge University Press, Edward Elgar, Elsevier, Kluwer Academics, and *Keio Economic Studies* for permitting me to include these previously published works in this book.

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Chapter 1

AN OVERVIEW

1. Introduction

1. This book deals with the two topics cited in the title: biased technical change and economic conservation laws. At first glance, the two might appear to have no connection with one another, yet when viewed from the perspective of the optimal control behavior of the primary agents in an economic analysis, both topics have much in common and are profoundly related as phenomena that are the result of optimized behavior.

The term “biased technical change,” needless to say, refers to a situation in which the factors of production (in this case, capital and labor) each achieve technical change at different rates. As a result, the efficiency of capital and the efficiency of labor exhibit different growth rates. Consequently, even though the ratio of capital to labor remains constant, the shares of each are affected by technical change. This is the essence of biased technical change.

2. The accepted postulate among economists has been that, as a special case of biased technical change, only Harrod neutral technical change is valid as a condition for balanced growth in a mature economy. In other words, for long-run macroeconomic stability, technical change must of necessity be Harrod neutral technical change. To be more precise, this means that the only hypothesis conducive to long-run balanced growth is one in which capital-augmenting technical change does not exist.

What economists have learned from serious empirical analysis over the past few decades, however, is that capital-augmenting technical change is not zero. On the contrary, it varies significantly in an economic upswing or downturn, increasing greatly during the former and decreasing during the latter. The trend is not zero, however. In short, technical change during an upswing is not completely canceled out by technical change during a downturn. Consequently, Harrod neutral technical change, which assumes a zero rate of capital-augmenting technical change not just over the short run but even in the long run, is an unrealistic hypothesis. It is merely the simplest mathematical hypothesis to ensure long-run macroeconomic stability.

3. It makes no sense to regard advances in information technology, computer functions, for example, which have been growing by leaps and bounds, as all the result of a rise in Harrod neutral labor efficiency. In the ten years that the author has used a computer, the technical change of his labor has remained virtually the same. Nevertheless, it is a fact that every time I buy a new computer, the efficiency of my work rises rapidly.

This fact is readily analyzable if one simply accepts that the computer's capital efficiency has risen. In order to explain why the author is able to make more expeditious use of the computer while his labor efficiency remains unchanged, an indirect explanation—the assumption that new computers are improving because the productivity of those workers involved in manufacturing them has gone up—is far-fetched. This would mean believing indirectly in a Ricardo-Marxian labor theory of value in which all values derive from labor.

4. If labor and capital are regarded as independent production factors, a company/agent might focus its energy on increasing the efficiency of labor over that of capital, depending on the economic situation at the time. Or, faced with too little capital, it might invest more resources into increasing the efficiency of capital rather than labor.

At the time of the oil crises of the 1970s, companies confronted by soaring oil prices focused on technical change as a way to economize on capital, i.e. oil. Indeed, technological development was precisely what saved Japan's resource-poor economy. In Japan, incidentally, this is called "sho-ene" or energy-saving technology development.

5. The differences between this book and other works are (1) its main theme is an analysis of precisely this sort of technical change in which the efficiencies of capital and labor rise at different rates, and (2) it shows that long-run economic stability is possible under a biased technical change other than Harrod neutral. The basic concept here is that, of the different types of technical change, capital-augmenting technical change occurs endogenously. To be more precise, the theory that an economy achieves stable balanced growth only under the existing Harrod neutral technical change is limited to a situation in which technical change of labor and of capital is exogenous; in short, the theory applies only to those situations in which technical change is bestowed upon a nation or one of its companies by another nation or another company or by Heaven without that nation or its company using any resources of its own. But technical change ought to be regarded as attainable only when investment is made in resources. This is the theory of endogenous technical change put forward in this book.

6. The essence of the theory of endogenous technical change is the question of optimization: i.e. how much of the resources that a nation or a company possesses should it invest in technical change? Specifically, this book attempts to analyze this problem using the Pontryagin-Hestenes-Bellman calculus of variation or an optimal control method.

7. The theory of economic conservation laws that makes up the second theme may be less familiar to many economists than the first concept of biased technical change. But in the fields of physics, applied mathematics and other modern sciences, it is becoming a tool for studying the question of optimization in greater depth.

The above mentioned calculus of variation or optimal control theory is a suitable tool to explain natural or economic phenomena; behind variables and systems that are mathematically observable are phenomena that cannot at first glance be observed. To take a well-known example in physics, there is a rule (conservation law) that, while the movement of a billiard ball is observable, the sum of the kinetic energy and the potential energy behind it, which are impossible to observe, is constant; this rule is constancy of the Hamiltonian function.

8. In endogenous occurrences of biased technical change or in the maximization of a company's long-run profits, we are able to observe fluctuations over time in technical change rates or in profit rates. Likewise, in models dealing with a country's optimum capital savings or its optimum technical change rate, we can observe variables in savings and investment, consumption and GDP, just as we can observe the movement of a billiard ball, but, of course, the more essential but invisible and hidden law behind these optimized phenomena cannot be observed. Yet, it is this law, which the eye cannot see, that embodies the essence of the endogeneity question.

Several hidden conservation laws may exist in a single optimized system. The methodology for making a close and accurate analysis of this sort of problem is the Emy Noether theory of invariance using Lie groups. To sum up, the two main themes are bound together by the common terms endogeneity and conservation laws.

9. A Short History of Biased Technical Change. The work that formally introduced the concept of technical change to modern economics was *The Theory of Wages* (1932) by Sir John Hicks. Hicks analyzed the impact of inventions and technical change on the shares of labor and capital in Britain's post-industrial-revolution economy and society. Even in situations where the ratio of capital to labor was constant and unchanging, if technical change occurred, it might have an effect on the shares of capital and labor or their distributive shares.

Even though the ratio of capital to labor remained unchanged, a new invention might make the share of capital increase and the share of labor decline. Or the opposite might occur. As the simplest case, Hicks conceived of a situation in which a new invention or technical change had absolutely no impact whatsoever on the shares of labor and capital. To be more precise, one in which, under a constant capital-to-labor ratio, a new invention or technical change would have no effect on the income distributions of labor or capital, or the effect would be neutral. This subsequently came to be called Hicks neutral technical change.

10. The 1930s when Hicks advanced his theory of technical change was the era of the Great Depression, which had started in the United States and spread throughout the globe. The world's economists paid no attention to Hicks' contribution to the analysis of questions related to growth and technical change. Keynes' *General Theory* came out in 1936, a few years after the publication of Hicks' book, at the very time when an academic theory of how to get through the depression was in high demand. Needless to say, Keynes' work attracted the attention not only of economists but of politicians and policy-makers.

After World War II at the beginning of the 1950s, economists primarily in the United States began to think that, by skillfully combining Keynes' prescription and market principles, it would be possible for the world economy to escape

recession and achieve stable growth. This was the theory of a mixed economy set forth by Paul Samuelson.

11. By the mid 1950s the belief had become prevalent that the world economy, and especially the American economy, would never experience another Great Depression like the one in the 1930s. On the contrary, given the prosperity and growth of the US economy resulting from postwar technological advances, there was a growing view that, in addition to the growth of labor and capital, reconsideration needed to be given to the significance of the contribution of technical change. For the first time, the theory of technical change that Hicks had analyzed in the 1930s began to attract economists' interest. Representative of this trend was Professor Robert Solow of MIT.

12. Solow analyzed growth trends in the non-agricultural private sector using Kendrick data for the US economy between 1909 and 1949 (Review of Economics & Statistics, 1957). Using a neoclassical production function of constant returns, he hypothesized that the growth rate for income in the non-farm private sector would also be affected by the growth rate for technical change over and above the growth rates for capital and labor. The technical change that Solow used here was the very same neutral type that Hicks had analyzed in the 1930s.

Solow ascertained that part of the income growth rate which is not dependent on the growth rates of labor and capital occurs as a result of technical change. Moreover, he succeeded in estimating the growth rate of technical change simply by subtracting the sum of the growth rates of labor and capital weighted by their distributive shares from the growth rate of income. According to Solow's estimates, approximately one-third of the income growth rate is dependent on technical change. Thus, Hicks' technical change of the 1930s was revived by Solow in the 1950s.

13. Solow's achievement had an enormous impact on young economists at the time. In particular, Solow's method opened the way to estimate from the same data both the production function, which is hard to estimate, and the rate of technical change. The paper by Professor John Kendrick and the author (American Economic Review, 1963) brought this work to completion.

Using the same Kendrick data as Solow had, it was possible to estimate the technical change rate and the production function simultaneously by estimating the elasticity of factor substitution, a concept that Professor Hicks had developed in the 1930s. It was estimated that the technical change rate in the period 1909–1920 was an average of 2.1 percent, and the elasticity of factor substitution was 0.6. Thus, it was proposed that the most suitable theory for explaining the growth of the American economy in that period is a production function with an elasticity of factor substitution of 0.6 (a CES production function).

Another important point made in this article was that the estimated value of the elasticity was smaller than the Cobb-Douglas production function (=1) emphasized in the Solow estimate, i.e. a production function with an elasticity of factor substitution of 0.6, more approximately explained the growth of the American economy.

14. There were doubts at the time whether the Hicks neutral concept might be too unrealistic to explain technical change. Economists began to perceive a need to reconsider once again the concept of neutral technical change and to analyze how technical change is related to a neoclassical production function.

In addition to the Hicks' concept of neutral technical change, in the late 1940s Harrod had studied technical change which would enable a mature economy to achieve a long-run, stable, balanced growth path, one in which a constant income-capital ratio is maintained. In other words, technical change in which the shares of labor and capital are unchanged when the ratio of income to capital is constant. This came to be called Harrod neutral technical change.

Analysis was also made of a type of technical growth which is the antithesis or mirror image of this and which applied to the economies of developing countries, i.e. one in which there is absolutely no impact on the shares of capital and labor both before and after technical change when the ratio of income to labor is constant. This is Solow-Ranis-Fei neutral technical change.

15. In the empirical field of technical change, on the other hand, a multiplicative type of technical change, in which the efficiencies of capital and labor are different, is applicable to empirical data. This is the so-called factor augmenting technical change.

Although factor augmenting technical change was known to include Hicks neutral, Harrod neutral as well as Solow-Ranis-Fei neutral technical change, the theoretical grounds under which it maintained neutrality were completely unknown. To put it another way, it was not known under what conditions the distributive shares of capital and labor would remain constant, as in the three other types of neutrality.

The First answer to this question was the article that the author and Professor Beckmann published in the *Review of Economic Studies* (1968). The note that Professor Rose published a year later in the *Economic Journal* showed the same results as the Sato-Beckmann paper. This is the theory that was later called Sato-Beckmann-Rose technical change. It is also an explanation of the above mentioned factor augmenting technical change. Hicks, Harrod and Solow-Ranis-Fei neutral are all defined as special cases of Sato-Beckmann-Rose neutral. Hicks neutral is the case in which the efficiencies of capital and labor remain the same; Harrod neutral is one in which only the efficiency of labor goes up; and Solow-Ranis-Fei neutral is one in which only the efficiency of capital goes up.

16. What is the neutrality principle that justifies Sato-Beckmann-Rose? It is this: technical change takes the form of Sato-Beckmann-Rose factor augmenting technical change if it is neutral in the sense that it has no effect on the distributive shares of labor and capital, as long as the elasticity of factor substitution remains constant. Technical change of this type is most frequently used in empirical studies; its suitability for theoretical use is explained by the fact that it is derived from the neutrality principle of technical change.

Production functions that include Sato-Beckmann-Rose factor augmenting technical change, as well as estimates of technical change and an empirical analysis had first been carried out by the author in an article in the *International Economic Review* of 1963. The main point of this paper was to demonstrate that, by measuring the elasticity of factor substitution on using Kendrick data and then using that value, it was possible to measure the technical growth rates of both capital and labor in two equations. In contrast to the paper by Kendrick and Sato, it explained that there is a large gap between the rate of technical change for labor and the rate of technical change for capital; hence, the Hicks' technical change hypothesis did not apply to the US economy.

It also explained that the rate of technical change for labor is higher than that of capital, and that the latter is not zero as had been assumed by Harrod neutral technical change. It might be noted in passing that the rate of technical change for labor was 2.7 percent and the rate of technical change for capital was 0.7 percent (see Sato, *International Economic Review* 1963).

What is noteworthy here is that, except in cases where neutrality in one model is combined with a special production function (like the Cobb-Douglas type), it does not mean neutrality in the other models. In other words, if the model adopts Hicks neutral, then by definition, technical change of the Harrod, Solow-Ranis-Fei and Sato-Beckmann-Rose types are all not neutral; in short, they are biased technical change. The technical dealt with in this book will, for the most part, be of the Sato-Beckmann-Rose type.

17. An analysis of Sato-Beckmann-Rose technical change using Lie group transformations.

Sato-Beckmann-Rose neutral factor augmenting type of technical change under a neo-classical production function of constant returns to scale is expressed as follows:

$$Y(t) = F[A(t) K(t), B(t) L(t)]$$

$$\bar{Y}(t) = F[\bar{K}(t), \bar{L}(t)]$$

Capital and labor, which we here express in terms of the efficiency of technical change, are

$$\bar{K}(t) = A(t) K(t) \bar{L}(t) = B(t) L(t)$$

This can be seen as the transformation of $K(t) \rightarrow \bar{K}(t)$

$$L(t) \rightarrow \bar{L}(t)$$

If $A(t) = A_0 e^{\alpha t}$, where $\alpha \geq 0$ and $B(t) = B_0 e^{\beta t}$, where $\beta \geq 0$, the transformation of $K(t)$ and $L(t)$ is a magnificent type of a Lie group (Sato [1981, 1999]). I will not go more deeply into this question here.

18. A short history of economic conservation Laws. Ramsey's article in the *Economic Journal* of 1928 was the first in the long history of economics to introduce a dynamic method, i.e. a calculus of variation. Ramsey examined the question of how much a country would need to save and invest in order to maximize welfare, which he measured in terms of a mature economy's long-run rate of consumption. This was the first attempt at an optimum savings theory, which neo-classical growth theories have often dealt with from the 1960's on.

Ramsey intentionally did not introduce the concept of the discount rate. This marks a clear distinction from the optimum savings theories of today. Consequently, he was able to use a concept similar to the one mentioned earlier, the Hamilton energy conservation law used in physics. In the law of energy conservation, at each point of a movement in time kinetic energy + potential energy = constant; similar to this, in Ramsey's case, at each point in time the sum of net welfare and the value of saving is always constant, i.e. net welfare + value of saving = constant. Moreover, this value is the highest that nation's economic system can achieve. Ramsey called this value "bliss." Total energy in

the energy conservation law and Ramsey's Bliss are the values of a maximized Hamilton function in a dynamic system. In short, Ramsey knew the conservation law that, assuming a discount rate of zero, a maximized Hamilton discount rate at each point in time is always constant. Although the concept itself was not used, for all practical purposes, this was the first use of a conservation law in economics.

19. Economists did not revisit Ramsey's conservation law until the 1970s. Samuelson's paper in 1970 was the first in that era to introduce an economic conservation law into modern theoretical economics. Like Ramsey, Samuelson analyzed a von Neumann optimum problem using the analogy of the total energy law. The conclusion derived from this was the discovery that an economic conservation law is at work: aggregate capital-output ratio = constant. This was demonstrated by Sato [1981, 1999] with a detailed derivation.

Although, unlike the present book, Samuelson's article did not make use of the Noether theorem using Lie groups as his principle methodology, it was an outstanding accomplishment for both its accuracy of intuition and its economic significance.

20. Ramsey's and Samuelson's achievements were not generally known at the time the author received grants from the National Science Foundation and the Guggenheim Foundation to engage in a study of Lie groups and economic conservation laws. That was in the mid 1970s.

The author's comprehensive survey of the field, *The Theory of Technical Change and Economic Invariance: Application of Lie Groups* (Academic Press, 1981; revised edition, Elgar, 1999), not only made a representation of technical change using Lie groups, it conducted a thorough, full-scale analysis of economic conservation laws using the Noether theorem. It showed that Ramsey's Bliss conservation law and Samuelson's capital-output ratio conservation law are derived as special cases of the Noether theorem. (The Noether theorem, simply put, is that if a dynamic system including integral calculus is constant under Lie transformation groups that have r -parameters, r conservation laws exist in it).

The Ramsey model and other neo-classical long-run growth models were analyzed in detail in Chapter 7 of the above mentioned book. Samuelson's conservation law was also thoroughly analyzed there, and it was discovered to be the only conservation law with a von Neumann model. In addition, using the Noether theorem in standard neo-classical growth theory led to the discovery that, when consumption or the utility of consumption is maximized from the present into the infinite future, the "income-wealth ratio is constant" (for further details see *Journal of Econometrics* 1985; also found in *The Selected Essay of Ryuzo Sato* [Elgar, 1999], vol. II, chapter 18). This income-wealth conservation law was also derived by M. Weitzman, Kemp and others. But Weitzman's law was obtained using Bellman's principle of optimality not the general Noether theorem. Weitzman never mentioned the possibility that conservation laws other than the income-wealth law might also exist. In addition, unlike Sato (1985), he gave no answer to the question of whether the income-wealth conservation law would be found in a more realistic model such as one in which the discount rate changes over time.

As a matter of fact, the question of whether the income-wealth ratio is constant when the discount rate changes with time was first raised by Samuelson.