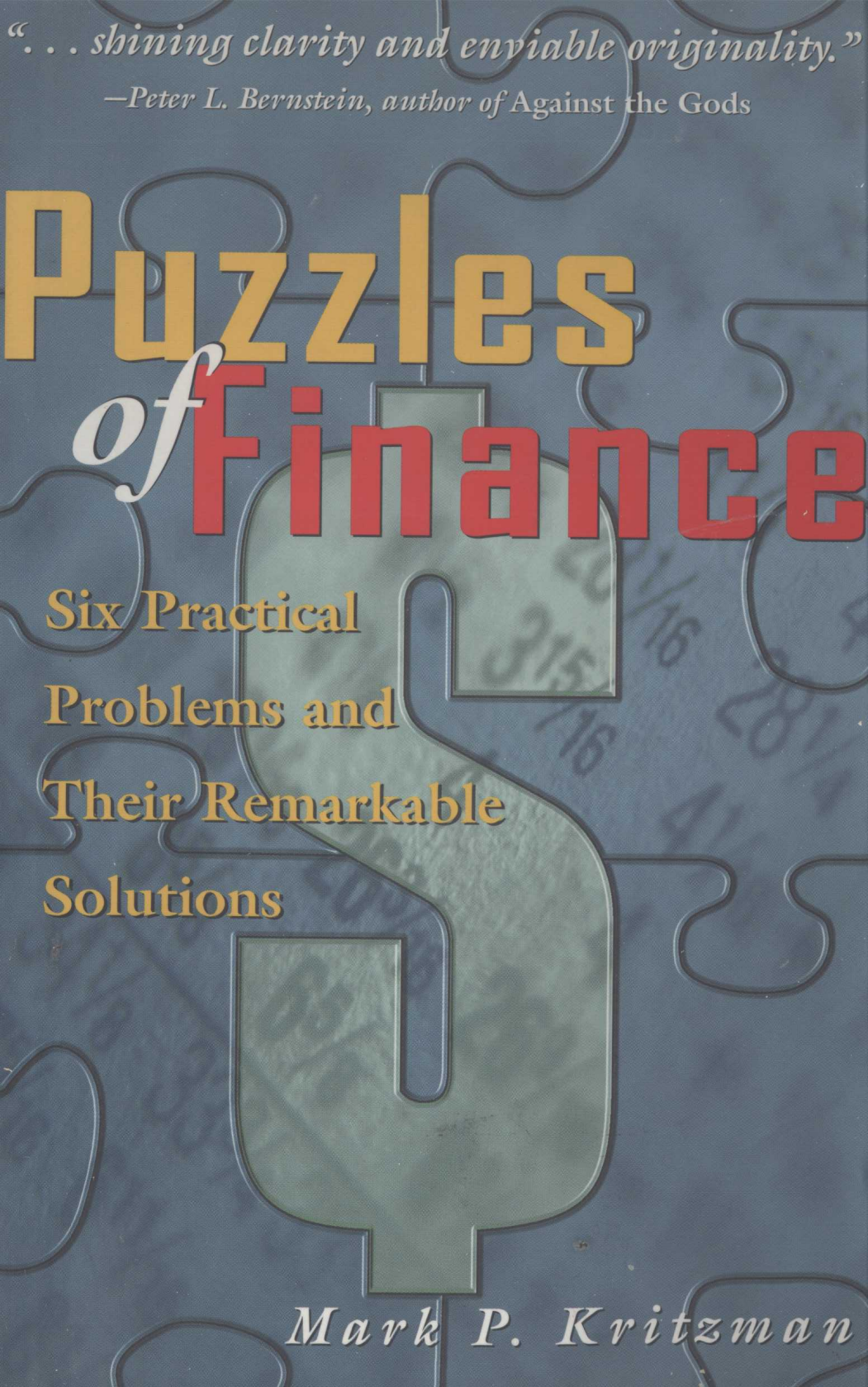
"... shining clarity and enviable originality."

—*Peter L. Bernstein, author of* Against the Gods

Puzzles of Finance

Six Practical Problems and Their Remarkable Solutions



Mark P. Kritzman

PUZZLES OF FINANCE

Six Practical Problems and Their Remarkable Solutions

MARK P. KRITZMAN

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FOREWORD

This book is an extraordinary combination of the elements of finance, common sense, wisdom, sparkling humor, shining clarity, and enviable originality. That is a potent blend by any standard of measurement. Longtime Kritzman-watchers, however, would anticipate nothing less.

I first came to know Mark Kritzman almost 20 years ago, in 1980, when he submitted an article to *The Journal of Portfolio Management,* of which I was then editor, under the title of "A Short-Term Approach to Asset Allocation." The paper provided a market-timing model that functioned within the constraints of investor utility and the longer term preferred asset mix. No one could read that article without sensing that here was a young man whose originality and analytical power would in time lead him to make a major contribution to the field. Indeed, this paper was only the first of a long series of articles by Kritzman that the *Journal* has had the honor to publish, covering a wide range of topics such as issues in performance measurement, currency management, and portfolio insurance. These, and Kritzman's other published materials, reveal the extent of his understanding of finance and his ability to translate difficult ideas into comprehensible language.

This new book is a further exploration into the many paths that Kritzman's inexhaustible intellectual curiosity has

led him over the years. He rejoices in puzzles: He appreciates the fun that puzzles provide and leaps at the opportunity to use them for profound theoretical or practical generalization. He enjoys the challenge of finding complexity in what appears to be utmost simplicity and revealing simplicity in what appears to be hopelessly complex. He approaches the solutions to these puzzles like an explorer who penetrates into a deep forest with the confident knowledge that somewhere in the shadows he will find the light to lead him out at the other side.

The most important feature of Kritzman's chosen puzzles is their relevance. He is not just an intellectual acrobat who delights in conundrums. He spends the largest portion of his daily life on the firing line, managing money in some of the most sophisticated environments of the world of finance. After you have completed all the mental handsprings that Kritzman asks you to perform, you will find that the effort pays off in hard, practical insights that you can apply in your own daily responsibilities.

Kritzman has reserved one of the great treats of the book for the two final sections: his enlightening review of the basic concepts of finance and his invaluable glossary. These two sections alone are worth the price of admission, even though they appear at the end. Many readers, I suspect, would benefit from reading the book backward instead of in the conventional manner! I suggest that you begin with at least a look at the Primer, to make certain that you will be up to speed in working your way through the six chapters that precede it.

I cannot close this Foreword without adding that my long professional association with Mark Kritzman has developed into a rich personal friendship, embellished by a

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shared love for the state of Vermont, where we are neighbors during part of the year. I have leaned on him mercilessly when my own understanding of some matter was cloudy. It is gratifying to note that the generosity and steadfast integrity I have come to know so well shine through in the pages of this fine book.

PETER L. BERNSTEIN

PREFACE

This book presents six puzzles of finance and their solutions. These puzzles are interesting for two reasons. First, they address important financial concepts that continue to confound academics and investment professionals alike, and second, the intellectual paths that lead to their solutions are entertaining.

These puzzles are unlike some of the better-known finance puzzles, such as the dividend puzzle and the equity risk premium puzzle. The dividend puzzle questions why most companies pay dividends even though there is no apparent economic motivation to do so. The equity risk premium puzzle deals with the fact that the historical spread between stock returns and the return on a riskless asset is too large to accord with commonly accepted views about risk aversion. These two puzzles and others like them are technically unsolvable because they deal with human behavior. The best we can hope for are plausible conjectures supported by persuasive evidence. The puzzles presented in this book, by comparison, deal with logical and mathematical subtleties; hence the solutions I present are incontrovertible . . . although I'm not always right.

Chapter 1 presents the first puzzle, which is called *Siegel's Paradox.* Jeremy Siegel, professor of finance at Wharton, drew attention to the mathematical fact that the expected value of the reciprocal of an exchange rate is greater than

the reciprocal of the expected value of the exchange rate. This relationship implies that a particular percentage increase in a given currency corresponds to a smaller percentage decrease in the currency on the other side of the exchange rate. Does this mathematical truism have any economic relevance?

The next notion, *Likelihood of Loss,* is puzzling because, given the same information *and agreement* about a particular investment's expected return and risk and identical views about the process that generates its returns, the likelihood that this investment will produce a loss varies from hardly at all to very likely. The critical determinant is how we frame the question, a fact that is significantly under appreciated by most investment professionals and even world-class economists engaged in restructuring our social security system.

The puzzle *Time Diversification* is related to *Likelihood of Loss* but addresses a broader and more fundamental issue. How does time influence an investment's expected utility? The concept of utility, by itself, is intriguing and rich in intellectual history. The discovery of utility by the prominent mathematician, Daniel Bernoulli, solved the famous St. Petersburg Paradox, which was proposed by Daniel's cousin, Nicholas Bernoulli. *Time Diversification* embraces Bernoulli's description of utility and also has an amusing history. Paul A. Samuelson, America's first Nobel Prize winner in economics and arguably its greatest economist, challenged one of his MIT colleagues to a wager. His colleague's response inspired Samuelson to present the fallacy of large numbers, which provides the elegant solution to the *Time Diversification* puzzle.

Next is one of the more confounding and important puzzles of finance, *Why the Expected Return Is Not to Be Expected.*

Although the solution to this puzzle, once it is revealed, is astoundingly clear, intelligent and well-trained investment professionals continue to challenge its validity with annoying persistence.

The next puzzle, at first glance, seems rather straightforward, but on closer scrutiny, reveals surprising layers of complexity: *All Stocks Half the Time or Half Stocks All the Time?* This question appeared on the late Fischer Black's list of 50 questions that he would use to motivate discussions in his classes at MIT. More explicitly, Fischer posed the question: Are you better off by investing 100% of your savings in stocks 50% of the time and in a riskless asset the other 50% of the time, or by investing 50% of your savings in stocks and 50% in a riskless asset 100% of the time? It has been rumored that students would repeat Fischer's class even though his questions never changed. They returned because his answers changed. Paul A. Samuelson has since provided the dispositive answer to this intriguing puzzle.

The final and perhaps most profound puzzle is *The Irrelevance of Expected Return on Options.* The solution to this puzzle resulted in the famous Black-Scholes options pricing formula, which earned a Nobel Prize for Robert Merton and Myron Scholes. Sadly, Fischer Black passed away before the Nobel Committee officially recognized their achievement. This puzzle refers to the fact that the fair value of an option is invariant to the expected return of the underlying asset. The solution to this puzzle is remarkable for several reasons. Some of the greatest minds in science have contributed along the way to its ultimate solution, including no fewer than six Nobel laureates. Moreover, critical components of the solution are derived from seemingly unrelated fields, such as botany and thermodynamics.

I also include a primer on basic financial concepts and quantitative methods. Much of this material will be familiar, but no doubt, some of these topics had a soporific effect on you during your school days; thus, you may wish to refresh your memory before tackling the puzzles of finance. If you choose to ignore this material, however, it should not matter much. Each puzzle is presented in a self-contained manner with references to the relevant sections of the book.

Finally, I include a glossary so that you can quickly review the definitions of technical terms whose meanings may not be obvious from the context in which they are used.

MARK P. KRITZMAN

Cambridge, Massachusetts
March 2000

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M. P. K.

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CHAPTER 1

Siegel's Paradox

When the euro was introduced to the world on January 1, 1999, it was valued at 1.1800 dollars. By June 30th, it had depreciated 12.60% against the dollar to 1.0313. During the same six-month span, the dollar appreciated 14.42% against the euro from 0.8475 euros to 0.9697. Thus, the combined change in value of these two currencies was positive 1.82%. Is this number economically relevant or merely an artifact of currency accounting?

Exchange rates are annoying, not so much because they introduce uncertainty but because they are confusing. For example, each day the *New York Times* reports the change in the value of the euro and the yen on the front page of its business section. If the *Times* reports an increase in the euro, they mean that the euro appreciated, and it will cost Americans more for a pilgrimage to Lourdes. However, if the *Times* reports an increase in the yen, they really mean that the yen lost value relative to the dollar and Americans can afford more cameras. The reason for this annoying contradiction is that currency traders are a strange lot. They quote

the value of the euro as the number of dollars per euro, while they quote the value of the yen as the number of yen per dollar.

This little quirk in the way currency traders communicate is trivial, however, when compared to the confusion introduced by exchange rate arithmetic. For example, if the dollar were to rise 25% against the euro, I could buy 25% more French wine. Of course, my friend Edouard in France wouldn't be so happy, because he prefers wine from Napa Valley. But he would not be disadvantaged by the exchange rate change to the same extent that I benefit from it. Given a 25% increase in the dollar, the euro only decreases 20% against the dollar (1/1.25 = 0.80). Can it be true that Edouard and I are collectively better off?

This asymmetry in exchange rate changes is a feature of *Siegel's paradox*. Although most investors find Siegel's paradox bothersome because it complicates financial analysis, I take the view that we should be grateful for the opportunity it affords us to consume more wine. But before I explain how we can consume more wine, let me describe Siegel's paradox a little more precisely.

Siegel's paradox refers to the mathematical fact that the expectation of the reciprocal of an exchange rate is greater than the reciprocal of the expectation of the exchange rate. If we let E represent the expectation and S represent the exchange rate, then we can write Siegel's paradox as:

$$E\left(\frac{1}{S}\right) > \frac{1}{\left[E(S)\right]} \tag{1.1}$$

Suppose, for example, that the euro is currently valued at 1.0600 dollars and we expect it will either increase 25% to

Table 1.1 Siegel's paradox.

	Dollars per Euro	Euros per Dollar
Current exchange rate	1.0600	0.9434
Euro appreciates 25%	1.3250	0.7547
Euro depreciates 25%	0.7950	1.2579
Expected value	1.0600	
Expected value of reciprocal		***1.0063***
Reciprocal of expected value	***0.9434***	

1.3250 dollars or depreciate 25% to 0.7950 dollars with equal probability. The expectation of the reciprocal of these potential outcomes is 1.0063 euros. If the euro rises 25%, the dollar falls to 0.7547 euros (1/1.3250), and if the euro falls 25%, the dollar rises to 1.2579 euros (1/0.7950). The expectation of these two outcomes equals 0.7547 times 50% plus 1.2579 times 50%, which is 1.0063.

The reciprocal of the expectation of the euro, however, equals only 0.9434. Because the euro has an equal chance of increasing to 1.3250 or decreasing to 0.7950, its expectation is 1.0600 and the reciprocal of this value is 0.9434. This relationship is presented in Table 1.1.

This feature of reciprocal relationships is known as Jensen's inequality.[1] Jeremy Siegel showed that as a result of Jensen's inequality, the currency forward rate cannot be an unbiased estimate of the future spot rate because an expected increase in one exchange rate implies an expected

[1] Jensen's inequality is discussed by C. Radhakrishna Rao in *Linear Statistical Inferences and Its Applications* (New York: John Wiley & Sons, 1965), p. 46.

decrease of smaller magnitude in its reciprocal.[2] Therefore, even if expected changes in the spot rate are distributed symmetrically around the forward rate from the perspective of one investor, Siegel's paradox guarantees that the forward rate will be biased from the perspective of the investor on the other side of the exchange rate. To understand why the forward rate must be biased, let's review covered and uncovered interest arbitrage.

COVERED INTEREST ARBITRAGE

Covered interest arbitrage describes the economic forces that relate the difference between the domestic and foreign interest rate to the difference between the spot exchange rate and the forward exchange rate. The spot rate is the rate at which currencies are exchanged at the present time as opposed to a future date. By contrast, the forward rate is a previously agreed on rate at which currencies are exchanged on a future date. Covered interest arbitrage ensures that a currency's forward rate will equal its spot rate multiplied by the ratio of one plus the domestic interest rate to one plus the foreign interest rate, as shown:

$$F = S \times \frac{(1 + R_d)}{(1 + R_f)}$$

[2]Jeremy J. Siegel, "Risk, Interest Rates, and the Forward Exchange," *Quarterly Journal of Economics* (February 1975), pp. 303–309.

where F = Forward exchange rate (domestic units per foreign unit)
S = Spot exchange rate (domestic units per foreign unit)
R_d = Domestic interest rate
R_f = Foreign interest rate

In liquid markets, this relationship must hold; otherwise arbitrageurs would earn riskless profits. Consider the following example: Suppose one-year interest rates in the United States are 5%, while one-year rates in the United Kingdom are 8%. Further, suppose that one pound can be exchanged in the spot market for 1.6000 dollars. It must follow that the one-year forward rate to exchange pounds for dollars equals 1.5556 ($1.6000 \times 1.05/1.08$). To see why this relationship must hold, imagine that a dealer agrees to exchange pounds for dollars one year hence at a rate of 1.5700 instead of 1.5556. An arbitrageur could borrow $1,600,000 in the United States at 5%, convert the dollars to 1,000,000 pounds, lend the pounds at 8%, and sell a forward contract to hedge the loan. The forward contract must be sufficient to cover the principal amount of the loan as well as the interest it generates; thus the arbitrageur would sell pounds forward in an amount equal to $1,695,600 ($1,000,000 \times 1.08 \times 1.5700$). These transactions would produce a riskless profit of $15,600 regardless of the level of the spot rate one year from now. Table 1.2 illustrates this point by assuming the spot rate either increases 10% to 1.7600 or falls 10% to 1.4400.

The gain or loss on the principal amount of the loan is calculated by subtracting the initial dollar value of the loan ($1.6000 \times 1,000,000$) from the dollar value of the loan when it matures ($1.7600 \times 1,000,000$ or $1.4400 \times 1,000,000$). The

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