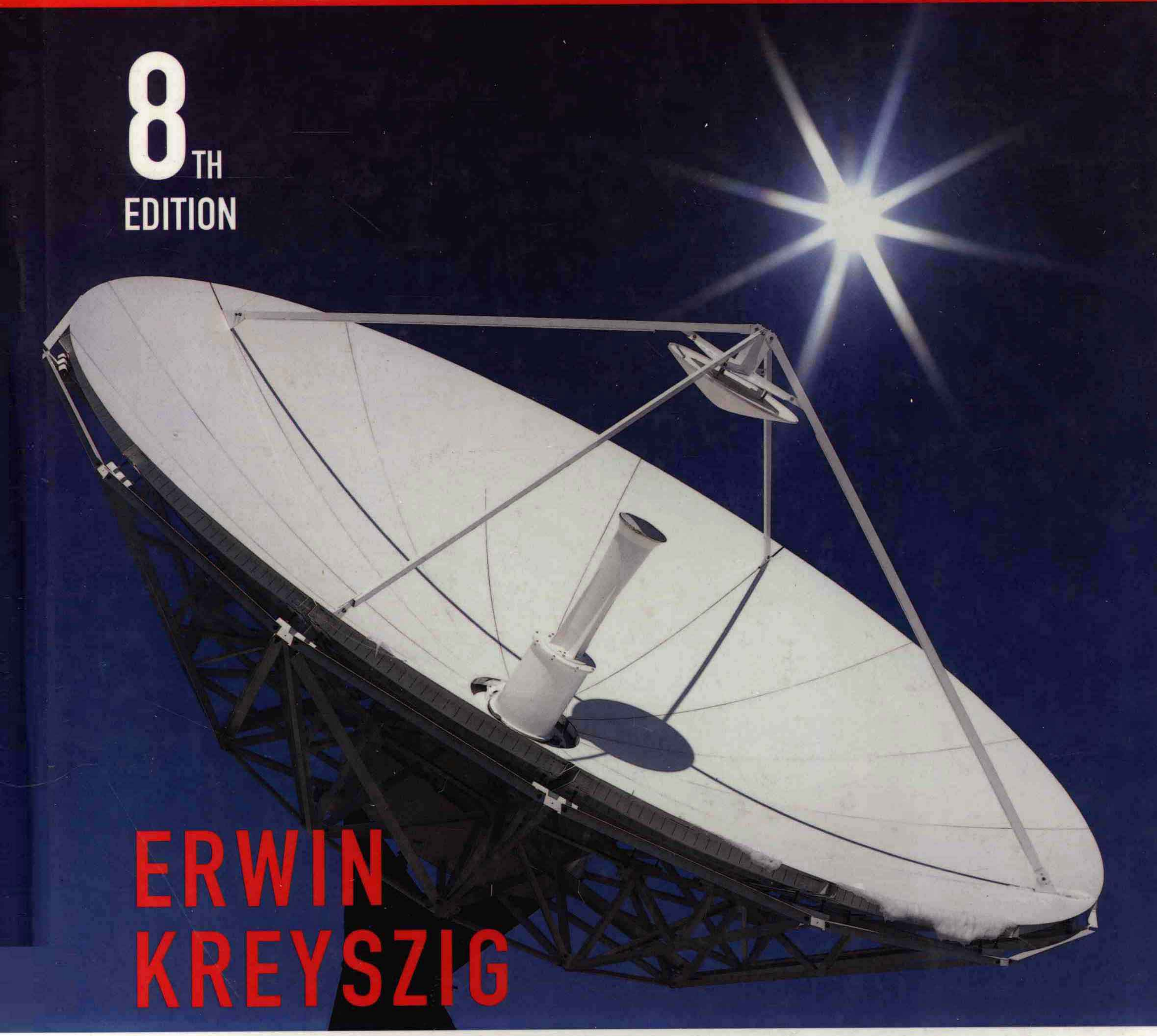
8TH EDITION



ERWIN KREYSZIG

ADVANCED ENGINEERING MATHEMATICS

EIGHTH EDITION

Advanced Engineering Mathematics

ERWIN KREYSZIG
Professor of Mathematics
Ohio State University
Columbus, Ohio

JOHN WILEY & SONS, INC.
New York Chichester Brisbane Toronto Singapore

Publisher: Peter Janzow
Mathematics Editor: Barbara Holland
Marketing Manager: Audra Silveric
Freelance Production Manager: Jeanine Furino
Lorraine Burke HRS Electronic Text Management
Designer: HRS Electronic Text Management
Illustration Editor: Sigmund Malinowski
Electronic Illustrations: Precision Graphics
Cover Photo: Chris Rogers/The Stock Market

This book was set in Times Roman by GGS Information Services
and printed and bound by Von Hoffmann Press.
The cover was printed by Phoenix Color Corp.

This book is printed on acid-free paper. ♾™

Kreyszig, Erwin.
Advanced engineering mathematics / Erwin Kreyszig.—8th ed.
p. cm.
Accompanied by instructor's manual.
Includes bibliographical references and index.
ISBN 0-471-15496-2 (cloth : acid-free paper)
1. Mathematical physics. 2. Engineering mathematics. 1. Title.

Printed in the United States of America
10 9 8 7 6 5 4 3

Advanced Engineering Mathematics

PREFACE

See also http://www.wiley.com/college/mat/kreyszig154962/

Purpose of the Book

This book introduces students of engineering, physics, mathematics, and computer science to those areas of mathematics which, from a modern point of view, are most important in connection with practical problems.

The content and character of mathematics needed in applications are changing rapidly. Linear algebra—especially matrices—and numerical methods for computers are of increasing importance. Statistics and graph theory play more prominent roles. Real analysis (ordinary and partial differential equations) and complex analysis remain indispensable. The material in this book is arranged accordingly, in seven independent parts (see also the diagram on the next page):

A Ordinary Differential Equations (Chaps. 1–5)
B Linear Algebra, Vector Calculus (Chaps. 6–9)
C Fourier Analysis and Partial Differential Equations (Chaps. 10, 11)
D Complex Analysis (Chaps. 12–16)
E Numerical Methods (Chaps. 17–19)
F Optimization, Graphs (Chaps. 20, 21)
G Probability and Statistics (Chaps. 22, 23)

This is followed by

References (Appendix 1)
Answers to Odd-Numbered Problems (Appendix 2)
Auxiliary Material (Appendix 3 and inside of covers)
Additional Proofs (Appendix 4)
Tables of Functions (Appendix 5).

This book has helped to pave the way for the present development and will prepare students for the present situation and the future by a modern approach to the areas listed above and the ideas—some of them computer related—that are presently causing basic changes: Many methods have become obsolete. New ideas are emphasized, for instance, stability, error estimation, and structural problems of algorithms, to mention just a few. Trends are driven by supply and demand: supply of powerful new mathematical and computational methods and of enormous computer capacities, demand to solve problems of growing complexity and size, arising from more and more sophisticated systems or production processes, from extreme physical conditions (for example, those in space travel), from materials with unusual properties (plastics, alloys, superconductors, etc.), or from entirely new tasks in computer vision, robotics, and other new fields.

The general trend seems clear. Details are more difficult to predict. Accordingly, students need solid knowledge of basic principles, methods, and results, and a clear perception of what engineering mathematics is all about, in all three phases of solving problems:

- **Modeling:** Translating given physical or other information and data into mathematical form, into a mathematical *model* (a differential equation, a system of equations, or some other expression).

Parts of the Book and Corresponding Chapters

Part A

Chaps. 1–5

Ordinary differential equations

Chaps. 1–3
Basic material

Chap. 4
Series solutions, Special functions

Chap. 5
Laplace transforms

Part B

Chaps. 6–9

Linear algebra, Vector calculus

Chaps. 6, 7
Vectors, Matrices, Eigenvalues

Chap. 8
Vector differential calculus

Chap. 9
Vector integral calculus

Part C

Chaps. 10, 11

Fourier analysis, Partial differential equations

Chap. 10
Fourier analysis

Chap. 11
Partial differential equations

Part D

Chaps. 12–16

Complex analysis

Chap. 12–15
Basic material

Chap. 16
Potential theory

Part E

Chaps. 17–19

Numerical methods

Chap. 17
General numerical methods

Chap. 18
Methods for linear algebra

Chap. 19
Methods for differential equations

Part F

Chaps. 20, 21

Optimization, Graphs

Chap. 20
Linear programming

Chap. 21
Graphs, Combinatorial optimization

Part G

Chaps. 22, 23

Probability, Statistics

Chap. 22
Probability theory

Chap. 23
Mathematical statistics

Manuals

MAPLE Computer Manual
MATHEMATICA Computer Manual

STUDENT SOLUTIONS MANUAL (New!)

- **Solving:** Obtaining the solution by selecting and applying suitable mathematical methods, and in most cases doing numerical work on a computer. This is the main task of this book.
- **Interpreting:** Understanding the meaning and the implications of the mathematical solution for the original problem in terms of physics—or wherever the problem comes from.

It would make no sense to overload students with all kinds of little things that might be of occasional use. Instead, it is important that students become familiar with ways to think mathematically, recognize the need for applying mathematical methods to engineering problems, realize that mathematics is a systematic science built on relatively few basic concepts and involving powerful unifying principles, and get a firm grasp for the interrelation between theory, computing, and experiment.

The rapid ongoing developments just sketched have led to many changes and new features in the present edition of this book.

In particular, many sections have been rewritten in a more detailed and leisurely fashion to make it a simpler book.

This has also led to a still better balance between applications, algorithmic ideas, worked-out examples, and theory.

Big Changes in This Edition

1 PROBLEM SETS CHANGED

The new problems place more emphasis on qualitative methods and applications. There is a (slight) reduction of formal manipulations in favor of problems that require mathematical thinking and understanding, as opposed to a routine use of a CAS (Computer Algebraic System).

2 PROJECTS

Modern engineering work is team work, and TEAM PROJECTS will help the student to prepare for this. (These are relatively simple, so that they will fit into the time schedule of a busy student.) WRITING PROJECTS will help in learning how to plan, develop, and write coherent reports. CAS PROJECTS and CAS PROBLEMS will invite the student to an increased use of computers (and programmable calculators); these projects are not mandatory, simply because ***this book can be used independently of computers or in connection with them*** (see page x).

3 NUMERICAL ANALYSIS UPDATED

Details are given below.

Further Changes and New Features in Chapters

Ordinary Differential Equations (Chaps. 1–5)

- **First-Order Differential Equations** (Chap. 1). Qualitative aspects emphasized by discussing direction fields early (Sec. 1.2). Presentation streamlined by combining exact equations and integrating factors into one section (Sec. 1.5) and moving Picard's iteration to Sec. 1.9 on existence and uniqueness.

- **Linear Second and Higher Order Differential Equations** combined into one chapter (Chap. 2), to save some time by avoiding duplications.
- **Systems of Differential Equations** (Chap. 3). Lotka–Volterra predator–prey population model included.
- **Frobenius Method** (Chap. 4). Material on Bessel functions slightly reduced.
- **Laplace Transforms** (Chap. 5). New section on systems of differential equations included.

Linear Algebra, Vector Calculus (Chaps. 6–9)

- **Matrices, Linear Systems** (Chap. 6). Slightly more rapid start by combining the first two sections from the last edition. Applications appearing earlier. Cramer's rule absorbed into the (slightly condensed) section on determinants (Sec. 6.6). Inverse treated more compactly (Sec. 6.7).
- **Eigenvalue Problems** placed in a separate chapter (Chap. 7), to have the basic material on vectors and matrices in a chapter of its own.

Fourier Analysis and Partial Differential Equations (Chaps. 10, 11)

- **Fourier Series** (Chap. 10) streamlined by moving half-range expansions into the section on even and odd functions (Sec. 10.4).
- **Partial Differential Equations** (Chap. 11). Fourier transform method absorbed into the section on Fourier integrals for heat problems (Sec. 11.6).

Complex Analysis (Chaps. 12–16)

- **Complex Numbers and Functions** (Chap. 12). The old chapter on conformal mapping no longer exists. Its (slightly reduced) material has been distributed in Sec. 12.5 on conformality, Secs. 12.6–12.8 on special functions, and Sec. 12.9 on linear fractional transformations. Hence for a better understanding we now discuss geometric properties of functions simultaneously with their analytic formulas, as we do all the time in calculus.
- **Complex Integration** (Chap. 13). Integration methods right after the definition of the integral.
- **Laurent Series,** formerly in a chapter jointly with power series, combined with residue integration in Chap. 15.

Numerical Methods (Chaps. 17–19)

- **Numerical Methods in General** (Chap. 17). Updated in the light of computer requirements and developments. Idea of error estimation by halving. Changes in Sec. 17.4 on splines, in Sec. 17.5 on error estimates in integration. Adaptive integration and Romberg integration included (Sec. 17.5).
- **Methods for Differential Equations** (Chap. 19). Automatic variable step size selection in modern codes, Runge–Kutta–Fehlberg method (Sec. 19.1), extension of Euler and Runge–Kutta methods to systems and higher order equations (Sec. 19.3) included.

Optimization, Graphs (Chaps. 20, 21)

- **Linear Programming** (Chap. 20). Simplex method completely rewritten in terms of matrix language and techniques.

Probability and Statistics (Chaps. 22, 23)

- **Probability** (Chap. 22) beginning with a section on data analysis, explaining stem-and-leaf plots and boxplots and motivating probability by relative frequency (Sec. 22.1).
- **Statistics** (Chap. 23) beginning with a section on the use of random number generators (Sec. 23.1). Introduction to correlation added (Sec. 23.10).

Appendices

- **Appendix 1** (References) updated.

Suggestions for Courses: A Four-Semester Sequence

The material may be taken in sequence and is suitable for four consecutive semester courses, meeting 3–5 hours a week:

First semester.	Ordinary differential equations (Chaps. 1–4 or 5)
Second semester.	Linear algebra and vector analysis (Chaps. 6–9)
Third semester.	Complex analysis (Chaps. 12–16)
Fourth semester.	Numerical methods (Chaps. 17–19)

For the remaining chapters, see below. Possible interchanges are obvious; for instance, numerical methods could precede complex analysis, etc.

Suggestions for Courses: Independent One-Semester Courses

The book is also suitable for various independent one-semester courses meeting 3 hours a week; for example:

Introduction to ordinary differential equations (Chaps. 1–2)
Laplace transform (Chap. 5)
Vector algebra and calculus (Chaps. 8, 9)
Matrices and linear systems of equations (Chaps. 6, 7)
Fourier series and partial differential equations (Chaps. 10, 11, Secs. 19.4–19.7)
Introduction to complex analysis (Chaps. 12–15)
Numerical analysis (Chaps. 17, 19)
Numerical linear algebra (Chap. 18)
Optimization (Chaps. 20, 21)
Graphs and combinatorial optimization (Chap. 21)
Probability and statistics (Chaps. 22, 23)

General Features of This Edition

The selection, arrangement, and presentation of the material has been made with greatest care, based on past and present teaching, research, and consulting experience. Some major features of the book are these:

The book is **self-contained,** except for a few clearly marked places where a proof would be beyond the level of a book of the present type and a reference is given instead. Hiding difficulties or oversimplifying would be of no real help to students.

The presentation is **detailed,** to avoid irritating readers by frequent references to details in other books.

The examples are **simple,** to make the book teachable—why choose complicated examples when simple ones are as instructive or even better?

CONTENTS

PART A

Ordinary Differential Equations

Differential equations are of fundamental importance in engineering mathematics because many physical laws and relations appear mathematically in the form of such equations. In Part A, which consists of five chapters, we shall consider various physical and geometrical problems that lead to differential equations, and we shall explain the most important standard methods for solving such equations.

Modeling. We shall pay particular attention to the derivation of differential equations from given physical (or other) situations. This transition from the given physical problem to a corresponding "mathematical model" is called *modeling*. This is of great practical importance to the engineer, physicist, and computer scientist, and we shall illustrate it using typical examples.

Computers. Differential equations are very well suited for computers. Corresponding **NUMERICAL METHODS** for solving differential equations are explained in Secs. 19.1–19.3. These sections are independent of other sections on numerical methods, so that they can be studied directly after Chaps. 1 and 2, respectively.

Evaluating Results. We must make sure that we understand what a mathematical result means in physical or other terms in a given problem. If we obtained the result using a computer, we must check the result for reliability—the computer can sometimes give us nonsense. This applies to all the work with computers.

CHAPTER 1

First-Order Differential Equations

In this chapter we begin our program of studying ordinary differential equations and their applications. This includes the derivation of differential equations from physical or other problems **(modeling)**, the solution of these equations by methods of practical importance, and the interpretation of the results and their graphs in terms of a given problem. We also discuss the questions of existence and uniqueness of solutions.

We start with the simplest equations. These are called **differential equations of the first order** because they involve only the ***first*** derivative of the unknown function. Our usual notation for the unknown function will be $y(x)$ or $y(t)$.

Numerical methods for these equations follow in Secs. 19.1 and 19.2, which are totally independent of other sections in Chaps. 17–19, and can be taken up immediately after this chapter.

Prerequisite for this chapter: integral calculus.
Sections that may be omitted in a shorter course: 1.7–1.9.
References: Appendix 1, Part A.
Answers to Problems: Appendix 2.

1.1 Basic Concepts and Ideas

An **ordinary differential equation** is an equation that contains one or several derivatives of an unknown function, which we call $y(x)$ and which we want to determine from the equation. The equation may also contain y itself as well as given functions and constants. For example,

$$y' = \cos x, \tag{1}$$

$$y'' + 4y = 0, \tag{2}$$

$$x^2 y''' y' + 2e^x y'' = (x^2 + 2)y^2 \tag{3}$$

are ordinary differential equations. The word ***"ordinary"*** distinguishes them from ***partial*** differential equations, involving an unknown function of two or more variables and its *partial* derivatives; these equations are more complicated and will be considered later (in Chap. 11).

Differential equations arise in many engineering and other applications as mathematical models of various physical and other systems. The simplest of them can be solved by remembering elementary calculus.