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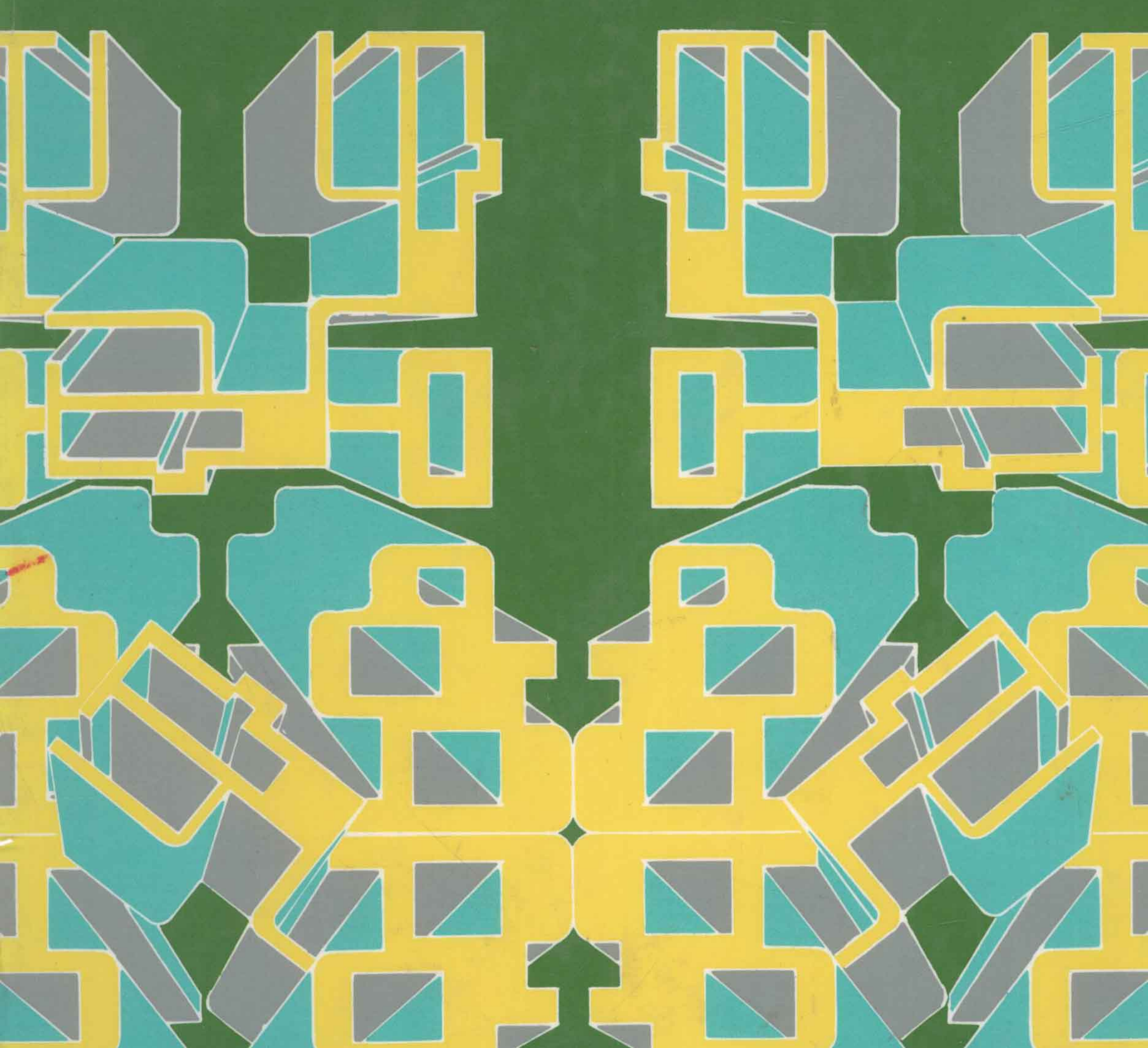
GROUP THEORY AND G-VECTOR SPACES IN STRUCTURAL ANALYSIS

vibration, stability and statics

George M. Zloković



GROUP THEORY AND *G*-VECTOR SPACES IN ENGINEERING STRUCTURES
Vibration, Stability and Statics

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GROUP THEORY AND *G*-VECTOR SPACES IN ENGINEERING STRUCTURES

Vibration, Stability and Statics

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Preface

The object of this work was to explore and formulate the theoretical basis and algorithmic procedures of group theory and G-vector spaces in the theory of structures. Besides substantial new material, this present volume incorporates my book *Group theory and G-vector spaces in vibration, stability and statics of structures* which was published in English and in Serbo-Croatian (ICS, Belgrade, 1973) and in Russian (Stroizdat, Moscow, 1977). Developments of space patterns by group-theoretical methods, which I suggested in my book *Space structures* (Gradjevinska knjiga, Beograd, 1969), induced me to devote attention to unexplored possibilities of innovations of mathematical descriptions in the theory of structures. The encouragement to write these books came mostly from my study of inspiring applications of group theory to physical problems in quantum mechanics and mathematical crystallography.

Group theory is a highly developed and abstract subject that can be applied in various circumstances, many of these concerning symmetry. The prime importance of applications of group theory in symmetry lies in the way in which the symmetry governs and simplifies the analysis of quantitative properties of the system. The quantitative side is inherent in group representation theory since symmetry groups are represented by groups of matrices acting as linear operators in a vector space.

As the basis for group-theoretical formulations in the theory of structures, this present volume contains results of abstract group theory, symmetry groups and vector spaces which are necessary for the development of group representations and G-vector spaces. The G-vector space is constructed as a symbiosis of groups, group algebra, linear operators and vector spaces. The properties of the G-vector space enable it to be decomposed into G-invariant subspaces with their independent systems of equations. Thus the matrix of the system of equations that describe the behaviour of the structure is obtained in block diagonal form, where no change of basis can reduce the dimensions of its submatrices. In this volume, theoretical bases and algorithmic procedures in G-vector spaces are formulated for elements with complex symmetry in general, and, specifically, in methods employed in the vibration, stability and statics of structures. This method, designed here for problems in the theory of structures, is called *G-vector analysis*.

In the general procedure of G-vector analysis of surface and space elements with complex symmetry, sophisticated sets of basis vectors of G-invariant subspaces are obtained as unique solutions with maximum performance. Analyses of the applications of various groups on an element disclose a dependence of the efficacy of the G-invariant analysis not only on the form of the element but also on the number of the nodes and their pattern. In comparison with conventional utilizations of the symmetry properties of the element, G-vector analysis provides a systematic procedure, simplification and facilitation of the analysis with a large reduction in the amount of computation.

In applications of G-vector analysis to eigenvalue problems of vibration the matrix of equations of the vibrating system appears in block diagonal form. The critical force is found in each G-invariant subspace separately by solving polynomial equations of degrees which are fractions of the degree of the polynomial in the conventional analysis.

In the method of dynamic analysis of cable networks, where the solution of differential equations is sought in the form of Maclaurin's series, G-vector analysis solves the problem of the vibration of a cable network by computing a small number of simple fractions, whereas conventional utilization of symmetry requires the solution of systems of equations to determine the coefficients of Maclaurin's series, the solution of systems of equations to determine the coefficients of a polynomial and a polynomial equation, and evaluation of the deflections and amplitudes by means of substantial expressions. Besides enormous quantitative reductions, G-vector analysis has here significant qualitative advantages by simplifying derivations of expressions and by facilitating the analysis as a whole.

In the stability of structures, where the critical force is determined from conditions for the minimum potential energy, G-vector analysis solves the eigenvalues problem for each G-invariant subspace separately. In this way, as in vibration, the critical force is found by solving polynomial equations of degrees which are fractions of the degree of the polynomial in the conventional solution.

In plate buckling, in the case of a square plate subjected to an in-plane load, conventional analysis by the finite-difference method requires the solution of a polynomial of fourth degree, while by G-vector analysis the same problem is solved by equations of first degree.

In the force method in the statics of structures the choice of the statically determined basic system with its redundant generalized forces is empirical and in complex cases it is unlikely that the best solution can be reached only by intuition. On the contrary, the procedure of G-vector analysis in the force method provides one unique solution in the choice of the statically determinate basic system with redundants, and the matrix of equations of deformation conditions appears in the most reduced diagonal block form. Symmetry-adapted redundants and symmetry-adapted loads are determined for each subspace, while redundants for arbitrary loads are obtained from linear combinations of symmetry-adapted redundants which were found for symmetry-adapted loads. The case of the pin-jointed space frame with pentagonal symmetry is a remarkable example of a very intricate and empirically undetectable set of basis vectors which provides maximum utilization of the symmetry properties of the structure.

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In G-vector analysis in the direct stiffness method in the statics of structures, many standard conventions are modified to correspond to the symmetry types of the group that describes the symmetry of the structure. Thus the numbering of the nodes of the structure, the positions of origins and positive directions of coordinate axes of the local and the global coordinate systems, and the sets of positive directions of displacements and rotations of the nodes are determined in an exact way by specific G-vector analysis procedure in the direct stiffness method. Then the derived symmetry-adapted system stiffness matrix can be put in block diagonal form where submatrices are the system stiffness matrices of G-invariant subspaces. In comparison with the standard direct stiffness method, G-vector analysis provides large reductions in the amount of computation and a systematic procedure with many advantages concerning input of data, checking and survey of the analysis.

It may be concluded that applications of group theory and G-vector spaces to structures give significant qualitative and quantitative results, especially for those with complex configurations.

In some modern methods in the theory of structures, new group-theoretical formulations, with innovations of mathematical descriptions, may provide interesting results. This I intend to present in my next book.

Finally, I should like to express my gratitude to the publisher Ellis Horwood.

George Zloković

Introduction

An important fact about a structure that can be represented by a mathematical model, which consists of an interconnected system of points, is that the positions of these points, relative to each other, define a framework which may be considered as a geometrical point system. In this way, one can describe structures as simple or complex patterns of points with precise figures for their distances.

The approximations in the vibration, stability and statics of structures allow the total motion of the structure to be decomposed into independent displacements, which makes linearity possible. Consideration of the symmetry properties of a framework by group theory and representation of vibrational, stability and static phenomena in G-vector spaces provide a most systematic way to describe the symmetry of the structure and enable the maximum utilization of its symmetry properties. In the analyses, besides important qualitative advantages, it is possible to secure significant quantitative results in the form of facilitations and substantial reductions in the amount of computations in comparison with conventional utilization of the symmetry properties of the structure.

Chapter 1 contains some basic results of abstract group theory and symmetry groups which are necessary for derivations of group representations and G-vector spaces in Chapter 3. In group theory, consideration of the symmetry properties of a space object is usually based on the geometrical framework formed by the points or nodes of the system. The object may contain identical nodes occupying physically identical positions in the nodal framework. A symmetry operation (rotation, reflection or rotary reflection) may rearrange the physically equivalent nodes and move the nodal framework to a new position which coincides exactly with the original position and is physically indistinguishable from it. A symmetry operation has no effect on any physical property of the structure or its state with respect to forces and displacements.

Chapter 2 deals with n-dimensional vector spaces, linear operators, basis transformations, unitary spaces, self-adjoint operators and quadratic forms to the extent which is necessary for derivations of group representations and G-vector spaces in Chapter 3. Also, some expressions concerning characteristic equations and quadratic forms will be used in Chapters 5 and 6.

Chapter 3 describes representation theory, containing group representations, group algebra, idempotents of the centre of group algebra, G-vector spaces, the eigenvalue problem for linear operators which commute with symmetry operations and a systematic tabular survey of properties of the vector (unitary) space, the function space, linear operators in a vector space, mappings by linear operators in a vector space, groups, group algebra and G-vector space. The G-vector space was derived on the basis of the following results:

(1) in an n-dimensional vector space with an established basis a group representation is formed by mapping group elements on a set of $n \times n$ matrices;
(2) there is always a basis with respect to which the matrices of the group representation are expressed as direct sums of submatrices that no change of basis can reduce to matrices of smaller dimensions; this is the **irreducible group representation**;
(3) the **group algebra** with the properties of a vector space is established by introducing linear combinations of group elements into groups;
(4) the **centre of group algebra** consists of all elements that commute with all elements of the group algebra;
(5) sums of group elements that belong to classes of conjugate elements form a basis of the group algebra;
(6) the idempotents of the centre of group algebra, as linear combinations of class sums, act as projection operators that generate subspaces of definite symmetry types;
(7) the G-vector space is formed by introducing group elements as linear operators in the n-dimensional space V.

In the case of symmetry groups an idempotent of the centre of group algebra, acting as a linear operator of a vector space, does not change the vectors of its symmetry type and nullifies all vectors of other symmetry types. In this way it is possible to generate G-invariant subspaces with definite symmetry types, while all matrices of equations that describe the behaviour of the structure appear in block diagonal form. The procedures in G-vector spaces which are developed in Chapters 4, 5, 6 and 7 in general and as various methods in the vibration, stability and static of structures, are called ***G*-vector analysis**.

In Chapter 4, G-vector analysis of surface or space elements with complex symmetry performs decompositions into G-invariant subspaces and solves the problems with large reductions in the number of mathematical operations in comparison with solutions by conventional utilizations of the symmetry properties of the structure. The efficancy and the extent of the decomposition depend not only on the form of the element but also on the number of nodes and their pattern. Comparative analyses of applications of various groups on a configuration with different patterns of nodes exhibit criteria for the choice or composition of node patterns. On partition of the continuum of a structure into surface or space elements, there is a preference for geometrical configurations with a small number of different types of nodal joint, which may be realized by patterns composed of suitable polygons or polyhedra. In comparison with conventional utilizations of one or two planes of symmetry, G-vector analysis makes use of eight symmetry operations for a square, 48 operations for a cube and 120 operations for a regular icosahedron. To derive the basis vectors of G-invariant subspaces of the vector space of an element, G-vector analysis is formulated as a procedure and as an algorithmic scheme for computer programs.

Chapter 5 deals with G-vector analysis in eigenvalue problems on the vibration of beams and taut strings and as a method of dynamic analysis of cable networks. The vibration of structures with n degrees of freedom, which are described by systems of homogeneous linear differential equations with constant coefficients, is solved by evaluation of eigenvalues and eigenvectors of an $n \times n$ matrix or an $n \times n$ matrix pair. When symmetry operations of a vibrating system form a group, G-vector analysis ensures that the matrix of equations of the vibrating system appears in block diagonal form as a direct sum of matrices of G-invariant subspaces of definite symmetry types. This results in substantial reductions in the amount of computation since, instead of a polynomial equation of degree n, it is necessary to solve polynomial equations of degrees which are a fraction of n. The specific G-vector analysis procedure for problems of vibration consists of determination of the symmetry group, idempotents of the centre of group algebra, the set of n functions that span the n-dimensional space, G-invariant subspaces U_i, symmetry-adapted group functions, the deflection matrix, eigenvalues and eigenvectors.

G-vector analysis was also applied as a method of dynamic analysis which considers a cable network as a pin-jointed elastic system of members, where the solution of differential equations is sought in the form of Maclaurin's series. Comparative solutions of the vibration of a cable network show drastic reductions in the amount of derivation and computations achieved by G-vector analysis compared with conventional utilization of the symmetry properties of the structure. In G-vector analysis, the complete calculation contains only a small number of simple fractions while, for solution by conventional application of the diagonal plane of symmetry, there are six systems with four equations to determinate recurrent 24 coefficients of Maclaurin's series, one system with three equations to determine the coefficients of the polynomial, a cubic equations to evalute the frequencies of vibration, and substantial expressions to evaluate static deflections and amplitudes of vibration.

Chapter 6, on the stability of structures, deals with G-vector analysis as a method for determining the critical forces of compressed systems. In the analyses the critical force is determined from the conditions for the minimum potential energy in the deflected state of the compressed system, which is expressed as a function of n ordinates of the elastic line. By partial differentiations of the equations of the potential energy, one obtains a system of homogeneous linear equations which appears in the form of an eigenvalue problem, where its solution yields the critical force of the compressed system. The matrix of this system of homogeneous equations can also be obtained by symmetrization of the matrix of quadratic form. When symmetry operations of the compressed system form a group, the n-dimensional space of the system is decomposed into G-invariant subspaces U_i, so that the conditions for the minimum potential energy can be stated for each subspace separately. The specific G-vector analysis procedure for stability problems consists of determination of the symmetry group, idempotents of the centre of group algebra, the bases of G-invariant subspaces U_i, symmetry-adapted deflection diagrams, matrices of conditions for the minimum potential energy of subspaces U_i and the eigenvalues.

G-vectors analysis was also applied in the finite-difference method to plate buckling. When a square plate is subjected to in-plane uniform loads, standard analysis of the eigenvalues problem requires solution of a polynomial equation of fourth degree. By means of G-vector analysis the same problem is solved by equations of first degree.

Chapter 7 deals with G-vector analysis in the force method and in the direct stiffness method in the statics of structures. In the force method the amount of computation may be minimized through judicious choice of the statically determinate basic system with its redundant generalized forces. Since this choice is quite empirical, it is possible to miss better solutions, especially in complex cases when satisfactory sets of redundants are too sophisticated to be found by conventional considerations. This is striking in the example of the pin-jointed space frame with its symmetry properties described by the group C_{5v} where, by G-vector analysis, the best possible set of redundants, expressed by the constant φ of the golden section in a highly intricate way, is determined in a unique way which is impossible using intuitive exploration. Application of G-vector analysis to the force method provides maximum utilization of the symmetry properties of the structure with the following advantages:

— the unique choice of the statically determinate basic system with its redundants is directed by an exact procedure;
— the system of equations of deformation conditions has the matrix in the most reduced block diagonal form, since its submatrices correspond to irreducible group representations;
— each subspace has its own independent system of equations with a matrix the order of which is a fraction of the order of the matrix of the system;
— redundants for an arbitrary load are obtained by linear combinations of symmetry-adapted redundants for respective symmetry-adapted loads; these are the only cases that require solution of the equations.

The G-vector analysis procedure in the force method consists of determination of the symmetry group, idempotents of the centre of group algebra, the statically determinate basic system, bases of G-invariant subspaces U_i, symmetry-adapted redundants, equations of deformation conditions and redundants for arbitrary loads. Applications of G-vector analysis are shown for continuous beams, girder grillage, a silo cell and pin-jointed space frames.

G-vector analysis in the direct stiffness method differs from the standard direct stiffness method in many aspects. In G-vector analysis the standard conventions concerning the numbering of the nodes, positions of origins and positive directions of coordinate axes of the local and the global coordinate systems, as well as the sets of positive directions of displacements and rotations of the nodes, must be modified to suit the symmetry types of the group that describes the symmetry properties of the structure. When these conditions are satisfied, it will be possible to formulate the symmetry-adapted system stiffness matrix and the system stiffness matrices of G-invariant subspaces of the problem, according to the G-vector analysis procedure

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