

HANDBOOKS IN ECONOMICS I

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HANDBOOK OF MATHEMATICAL ECONOMICS

VOLUME III

Editors :

Kenneth J. Arrow

Michael D. Intriligator

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NORTH-HOLLAND

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HANDBOOK OF MATHEMATICAL ECONOMICS

VOLUME III

Edited by

KENNETH J. ARROW

Stanford University

and

MICHAEL D. INTRILIGATOR

University of California, Los Angeles



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INTRODUCTION TO THE SERIES

The aim of the *Handbooks in Economics* series is to produce Handbooks for various branches of economics, each of which is a definitive source, reference, and teaching supplement for use by professional researchers and advanced graduate students. Each Handbook provides self-contained surveys of the current state of a branch of economics in the form of chapters prepared by leading specialists on various aspects of this branch of economics. These surveys summarize not only received results but also newer developments, from recent journal articles and discussion papers. Some original material is also included, but the main goal is to provide comprehensive and accessible surveys. The Handbooks are intended to provide not only useful reference volumes for professional collections but also possible supplementary readings for advanced courses for graduate students in economics.

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PREFACE TO THE HANDBOOK

The field of mathematical economics

Mathematical economics includes various applications of mathematical concepts and techniques to economics, particularly economic theory. This branch of economics traces its origins back to the early nineteenth century, as noted in the historical introduction, but it has developed extremely rapidly in recent decades and is continuing to do so. Many economists have discovered that the language and tools of mathematics are extremely productive in the further development of economic theory. Simultaneously, many mathematicians have discovered that mathematical economic theory provides an important and interesting area of application of their mathematical skills and that economics has given rise to some important new mathematical problems, such as game theory.

Purpose

The *Handbook of Mathematical Economics* aims to provide a definitive source, reference, and teaching supplement for the field of mathematical economics. It surveys, as of the late 1970's, the state of the art of mathematical economics. Bearing in mind that this field is constantly developing, the Editors believe that now is an opportune time to take stock, summarizing both received results and newer developments. Thus all authors were invited to review and to appraise the current status and recent developments in their presentations. In addition to its use as a reference, the Editors hope that this Handbook will assist researchers and students working in one branch of mathematical economics to become acquainted with other branches of this field. Each of the chapters can be read independently.

Organization

The Handbook includes 29 chapters on various topics in mathematical economics, arranged into five parts: *Part 1* treats *Mathematical Methods in Economics*, including reviews of the concepts and techniques that have been most useful for the mathematical development of economic theory. *Part 2* elaborates on *Mathematical Approaches to Microeconomic Theory*, including consumer, producer, oligopoly, and duality theory. *Part 3* treats *Mathematical Approaches to Competi-*

tive Equilibrium, including such aspects of competitive equilibrium as existence, stability, uncertainty, the computation of equilibrium prices, and the core of an economy. *Part 4* covers *Mathematical Approaches to Welfare Economics*, including social choice theory, optimal taxation, and optimal economic growth. *Part 5* treats *Mathematical Approaches to Economic Organization and Planning*, including organization design and decentralization.

Level

All of the topics presented are treated at an advanced level, suitable for use by economists and mathematicians working in the field or by advanced graduate students in both economics and mathematics.

Acknowledgements

Our principal acknowledgements are to the authors of chapters in the *Handbook of Mathematical Economics*, who not only prepared their own chapters but also provided advice on the organization and content of the Handbook and reviewed other chapters.

KENNETH J. ARROW

Stanford University

MICHAEL D. INTRILIGATOR

University of California, Los Angeles

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PART 4

**MATHEMATICAL APPROACHES TO
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