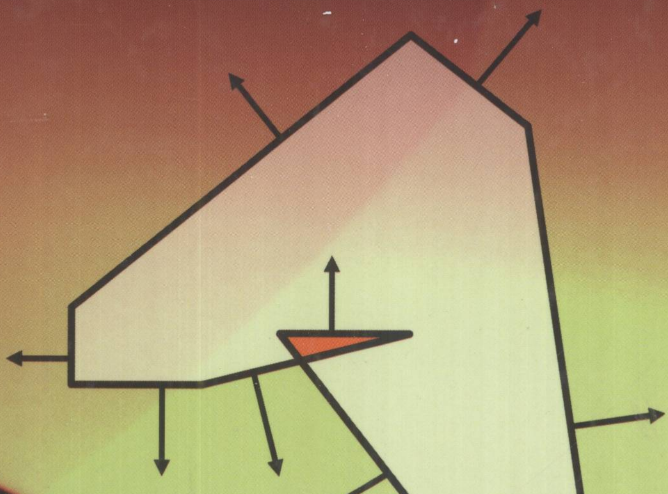
MATHEMATICS OF SHAPE DESCRIPTION

A Morphological Approach to Image Processing and Computer Graphics



Pijush K. Ghosh | Koichiro Deguchi

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A Morphological Approach to Image Processing and Computer Graphics

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MATHEMATICS OF SHAPE DESCRIPTION
A Morphological Approach to Image Processing and Computer Graphics

To
Gopa, Nairita,
and Kazuko

To the memory of Pijush K. Ghosh

The doors to knowledge were opened to me at an early age by my father. He taught me that the thirst for knowledge is unquenchable. He was like my very own magician, who made learning creative and fun. Today he is no longer by my side to guide me, but the fulfillment of his dream in the form of this book brings me immense joy. He is alive to me in the pages of this book, and this book is a ray of light in the darkness he has left behind in my life.

Daddy's Little Girl ...
Nairita Ghosh

Foreword

The computer description of shape and the computer manipulation of shape is complex simply because shape itself is complex. Of course, if the world of shape were limited to the Euclidean shapes, there would be no such complexity. However, shape includes all the varieties of biological shapes: from the shapes of trees and their leaves to fish, animals, flowers, and plants – and also natural shapes, such as those of coastlines, and of rocks and crystals.

Mathematical morphology is the mathematical study of shapes through a particular algebra of operations, known as the Minkowski set operations. Here, a shape can be thought of in the most general way possible, as a set of points in two or three dimensions. To fully understand the nature of the algebra of mathematical morphology requires: (1) an understanding of what an axiom system actually provides; (2) fluency in a variety of concepts associated with sets, including the set builder notation in mathematics; and (3) fluency in the concepts of algebraic structures. It is in this setting, formulated by Professor Deguchi, that the particulars of the concepts of mathematical morphology can most fully be appreciated.

Mathematics of Shape Description is the first book to devote half of its pages, in a tutorial fashion, to the basic background and/or essential preliminary concepts that lead up to the definitions of the mathematical morphological operators. This treatment of mathematical morphology simultaneously handles the discrete and the continuous domains, and is based on the mathematical morphology papers of Pijush Ghosh.

I knew Pijush Ghosh in the early 1990s, when he came to visit my laboratory at the University of Washington. His knowledge and understanding of mathematical morphology operations and what could be done with them, and what structures to use to implement continuous domain morphology in a computer program, was thorough and complete. I learned a great deal from him. He was a beautiful person, with a wonderful mind. He passed from this world prematurely, at an early age, only a few years after he returned to India, and he is greatly missed.

Robert M. Haralick
Distinguished Professor of Computer Science
Graduate Center, The City University of New York

Preface

In this book, the coauthors have set out to provide a shape description scheme that is a notational system for expressing the shapes of objects. This is also a way of writing the shape information symbolically to avoid both ambiguity and obscurity, just as we use notation to express music or electronic circuitry. We were interested in the algebraic structure hidden in the shapes, and we wanted to answer the following question: "Even if we identify that a given set of objects possesses an *algebraic structure*, how much is gained in practice from this discovery?" Of course, we have come to know that the set of objects is closed under some algebraic composition law, and that if it becomes possible to identify its set of generators, we may construct the whole set from that subset. However, can we conceive of some "stronger" kind of structure than this?

In this book, we take a morphological and set-theoretic approach to answering the above question. Then, we show the capability of this approach for image processing and computer graphics by presenting a simple shape model using two basic morphological and set-theoretic shape operators, which are called Minkowski addition and decomposition. The mathematical characteristics of these operators and their significance are explored in some detail, with the aim of eventually arriving at a formal theory of shape description.

We start the book with the mathematical basis of sets and functions, and next review modern algebra in general, thus highlighting the importance of the Minkowski operators. Then, on the basis of these preparations, we set out to construct a systematic method for the representation and analysis of shapes.

The first author, Pijush Ghosh, was a leading researcher in the area of Minkowski algebra and its applications to shape analysis and related problems.

His idea was to answer a simple question: *"Is it possible to do addition and subtraction (or multiplication and division) with geometric shapes as we do in ordinary arithmetic with numbers?"* In other words, given a geometric shape, does its inverse – that is, its *negative shape* – exist? If this were possible, then we might have obtained a remarkable insight into analyzing and synthesizing shapes, just as we have in the case of numbers.

This notion still fascinates me. We discussed the central problem in the understanding of shape, which can be compared with the analogous problem in number theory: *"Given a positive integer number n, are there integers k and $l > 1$ such that $n = k \times l$?"* As is well known, this question gave rise to one of the most fundamental concepts of number theory; namely, the concept of prime numbers.

Analogously, there exist sets of points in the plane or space that cannot be expressed as a Minkowski sum in any manner other than the most trivial one; that is, as the sum of a single point and the given set S itself – in other words, they cannot be decomposed further, as a Minkowski sum of two simpler shapes. Such point sets may be termed morphologically indecomposable shapes, or *prime shapes*.

Then, one may ask, what shape can be considered to be the "*prime shape*"?

Before he was able to solve this problem completely, Pijush K. Ghosh passed away in 1999, at the age of 47, due to a brain tumor.

This book is a collaborative work between a mathematician, Pijush Ghosh, and an engineering researcher, Koichiro Deguchi. We first met in the early 1990s, at Professor Haralick's laboratory at the University of Washington, where we were both visiting researchers. In the course of our discussions, it became clear that we both considered that image processing and computer vision were vitally important fields of information science and technology. Researchers in several areas of mathematics have contributed to the essential progress of these fields but, unfortunately, the ties between engineering and mathematics are not sufficiently strong, even in countries where image processing technologies have made considerable progress.

We decided to write a textbook to introduce a proper and well-defined algebra for image processing problems, and we began work in 1997. Sadly, we lost Pijush Ghosh halfway through our coauthorship, and his grand plan and our framework for the book were left in my hands. It took several years for me to restart the process of compiling and shaping his ideas in order to complete our task.

The first half of this book is my realization of Pijush's original idea, whereas in the latter half of the book I have reconstructed Pijush's original research.

Pijush's family, and many of his friends and colleagues, have given me great encouragement throughout. I thank Professor Robert M. Haralick for his Foreword to this book. Dr. S.P. Mudur, of the National Centre for Software Technology, India, very kindly provided me with Pijush's remaining manuscripts. The book would have been incomplete without the help of Pijush's former colleagues Dr. Vinod Kumar and Ms. Sandhya Desai; and his friends Professor Subir Kumar Ghosh, of the Tata Institute of Fundamental Research, India, and Professor Kokichi Sugihara, of the University of Tokyo, Japan, have also been very supportive. Students in my laboratory at Tohoku University have also helped by proof-reading the book. I am most grateful to all of them.

Koichiro Deguchi

Contents

1

In Search of a Framework for Shape Description

1.1 Shape Description: What It Means to Us

It is difficult to obtain a very precise meaning of *shape*. In the *Oxford English Dictionary*, the meaning of the word "shape" is given as follows:

> **Shape** – external form or contour; that quality of a material object (or geometrical figure) which depends on constant relations of position and proportionate distance among all the points composing its outline or its external surface.

The dictionary meaning of the word "description" is as follows:

> **Description** – setting forth in words; reciting the characteristics of; more or less complete definition.

The dictionary meaning of "shape" emphasizes the fact that we human beings are aware of shapes through *outlines* and *surfaces* of objects, both of which can be visually perceived. On the other hand, shape does not take into account the color or texture of a surface. In more technical terms, the shape of an object is: "Information about the geometrical aspects of the surface of the object." Shape description, therefore, involves specifying the information through a scheme or a system. A shape description scheme is a *notational system* for expressing the shapes of objects, a way of writing the shape information symbolically to avoid both ambiguity and obscurity, just as we use notation to express music or electronic circuitry.

It is well known that the discipline of shape description covers a very wide area, ranging from geometry to physics, and also to many other branches of science. For example, consider the task of describing the shape of a flat-faced solid cube. (a) A simple and direct scheme is to describe the shape by means of its vertices, edges, and faces. A vertex can be represented by a point (x_i, y_i, z_i) in a coordinate space, an edge by its end-point vertices, and a face by

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Mathematics of Shape Description Pijush K. Ghosh and Koichiro Deguchi

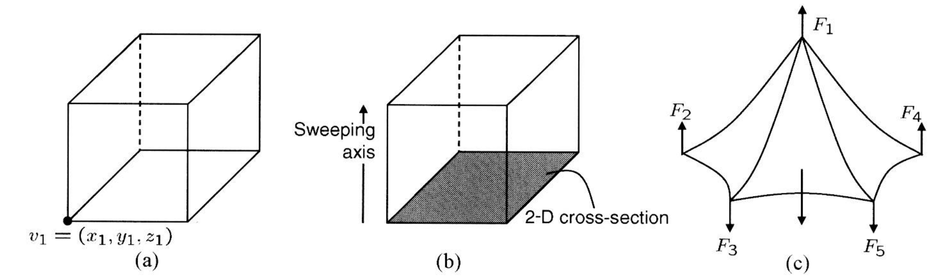

Figure 1.1 Shape can be described in a variety of ways

its bounding edges (Figure 1.1(a)). (b) The same cube can be described as the shape swept by a two-dimensional square cross-section when moved along a straight-line axis perpendicular to the cross-section (Figure 1.1(b)). This description is less direct than the previous one. (c) Sometimes, shape description may be more indirect. Consider the shape of a large suspension structure such as a tent. Its shape can be more conveniently described by means of the physical forces that act on various points of the tent (Figure 1.1(c)). The role of physical forces in describing the shapes of natural objects such as clouds, crystals, or trees is well known.

Such direct and indirect schemes are somewhat analogous to the *enumerative* and *generative* schemes in mathematics. A direct scheme is like writing down or enumerating all the elements of a set, such as

$$X = \{2, 4, 6, 8, 10, \ldots\}. \tag{1.1}$$

An indirect scheme, on the other hand, is the specification of a set by defining a generating function for its elements, such as

$$X = \{x \mid x = 2y \text{ for } y \in \{\text{Natural numbers}\} \text{ and } y \neq 0\}. \tag{1.2}$$

It is impossible for us to cover the whole range of shape description. We have decided to restrict ourselves only to that part of shape description that is connected to geometry and other closely related concepts. Thus modes of description such as those depicted in Figures 1.1(a) and (b) fall within the scope of this book, but not modes such as shown in Figure 1.1(c).

We can be a little more precise in delineating the scope of the book. The shapes around us can broadly be divided into two categories: (1) shapes of manufactured objects or potentially manufactured objects (that is, well-designed objects); and (2) shapes of natural objects. The reason for this subdivision is that the mathematical techniques that are very well suited for description of the shapes of manufactured objects turn out to be inadequate for the shapes of natural objects. It is obvious that, "clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line" [64]. Blum [7] wrote, "Euclid goes from triangles to more complex rectilinear objects, polygons. The only seriously considered nonpolygon is the circle. Where are the objects of biology? Where is the kidney bean, the tadpole? Note that the latter wiggles and is not congruent with or similar to even itself." When attempts are made to describe them in terms of classical geometry and

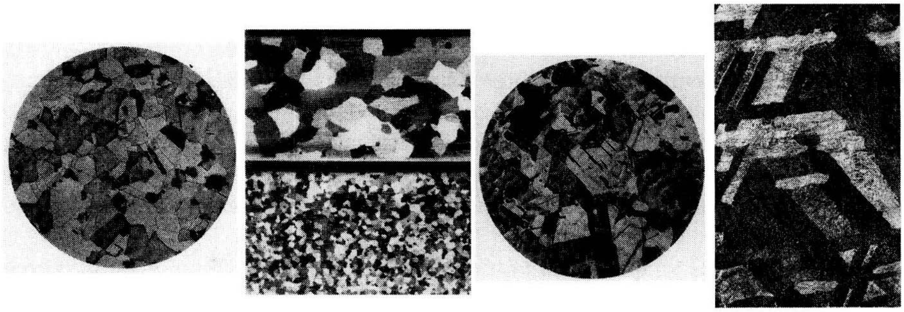

Figure 1.2 Shapes in crystals

mathematics, natural shapes turn out to be incredibly complex. Of course, there are some exceptions. For example, in nature, some crystals possess very regular geometric shapes, as shown in Figure 1.2.

But such exceptions are indeed very few in number. Thus if we restrict our domain to classical geometry and related mathematical areas, we are, in effect, addressing the question of shape description for manufactured objects. We, however, intend to discuss briefly the problems of shape description for natural objects, primarily to bring out the contrast.

1.2 Pure versus Pragmatic Approaches

Even after limiting ourselves to the geometrical aspects of shape, we find that the study of shape description starts from antiquity – from the geometry of Euclid or beyond, and extends to the geometric modeling, morphing, or fractal geometry of recent times.

The discipline of geometry itself has evolved from mankind's pursuit of describing and measuring the shapes that are of immediate importance. The original motivation for geometry was to describe and measure land and buildings (the word "geometry" comes from the Greek *γηøμετρια*, which means "earth-measuring"). The famous Rhind papyrus, a copy of which has been preserved from the Hyksos epoch (about 1700 B.C.), testifies that at that time the Egyptians, although empirically, were able to calculate the area of a plane figure or volume of a solid. Even the early Greek mathematicians, such as Thales of Milet or Pythagoras of Samos (6th century B.C.), were more interested in practical problems of surveying (means of determining the boundary, size, position, etc.) and mensuration (means measuring). And, as it often happens, a field of study that started with a motivation to solve immediate and concrete practical problems transcended to a more abstract branch of science. It became far more rigorous, but moved further away from concrete ideas. The chief contributor to this transcendence is certainly Euclid. His work entitled *Elements* (*στøιχετα*), written in Alexandria in about 300 B.C., is still considered to be one of the most valuable scientific books of all time. And then there is a long list of outstanding mathematicians – Descartes, Gauss, Lobachevski, Bolyai, Riemann, Klein, and Hilbert, to mention a few – who have all contributed in an essential way to the development of this branch of mathematics called geometry.

However, in recent times we have seen a revival of the trend toward solving immediate practical problems concerning shapes of objects. Because of the advent of the digital computer, it