College Algebra

SECOND EDITION

College Algebra

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PREFACE

PURPOSE In recent years some refreshing and practical developments have occurred in the organization and content of college algebra and in the assessment of student abilities and teaching methods. These developments have provided the framework for the second edition of this book. The aim of this edition of *College Algebra*, however, remains the same as the aim of the first edition—to provide the preparation necessary for students who intend to study calculus or take other subsequent mathematics courses. In the revision, the order of some topics has been changed, and the number of problems, examples, and applications has been increased. Several sections have been completely rewritten and new material has been added. We have been guided by surveys conducted by our publisher, by many constructive conversations with fellow teachers, and by feedback from students.

PREREQUISITES It is assumed that students who use this book have taken the equivalent of at least one year of high school plane geometry and one year of high school algebra, or that they have completed a college course in intermediate algebra.

OBJECTIVES Our book was written and revised with two goals in mind: first, that the book itself should provide all that is required for the student to learn the material; and second, that working through the book should provide students with a facility to *apply* the mathematical principles they have learned to the solution of a wide variety of specific problems. To help accomplish these objectives, discussions of new concepts always include numerous illustrative examples. Every attempt has been made to minimize the use of technical language and symbolism. However, those definitions, properties, and theorems that must be included in a book written at this level have been stated with care and precision. Theorems and properties are often preceded by a geometric discussion which will provide the student with motivation and some insight into why the formal statements are written as they are. There is a reasonable balance between theory, on the one hand, and technique, drill, and application, on the other. The many problems in this new edition are intended to help students gain confidence in their understanding of the material covered, but also to indicate those areas in need of additional study.

FEATURES *Color* is used to highlight theorems, properties, definitions, and important statements, phrases, and terms, as well as those parts of graphs that require emphasis. Since most of the explanations in the book build on intuition and geometric notions, we have made the tone of the presentation less formal and have placed more emphasis on *graphing*. In an effort to improve the graphs and illustrations, all of the artwork has been redrawn. Pertinent formulas from algebra and geometry are listed inside the front and back covers of the book for convenient reference.

There are more than 350 *examples* and over 2800 *problems* in the book. The examples are worked through in step-by-step detail, and the problems range from routine to challenging in order to test all levels of student understanding. Numerous examples and *applied problems* from physics, engineering, economics, business, biology, ecology, medicine, and the social sciences have been added throughout this edition.

The sets of problems at the end of each section have been organized to conform with the examples in that section. As a rule, a text example exists for each type of problem in a set, and each group of a particular type of problem is graded in difficulty. The odd-numbered problems are geared to the level of understanding expected of most students. Answers to these problems are at the back of the book. The even-numbered problems require a deeper, more conceptual understanding of the topics. This organization of problems will simplify assignment planning; an instructor can quickly choose whatever combination of computational and conceptual problems is desired. At the end of each chapter, a set of *review problems* provides a final test of the student's grasp of the topics.

With the availability of *hand calculators*, the answers to certain examples and problems can be computed more easily. In Chapter 5, for instance, alternative solutions which involve the use of a hand calculator have been included in some of the examples and in the appropriate problem sets. However, the availability of a calculator is *not* required for effective use of this book.

MAJOR CHANGES Chapter 1 has been rewritten to improve the clarity of the presentation. A new subsection on simplifying algebraic expressions has been added.

Chapter 2 has been reorganized to include material on equations and inequalities. A more logical development of quadratic equations and inequalities has been obtained by incorporating these topics into this chapter. A new section covering applications of first-degree equations has also been added.

Chapter 3 has been expanded to include a detailed discussion of graphing techniques and a unique subsection on shifting the graphs of functions. We define a function initially as a correspondence and then use this idea later to present the concept of a function as a set of ordered pairs. In addition, the section on linear functions has been moved into this chapter.

Chapter 4 has been completely rewritten and rearranged to include quadratic functions, power functions, and rational functions. The chapter on conic sections from the first edition has been condensed into one section here, and an expanded section on variation has been added.

Chapter 5 now includes a subsection on natural logarithms and a new section on applications of logarithms.

Chapter 6 is an expanded version of Chapter 8 in the original edition. The section on matrices has been expanded to include the algebra of matrices. We have added new sections on partial fractions, systems of linear inequalities, and linear programming. A section on systems of second-degree equations is also now included in this chapter.

Chapter 7 is essentially a rewritten version of Chapter 5 from the first edition.

Chapter 8 has been reorganized to include sequences, mathematical induction, the binomial theorem, permutations, and probability.

PACE The book allows for considerable flexibility in the pace of the course, as well as in the choice of topics. With adequately prepared students, the second edition of *College Algebra* can be covered in a three-hour, one-semester course, or in a four-hour, one-quarter course.

ADDITIONAL AIDS An accompanying *Study Guide*, also substantially revised, is available for students who may need or desire additional drill or assistance. The *Study Guide* is written in a semi-programmed format, and its organization conforms with the arrangement of the topics in the book. It contains a great many carefully graded, fill-in statements and problems. Each topic in the textbook has been broken down in the *Study Guide* into simpler units, to help students build confidence at each step in the learning process. A test is included for each chapter; to encourage self-testing, all answers are provided in the *Study Guide*.

ACKNOWLEDGMENTS Many people who used the first edition have contributed to the development of this revision. We are especially grateful for critical and constructive reviews from the following professors: David Carlson of the University of Missouri at Columbia; Kay Hudspeth of Pennsylvania State University at University Park; Richard S. Hyman of Everett Community College; Carl C. Maneri of Wright State University; Douglas Proffer and Wayne Mackey of Johnson County Community College; George W. Schultz of St. Petersburg Junior College, Clearwater Campus; Howard Taylor of West Georgia College; and Brian Wesselink of the College of Charleston. We also appreciate the comments of Professors Marguerite Gravez of Pennsylvania State University at Allentown; W. Frank McGrath of Florida Junior College at Jacksonville; and Jean Newton and Edgar R. Yarn, Jr., of St. Petersburg Junior College, Clearwater Campus. We also thank our colleague, Professor Wayne Hille, for painstakingly working through the problems in the book. Finally, we wish to express our sincere gratitude to the entire staff of Worth Publishers, especially Bob Andrews, and to our editor, Gordon Beckhorn, who was a constant source of encouragement during the writing and production of this edition.

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Warren, Michigan
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CONTENTS

CHAPTER 1

Basic Concepts of Algebra

In this chapter we review some important concepts of algebra that will be required in later chapters of this book. The topics include sets, real numbers, polynomials, rational expressions, exponents, and radicals.

1 Sets and Real Numbers

The study of algebra is concerned primarily with the properties of real numbers. Before examining these properties, it is worthwhile to introduce the concept of sets.

1.1 Sets

People communicate with each other by means of a language that consists of written or spoken symbols. In order to communicate mathematical ideas, it is essential to learn the language of mathematics. The vocabulary of this language has its beginnings with the notion of sets. Intuitively, we think of a *set* as a collection of objects. For example, we speak of the set of students in a physics class or the set of passengers on an airplane. Mathematically, we speak of the set of all counting numbers or the set of even numbers between 4 and 46. The objects that belong to a set are normally called the *elements*, or *members*, of that set.

Sets are often symbolized by capital letters, A, B, C, D, and so on, and elements of sets by lowercase letters, a, b, c, d, and so on. Another symbolism for sets uses braces to enclose either a list of elements of the set or some phrase that describes the elements. Thus, the set $B = \{1, 2, 3, 4\}$ represents the set whose elements are the numbers 1, 2, 3, and 4. The method of representing a set by listing its elements is called *enumeration*. The fact that the number x is an element of the set A is denoted by writing $x \in A$. To indicate that x is *not* an element of A, we write $x \notin A$. In our example above, $1 \in B$, $2 \in B$, $3 \in B$, and $4 \in B$, whereas $5 \notin B$. The set that has no elements is called the *empty set* or *null set* and is denoted by $\varnothing$ or $\{\ \}$. For example, the set of all Presidents of the United States who died before their thirty-fifth birthday is the empty set, because one must be at least 35 years old to be the President of the United States.

A set is said to be *finite* if it is possible to list or enumerate all the elements of the set; a set that is neither finite nor empty is an *infinite set.* For example, if A is the set of male students in a mathematics class, A is a finite set, because all its elements can be enumerated. On the other hand, if C is the set of all counting numbers, C is an infinite set because it is impossible to enumerate *all* the elements in the set. The empty set is considered to be a finite set. It is important to realize that $\varnothing$ is different from $\{0\}$, because $\{0\}$ is a set with one element, 0, whereas $\varnothing$ is the set that contains no elements.

Besides enumeration there is another method that is used to describe sets. This method consists of describing some identifying property of the set. A standard notation, called *set-builder notation,* is used for this description as follows:

$$A = \{x \mid x \text{ has the property } p\}$$

which is read "A is the set of all elements x that have property p." The letter x is called a *variable* because it denotes an unspecified element of A, and the vertical line is read "such that." For example,

$$E = \{x \mid x \text{ is an even counting number}\}$$

is read "E is the set of all elements x such that x is an even counting number." Because E is an infinite set, it is impossible to enumerate all of its elements; however, we can use the fact that the members of E form a generally known pattern to write E as $E = \{2, 4, 6, \ldots\}$, where the three dots mean the same as "and so on."

Suppose that F is the set of all female students in a mathematics class and that S is the set of all students in the class. Clearly, all the elements of F are also found in S. The set F is considered to be a *subset* of the set S. More precisely, we have the following definition.

DEFINITION 1 SUBSET

Set A is a *subset* of set B, written $A \subseteq B$, if every element of A is also an element of B. If, in addition, B contains at least one element not in A, then A is said to be a *proper subset* of B, written $A \subset B$ (notice that the horizontal bar is left off). The empty set $\varnothing$ is considered to be a subset of every set.

For example, if $A = \{2, 3, 5\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, then $A \subseteq B$; in fact, A is a proper subset of B.

Also, if $C = \{2, 4, 6\}$ and $D = \{x \mid x \text{ is an even counting number less than } 8\}$, then $C \subseteq D$ and $D \subseteq C$. This example illustrates the concept of *equality* of sets, because if $C \subseteq D$ and $D \subseteq C$, then the sets C and D are equal and we can write $C = D$.

EXAMPLE List all the subsets of $A = \{1, 2, 3\}$.

SOLUTION $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$ and $\varnothing$ are all subsets of A. Note that each of these subsets, with the exception of $\{1,2,3\}$, is a proper subset of A.

The *universe* or *universal set* is the set from which all other sets in a particular problem or discussion are formed. The choice of the universal set depends on the problem being considered. For example, in one case it may be the set of all people in the United States, and in another case, it may be the set of all people in Michigan.

Consider a universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$. From U we can form the sets $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 7\}$. How can sets A and B be used to form other sets? One way is simply to combine all the elements of A and B to form $\{1, 2, 3, 4, 7\}$. The operation suggested by this example is that of *set union*. More precisely, we have the following definition.

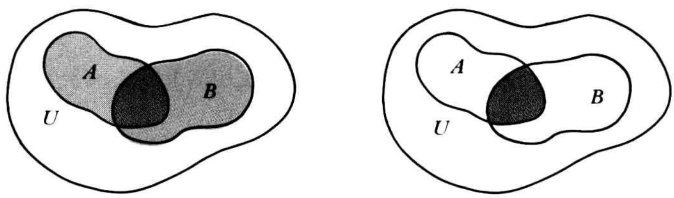
DEFINITION 2 UNION OF SETS

The *union* of A and B is the set of objects that are elements of *at least one* of the sets A and B (Figure 1a). Symbolically, it is written as $A \cup B$ and is defined by

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Hence, $A \cup B = \{1, 2, 3, 4\} \cup \{1, 3, 7\} = \{1, 2, 3, 4, 7\}$.

Figure 1



$A \cup B$ is represented by the entire shaded region

(a)

$A \cap B$ is represented by the shaded region

(b)

Another way to use $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 7\}$ to form another set is to form $\{1, 3\}$, the set of all elements common to A and B. This is an example of *set intersection*. More formally, we have the following definition.

DEFINITION 3 INTERSECTION OF SETS

The *intersection* of A and B is the set of *all* elements common to sets A and B (Figure 1b). In symbols, it is written as $A \cap B$ and is defined by

$$A \cap B = \{x \mid x \in A \text{ and (simultaneously) } x \in B\}$$

Thus, $A \cap B = \{1, 2, 3, 4\} \cap \{1, 3, 7\} = \{1, 3\}$.

If $C = \{1, 2, 3, 4\}$ and $D = \{5, 6, 7\}$, then $C \cap D = \varnothing$, and we say that C and D are disjoint sets. In general, A and B are *disjoint sets* if $A \cap B = \varnothing$.

The union of two sets, then, is simply the set composed of all elements that are in at least one of the two sets; the intersection is the set of all elements common to the two sets. Note that when the union of two sets containing common elements is enumerated, the common elements are *not* listed twice; therefore, $\{2,3,4\} \cup \{1,4,8\}$ is *not* written as $\{2,3,4,1,4,8\}$, but rather as $\{2,3,4,1,8\}$.

EXAMPLE Determine $A \cup B$ and $A \cap B$ if $A = \{1,2,3,4,5\}$ and $B = \{2,5,6,7\}$.

SOLUTION $A \cup B = \{1,2,3,4,5,6,7\}$ and $A \cap B = \{2,5\}$.

1.2 Sets of Numbers

The language of sets is used to describe the following number sets in algebra.

1 *Natural numbers* or *counting numbers* (N): The set of *natural numbers* or *counting numbers*, 1, 2, 3, . . . , forms the fundamental number set of algebra for both historical and logical reasons. This set, which we denote by N, is most often referred to as the set of *positive integers*. Thus, $N = \{1, 2, 3, \ldots\}$ is the set of positive integers.

2 *Negative integers* (I_n): The set of all *negative integers* is denoted by the symbol I_n. Thus, $I_n = \{-1, -2, -3, \ldots\}$.

3 *Integers* (I): The set of *integers* is the set formed by the union of the positive integers, the negative integers, and $\{0\}$. If we use I to denote the set of integers, we have

$$I = \{1,2,3,4,\ldots\} \cup \{-1,-2,-3,-4,\ldots\} \cup \{0\}$$

or

$$I = \{\ldots,-4,-3,-2,-1,0,1,2,3,4,\ldots\}$$

4 *Rational numbers* (Q): A *rational number* is any number that can be expressed in the form $\frac{a}{b}$, where a is an integer and b is a nonzero integer. For example, 5, $3\frac{2}{7}$, $-\frac{5}{8}$, and 21 percent are rational numbers, because they can be written as $\frac{5}{1}$, $\frac{23}{7}$, $\frac{-5}{8}$, and $\frac{21}{100}$, respectively. The set of *rational numbers* Q can be written symbolically as

$$Q = \left\{q \,\middle|\, q = \frac{a}{b}, \quad a, b \in I \text{ and } b \neq 0\right\}$$

Because any integer a can be expressed as $a/1$, every integer is also a rational number, so that $I \subseteq Q$. More precisely, because $I \neq Q$ (for instance, $\frac{2}{3} \in Q$, but $\frac{2}{3} \notin I$), I is a proper subset of Q; in symbols, $I \subset Q$. That is, all integers are rational numbers, but not all rational numbers are integers.

EXAMPLE Let $A = \{x \mid x \text{ is a counting number}\}$ and let $B = \{x \mid x \text{ is an even counting number}\}$, that is, $B = \{2,4,6,8,\ldots\}$. Find $A \cup B$ and $A \cap B$.

SOLUTION Note that $B \subset A$.

$A \cup B = \{x \mid x \text{ is a counting number or } x \text{ is an even counting number}\}$. Therefore, $A \cup B = \{x \mid x \text{ is a counting number}\} = A$.

$A \cap B = \{x \mid x \text{ is a counting number and (simultaneously) } x \text{ is an even counting number}\}$. Therefore, $A \cap B = \{x \mid x \text{ is an even counting number}\} = B$.

By using division, every rational number can be expressed as a decimal in which a block of one or more digits in the decimal repeats itself during the division process. For example, $\frac{2}{5} = 0.4000\ldots = 0.4\overline{0}$, $\frac{1}{3} = 0.333\ldots = 0.\overline{3}$, and $\frac{2}{11} = 0.18181818\ldots = 0.\overline{18}$. (In each example, the bar over one or more digits is used to indicate the block of digits that repeats itself.) Hence, it follows that every rational number can be represented by an eventually repeating decimal. The converse of this statement also holds; that is, *every eventually repeating decimal represents a rational number.* This latter statement means that any decimal number that eventually has a repeating block of digits in its decimal part can be represented by the ratio of two integers.

EXAMPLES

1 | Express each rational number in decimal notation.

(a) $\frac{3}{4}$ (b) $\frac{7}{9}$

SOLUTION By using division, we get

(a) $\frac{3}{4} = 0.75$ (b) $\frac{7}{9} = 0.777\ldots = 0.\overline{7}$

(Rational numbers, such as $\frac{3}{4}$, $-\frac{1}{8}$, or 3, in which the repeating block is the digit 0, are sometimes called *terminating decimals*.)

2 | Express each rational number in the form a/b, where a and b are integers and $b \neq 0$.

(a) 3.67
(b) $0.\overline{31}$

SOLUTION

(a) $3.67 = 3 + \frac{6}{10} + \frac{7}{100} = \frac{367}{100}$
(b) $0.31\overline{31}$ is another way of writing $0.\overline{31}$. Let $x = 0.31\overline{31}$. Multiplying both sides of this equation by 100 moves one of the repeating blocks of digits to the left of the decimal point, so that $100x = 31.\overline{31}$. Thus,

$$\begin{aligned} 100x &= 31.\overline{31} \\ -x &= -0.\overline{31} \\ \hline 99x &= 31 \end{aligned}$$

or $x = \frac{31}{99}$, so that $0.\overline{31} = \frac{31}{99}$.

5 *Irrational numbers* (L): The set of *irrational numbers* L is the set of numbers whose decimal representations are nonterminating and nonrepeating. For example, 0.01001000100001. . . , where there is one more "0" after each "1" than there is before the "1", is an irrational number. Other elements of this set are $\pi = 3.14159265358\ldots$, $\sqrt{2} = 1.4142135\ldots$, $\sqrt{3} = 1.7320508\ldots$, and $\sqrt{5} = 2.23606797\ldots$. An irrational number *cannot* be represented in the form a/b, where a and b are integers and $b \neq 0$.

6 *Real numbers* (R): The set of *real numbers* R contains all of the elements of the set of rational numbers Q and all of the elements of the set of irrational numbers L. In symbols,

$R = Q \cup L$ (Note that $Q \cap L = \varnothing$.)

1.3 Basic Properties of Real Numbers

Since the set of real numbers forms the basis for much of algebra, we now list the basic properties that the set of real numbers has under the operations of addition and multiplication, assuming that these operations are familiar to the reader.

1 *The Closure Properties*

(i) *Closure property for addition*: The sum of two real numbers is always a real number. That is, if a and b are real numbers, then $a + b$ is a real number.

(ii) *Closure property for multiplication*: The product of two real numbers is a real number. That is, if a and b are real numbers, then $a \cdot b$ is a real number.

2 *The Commutative Properties*

(i) *Commutative property for addition*: For all real numbers a and b,

$$a + b = b + a$$

(ii) *Commutative property for multiplication*: For all real numbers a and b,

$$a \cdot b = b \cdot a$$

3 *The Associative Properties*

(i) *Associative property for addition*: For all real numbers a, b, and c,

$$a + (b + c) = (a + b) + c$$

(ii) *Associative property for multiplication*: For all real numbers, a, b, and c,

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

4 *The Distributive Properties*: If a and b are real numbers, then

(i) $a \cdot (b + c) = a \cdot b + a \cdot c$

(ii) $(a + b) \cdot c = a \cdot c + b \cdot c$

5 *The Identity Properties*

(i) *Identity property for addition*: If a is a real number, then

$$a + 0 = 0 + a = a$$

(ii) *Identity property for multiplication*: If a is a real number, then

$$a \cdot 1 = 1 \cdot a = a$$

6 *The Inverse Properties*

(i) *Additive inverse*: For each real number a, there is a real number, called the *additive inverse* of a and denoted by $-a$, such that

$$a + (-a) = (-a) + a = 0$$

(ii) *Multiplicative inverse*: For each real number a, where $a \neq 0$, there is a real number, called the *multiplicative inverse* or *reciprocal* of a and denoted by $1/a$ or a^{-1}, such that

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$$

A set that satisfies the above properties is referred to as a *field*. For this reason, Properties 1–6 are sometimes called the *field properties* of real numbers.

Many other properties of the real number system can be derived from the above properties. For instance, if we let a, b, c, and d be real numbers, then it is possible to prove the following statements.

7 *The Equality Properties*

(i) *Addition property of equality*: If $a = b$ and $c = d$, then

$$a + c = b + d$$

(ii) *Multiplication property of equality*: If $a = b$ and $c = d$, then

$$a \cdot c = b \cdot d$$

8 *The Cancellation Properties*

(i) *Cancellation property for addition*: If a and b are real numbers and if $a + c = b + c$, then $a = b$.

(ii) *Cancellation property for multiplication*: If a and b are real numbers and if $ac = bc$, with $c \neq 0$, then $a = b$.

9 *The Zero Property of Multiplication*

$$a \cdot 0 = 0 \cdot a = 0$$

10 *Other Real Number Properties*

(i) $-(-a) = a$

(ii) $(-a)(b) = a(-b) = -(ab)$

(iii) $(-a)(-b) = ab$

(iv) If $ab = 0$, then either $a = 0$ or $b = 0$.

The operation of subtraction (denoted by $-$) is defined in terms of addition. If a and b are real numbers, the *difference* between a and b is the number c with the property that $b + c = a$; or, equivalently,

$$a - b = a + (-b) = c$$

For example,

$$3 - 2 = 3 + (-2) = 1 \quad \text{and} \quad 5 - (-3) = 5 + 3 = 8$$

The operation of division (denoted by $\div$) is defined in terms of multiplication. If a and b are real numbers with $b \neq 0$, then $a \div b$, the *quotient* of a and b, is the number c with the property that $b \cdot c = a$; or, equivalently,

$$a \div b = a \cdot \frac{1}{b} = c$$

For example,

$$4 \div 5 = 4 \cdot \frac{1}{5} = \frac{4}{5}$$

The symbol a/b is often used in place of $a \div b$, and it is referred to as the *fraction* of a over b. The quotient a/b is *not* defined if $b = 0$; that is, division by zero is *not* defined in the real number system. In particular, $0/0$ cannot be determined.

EXAMPLES

1| State the real number properties that justify each of the following equalities.

(a) $\frac{5}{3} \cdot (-\frac{5}{7}) = (-\frac{5}{7}) \cdot \frac{5}{3}$
(b) $(3 \cdot 7) \cdot b = 3 \cdot (7b)$
(c) $5(x+2) = 5x + 5 \cdot 2$
(d) $8 \cdot \frac{1}{8} = 1$
(e) $4 + (-4) = 0$
(f) $8 \cdot 0 = 0$
(g) $-(-6) = 6$
(h) $(-2)(-c) = 2c$
(i) If $5 + x = 5 + y$, then $x = y$
(j) If $3x = 3z$, then $x = z$.

SOLUTION

(a) $\frac{5}{3} \cdot (-\frac{5}{7}) = (-\frac{5}{7}) \cdot \frac{5}{3}$ (Commutative property for multiplication)
(b) $(3 \cdot 7) \cdot b = 3 \cdot (7b)$ (Associative property for multiplication)
(c) $5(x+2) = 5x + 5 \cdot 2$ (Distributive property)
(d) $8 \cdot \frac{1}{8} = 1$ (Multiplicative inverse property)
(e) $4 + (-4) = 0$ (Additive inverse property)
(f) $8 \cdot 0 = 0$ (Zero property of multiplication)
(g) $-(-6) = 6$ (Property 10i)
(h) $(-2)(-c) = 2c$ (Property 10iii)
(i) If $5 + x = 5 + y$, then $x = y$. (Cancellation property for addition)
(j) If $3x = 3z$, then $x = z$. (Cancellation property for multiplication)

2| Use Properties 1–6 to prove Property 10i. That is, prove that if a is a real number, then $-(-a) = a$. Justify each step.

PROOF

$a + 0 = a$ (Identity property for addition)
$a + \{-a + [-(-a)]\} = a$ (Additive inverse property)
$[a + (-a)] + [-(-a)] = a$ (Associative property for addition)
$0 + [-(-a)] = a$ (Additive inverse property)
$-(-a) = a$ (Identity property for addition)

PROBLEM SET 1

In Problems 1–6, indicate which statements are true and which are false.

1 $2 \in \{2, 5, 7\}$
2 $\varnothing \subseteq \{2, 3, 5\}$
3 $\{3, 5, 2\} = \{2, 3, 5\}$
4 $\varnothing \subseteq \{0\}$
5 $\{2, 3, 5\} \subseteq \{x \mid x \text{ is an odd number}\}$
6 $\{x \mid x \text{ is an integer}\} \subseteq \{x \mid x \text{ is a rational number}\}$.

7 List the elements in the set $\{x \mid x \text{ is an even positive integer less than } 13\}$.

8 List the elements in the set $\{x \mid x \text{ is a negative integer less than } -6\}$.

In Problems 9–12, list all the subsets of the given set. Indicate which subsets are proper subsets. How many subsets does each set have?

9 $\{7\}$
10 $\{5, 6\}$
11 $\{3, 7, 8\}$
12 $\{a, b, c, d\}$

In Problems 13–18, given that $A = \{1, 4, 7\}$, $B = \{1, 2, 5, 7\}$, $C = \{5, 6, 7, 8\}$, and $D = \{4, 5, 7\}$, list the elements of each set.

13 $A \cup C$
14 $B \cap C$
15 $A \cap B$
16 $B \cup D$
17 $A \cup D$
18 $B \cap \varnothing$