

IEEE Press Series on Electromagnetic Wave Theory
Andreas Cangellaris, Series Editor

Discontinuities in the Electromagnetic Field



M. Mithat Idemen

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IEEE Antennas and Propagation Society, *Sponsor*



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Preface

Without any hesitation we can claim that our civilization is largely based on the capacity and effectiveness of our knowledge pertaining to the natural phenomena collected under the name of *electromagnetism*. Our effective knowledge on these phenomena, with which we are familiar in fragments for nearly 2000 years, since the era of Thales, goes back only one and a quarter centuries ago. The years of 1873 and 1887 in the nineteenth century were two very important milestones in the intellectual evolution, as well as in the technological achievements of mankind. Indeed, the first one was the year in which the *electricity*, *magnetism*, and *optics*, which had been considered to be different natural phenomena until then, were unified under a common framework called the *electromagnetism*. To this end, Maxwell wrote a system of partial differential equations in England and claimed that those phenomena are all some particular aspects of a unique phenomenon which satisfies his equations. The equations introduced by Maxwell were not only what are reduced to the already known equations written 40 years ago by Faraday in England and 50 years ago by Ampère in France, but also claimed that the phenomenon in question consisted of a wave, which propagates with a finite velocity. What is very interesting is that physicists and engineers of those days were not familiar with such a kind of wave. To believe in the existence of the wave in question, one had to wait 14 years to witness the modest experiment carried out by Hertz in 1887 in Germany. That experiment, which revealed the so-called electromagnetic wave, showed also that energy transmission is possible through this wave. That was the indicator of a new direction for the civilization and thusly intensified the interest in the electromagnetic wave in both theoretical and experimental domains. Within the three years just after the Hertz experiment, the first radio receiver was realized by Branly in Germany in 1890, and soon after this, many crucial applications in

communication, control, remote sensing, medicine, radio-astronomy, space communication, heating, and so on, began to flow. Among them we can mention, for example, the radio communication between two sides of the Atlantic Ocean (in 1901), radar (in 1940), laser (in 1957), tomography (in 1984), etc.

By considering new products of the actual technology, which permit us to use extremely short waves whose frequencies increase day by day, we can hope that electromagnetic waves will continue to be the basis of new, even unexpected, applications. These applications will require solutions of the Maxwell equations under new supplementary relations (boundary conditions, edge conditions, tip conditions, etc). Therefore, a thorough investigation of these supplementary relations is very important from both theoretical and practical points of view. The aim of this short monograph is to fulfill this job. I think (and hope) that readers who are fond of the *theory* will find several enjoyable points in the book. However, the meticulous colleagues will certainly find many points to criticize. As a scientist who imagines science as a continuously evolving struggle, I look forward with deep gratitude to all constructive comments and critiques.

What seems also important to me is that by claiming the existence of the electromagnetic wave before its experimental observation, the theory of Maxwell epitomizes an example for the “*theory before experiment*.” The electromagnetic wave is not the unique example presented by the Maxwell equations in this sense. Indeed, by claiming a nonuniversal time concept, they became also the main instigator for the Special Theory of Relativity established by Einstein nearly 30 years later than the Maxwell’s Theory. The equations proposed by Maxwell conceal inside themselves too many secrets of the nature: reflected waves, refracted waves, creeping modes, whispering gallery modes, edge-excited waves, tip-excited waves, shadow boundaries, reflection boundaries, refraction boundaries, caustics, traveling waves, standing waves, Cherenkov waves, trapped waves, boundary conditions, edge conditions, tip conditions, space-time transformation under uniform motions, Doppler effect, aberration, and so on. They permit us to discover all these secretes as well as the interrelations among them if we interrogate with appropriate mathematical tools. I believe that the most effective tool for the investigation of the discontinuities in *natural phenomena* of any kind is the concept of *distribution*. This

concept, which was introduced by Schwartz in 1950 (three quarters of a century later than the Maxwell equations) in France to the contemporary mathematics, extended the meaning of the Maxwell equations. If one assumes that the differential operations taking place in the Maxwell equations are all in the sense of distributions, then the discontinuities can be discussed pretty easily and rigorously. This monograph adopts this approach. When the surfaces which carry the discontinuities are in uniform motion, formulas of the Special Theory of Relativity help to reveal behaviors of the discontinuities by transforming the expressions pertinent to the surfaces at rest.

With their elegances and powers, the above-mentioned two theories (i.e., the Theory of Electromagnetism—including the special theory of relativity—and the Theory of Distributions) excite feelings of admiration. For me, they are not a heap of spiritless mathematical symbols but rather the *self-consistent* intellectual and *fine-artistic* productions of human mind. Since the Theory of Distributions as well as that of the Special Theory of Relativity are not thoroughly included into the undergraduate curriculum, in order to offer a self-contained book, the necessary (and sufficient) material which helps to clarify the essentials of these theories, are also be included into the book.

It is a great pleasure for me to confess that the publication of this monograph could not have been realized if sincere encouragements and supports by certain colleagues had not existed. Among them I mention especially Professor Tayfun Akgül (Istanbul Technical University, Turkey), Professor Ross Stone (IEEE Antennas and Propagation Society Publishing Board member), Professor Robert Mailloux (IEEE Antennas and Propagation Society Press Liason Committee Chair) and Professor Andreas C. Cangellaris (editor of the IEEE Press Series on Electromagnetic Wave Theory). I am indebted to all of them for their invaluable interests and supports.

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Heybeliada, Istanbul
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M. MITHAT IDEMEN

Contents

Preface	ix
1. Introduction	1
2. Distributions and Derivatives in the Sense of Distribution	7
2.1 Functions and Distributions,	7
2.2 Test Functions. The Space C_0^∞ ,	9
2.3 Convergence in D ,	14
2.4 Distribution,	16
2.5 Some Simple Operations in D' ,	21
2.5.1 Multiplication by a Real Number or a Function,	21
2.5.2 Translation and Rescaling,	21
2.5.3 Derivation of a Distribution,	22
2.6 Order of a Distribution,	26
2.7 The Support of a Distribution,	31
2.8 Some Generalizations,	33
2.8.1 Distributions on Multidimensional Spaces,	33
2.8.2 Vector-Valued Distributions,	38
3. Maxwell Equations in the Sense of Distribution	49
3.1 Maxwell Equations Reduced into the Vacuum,	49
3.1.1 Some Simple Examples,	53
3.2 Universal Boundary Conditions and Compatibility Relations,	54
3.2.1 An Example. Discontinuities on a Combined Sheet,	57
3.3 The Concept of Material Sheet,	59
3.4 The Case of Monochromatic Fields,	62
3.4.1 Discontinuities on the Interface Between Two Simple Media that Are at Rest,	64

4. Boundary Conditions on Material Sheets at Rest 67

- 4.1 Universal Boundary Conditions and Compatibility Relations for a Fixed Material Sheet, 67
- 4.2 Some General Results, 69
- 4.3 Some Particular Cases, 70
 - 4.3.1 Planar Material Sheet Between Two Simple Media, 70
 - 4.3.2 Cylindrically or Spherically Curved Material Sheet Located Between Two Simple Media, 91
 - 4.3.3 Conical Material Sheet Located Between Two Simple Media, 93

5. Discontinuities on a Moving Sheet 109

- 5.1 Special Theory of Relativity, 110
 - 5.1.1 The Field Created by a Uniformly Moving Point Charge, 112
 - 5.1.2 The Expressions of the Field in a Reference System Attached to the Charged Particle, 114
 - 5.1.3 Lorentz Transformation Formulas, 115
 - 5.1.4 Transformation of the Electromagnetic Field, 118
- 5.2 Discontinuities on a Uniformly Moving Surface, 120
 - 5.2.1 Transformation of the Universal Boundary Conditions, 123
 - 5.2.2 Transformation of the Compatibility Relations, 126
 - 5.2.3 Some Simple Examples, 126
- 5.3 Discontinuities on a Nonuniformly Moving Sheet, 138
 - 5.3.1 Boundary Conditions on a Plane that Moves in a Direction Normal to Itself, 139
 - 5.3.2 Boundary Conditions on the Interface of Two Simple Media, 143

6. Edge Singularities on Material Wedges Bounded by Plane Boundaries 149

- 6.1 Introduction, 149
- 6.2 Singularities at the Edges of Material Wedges, 153
- 6.3 The Wedge with Penetrable Boundaries, 154
 - 6.3.1 The H Case, 156
 - 6.3.2 The E Case, 171
- 6.4 The Wedge with Impenetrable Boundaries, 174

- 6.5 Examples. Application to Half-Planes, 175
- 6.6 Edge Conditions for the Induced Surface Currents, 176

7. Tip Singularities at the Apex of a Material Cone 179

- 7.1 Introduction, 179
- 7.2 Algebraic Singularities of an H -Type Field, 185
 - 7.2.1 Contribution of the Energy Restriction, 185
 - 7.2.2 Contribution of the Boundary Conditions, 186
- 7.3 Algebraic Singularities of an E -Type Field, 191
- 7.4 The Case of Impenetrable Cones, 193
- 7.5 Confluence and Logarithmic Singularities, 195
- 7.6 Application to some Widely used Actual Boundary Conditions, 197
- 7.7 Numerical Solutions of the Transcendental Equations Satisfied by the Minimal Index, 200
 - 7.7.1 The Case of Very Sharp Tip, 200
 - 7.7.2 The Case of Real-Valued Minimal ν , 201
 - 7.7.3 A Function-Theoretic Method to Determine Numerically the Minimal ν , 203

8. Temporal Discontinuities 209

- 8.1 Universal Initial Conditions, 209
- 8.2 Linear Mediums in the Generalized Sense, 211
- 8.3 An Illustrative Example, 212

References 215

Index 219

IEEE Press Series on Electromagnetic Wave Theory

Introduction

A man may imagine things that are false, but he can only understand things that are true.

Isaac Newton

Almost all mathematical problems connected with the electromagnetic phenomena require solutions of the Maxwell differential equations

$$\text{curl} \mathbf{H} - \frac{\partial}{\partial t} \mathbf{D} = \mathbf{J}, \quad \text{curl} \mathbf{E} + \frac{\partial}{\partial t} \mathbf{B} \quad (1.1\text{a,b})$$

$$\text{div} \mathbf{D} = \rho, \quad \text{div} \mathbf{B} = 0 \quad (1.1\text{c,d})$$

under certain supplementary restrictions stipulated on certain surfaces. In (1.1a–d) \mathbf{E} , \mathbf{D} , \mathbf{H} , and \mathbf{B} stand for the *electric field*, *electric displacement*, *magnetic field*, and *magnetic induction*, respectively, while ρ and \mathbf{J} are the volume densities of the charges and currents. As to the parameter t that appears in (1.1a,b), it is, as usual, the time. The surfaces that bear the above-mentioned supplementary restrictions are either the interfaces between bodies of different constitutive parameters or surfaces that support surface charges and currents[†] or *material sheets* that model very thin layers. The supplementary restrictions in

*Michael White, *Isaac Newton: The Last Sorcerer*, Basic Books, New York, 1977, p. 5. See also *Sir Isaac Newton's Theological Manuscripts*, H. McLachlan (Ed.), Liverpool University Press, Liverpool, 1950, p. 17.

[†]Line and point charges are also carried by certain appropriately defined surfaces.

question are the relations that state the *physically admissible discontinuities* that may occur on these surfaces. They consist, in general, of the so-called *boundary conditions* that give the jump discontinuities on the surfaces in questions. If the discontinuity surfaces also involve sharp edges and/or sharp tips, then some components of the field become infinitely large at some points. In this case, in addition to the boundary conditions, one also has to know the physically admissible asymptotic behaviors of the field near those points because, as was shown more than 60 years ago by Bouwkamp [1], one can construct many solutions to the Maxwell equations under the given boundary conditions. Of course some of these solutions are not acceptable from physics point of view. Depending on the nature of the singular point, the relations that state the asymptotic behaviors in question are called the *edge conditions* or the *tip conditions*.

It is worthwhile to remark here that any relation written on a surface cannot be treated as a boundary condition for the electromagnetic field. In order to be so, it must also be compatible with the Maxwell equations. The spectrum of the electromagnetic waves used in the telecommunication is enlarging every day more and more toward very short waves. Hence many types of roughness, which had been assumed to be negligible in earlier investigations in order to reduce mathematical difficulties, became today unavoidable. In studies to be made in forthcoming days, one will have to consider the effects of these types of roughness on the propagation of waves. Therefore rigorous and detailed investigations of the boundary, edge, and tip conditions for various geometrical and physical structures are of crucial importance from both pure scientific and technological applications points of view. The aim of the present monograph is to study the discontinuities (i.e., singularities) in questions in their most general framework and discuss the validity of some particular relations that are in use in current literature. Our fundamental basis will be the so-called *distributions* (or *generalized functions*). In order to clarify the crucial role of this concept in the present study, it will be useful to reexamine the derivation of the *classical* boundary conditions in *almost* all textbooks.

On those days when the Electromagnetic Theory had been established, one had a huge heap of scientific knowledge in theoretical physics, especially in fluid mechanics. With this potential, the scientists of that time (i.e., mathematicians, physicists, and engineers) had formulated and solved many mathematical problems that could have

interesting and important interpretations in terms of the electromagnetic notions. First problems were those that needed to find the explicit expressions of the fields created by various sources distributed in the vacuum. They were very easy. Soon later, one had considered the problems connected with bodies having simple geometrical shapes such as infinite planes, infinitely long cylinders, whole spheres, whole ellipsoids, and so on. They were also rather easy and tractable with known techniques provided that the boundary conditions to be satisfied on the surfaces of the bodies were known beforehand. It was at this stage that the struggles to reveal the discontinuities of the electromagnetic field were started. The methods that seemed most propitious were what are based on the applications of the Gauss–Ostrogradski and Stokes theorems (see Smirnow [2, pp. 177, 197]). Although these applications are repeated in almost all textbooks, we want to recapitulate them here in order to clarify the philosophy of the method adopted in the present study. To this end, consider, for example, (1.1c) and integrate it inside a volume ϑ bounded by planar boundaries of very small areas (see Fig. 1.1). Assume that \mathbf{D} as well as its partial derivatives of the first orders are all *continuous inside ϑ except on a regular surface ΔS* . To avoid useless complexities, assume also that the field depends only on two space coordinates. Then a *careless* application (without regarding the validity conditions) of the Gauss–Ostrogradski theorem to

$$\int_{\vartheta} \operatorname{div} \mathbf{D} d\vartheta = \int_{\vartheta} \rho d\vartheta = Q \quad (1.2a)$$

yields

$$\int_{\Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4} \mathbf{D} \cdot \mathbf{n} dS = Q. \quad (1.2b)$$

Here Q stands for the total charge existing inside ϑ . If Q consists of the surface charge distributed with density ρ_S on S , then by making

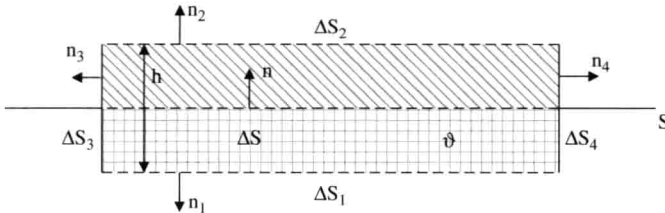


Figure 1.1. A neighborhood of a surface of discontinuity.

$h \rightarrow 0$ in (1.2b) one gets

$$\int_{\Delta S} [D_n^{(2)} - D_n^{(1)}] dS = \int_{\Delta S} \rho_S dS \quad (1.2c)$$

or

$$D_n^{(2)} - D_n^{(1)} = \rho_S. \quad (1.2d)$$

Here $D_n^{(1)}$ and $D_n^{(2)}$ show the values of the normal component of \mathbf{D} which are observed when one approaches S from lower and upper sides, respectively.

Quite similarly, from (1.1a,b,d) one gets

$$B_n^{(2)} - B_n^{(1)} = 0, \quad (1.2e)$$

$$\mathbf{n} \times \mathbf{E}^{(2)} - \mathbf{n} \times \mathbf{E}^{(1)} = 0, \quad (1.2f)$$

$$\mathbf{n} \times \mathbf{H}^{(2)} - \mathbf{n} \times \mathbf{H}^{(1)} = \mathbf{J}_S. \quad (1.2g)$$

The field \mathbf{J}_S appearing in (1.2g) is the density of the surface current that flows on S .

The relations in (1.2.d–g) are the *classical boundary conditions* that are satisfied on the interface between two regions filled with different materials. They are in use since the first days of the Theory to find solutions of the electromagnetic problems, which are in complete agreements with experiments. But the mathematical analysis made to reveal them is obviously *not legitimate*. Indeed, in order for the Gauss–Ostrogradski and Stokes theorems to be applicable, the fields \mathbf{E} , \mathbf{D} , \mathbf{H} , and \mathbf{B} have to be continuous inside ϑ . But, as the results themselves show, this is not the case. However, as we have already stated, the results are *correct from physics point of view*. A few writers who are familiar with this contrast warn readers by indicating that the results to be obtained by this kind of faulty applications are *assumed* to be acceptable for physics. D. S. Jones [3, p. 46] and S. A. Schelkunoff [4] epitomize this group of meticulous scientists.* It goes without saying that this assumption is nothing but an *additional postulate* to the Maxwell equations. Regarding the swift developments in the contemporary technology, which create and use very different materials and geometries, one can easily grasp that this approach cannot enable one

*Schelkunoff characterizes this kind of an approach to be “a proof which is not a proof but a swindle” (see Schelkunoff [4, Section 5]).

to overcome the difficulties completely. Hence one has to contrive a general and robust method.

One could also think that the difficulty in deriving (1.2d) resulted from the derivatives existing in (1.1c); if one started from the integral equation (Gauss' law)

$$\oint_S \mathbf{D} \cdot \mathbf{n} dS = Q, \quad (1.3a)$$

which is equivalent to (1.1c), one would not have the same difficulty. Indeed, in this case from (1.3a) one writes directly

$$\int_{\Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4} \mathbf{D} \cdot \mathbf{n} dS = \int_{\Delta S} \rho_S dS, \quad (1.3b)$$

which, after applications of the Gauss–Ostrogradski theorem to the partial regions lying above and below the surface ΔS , yields (1.2d). But this approach too, which at first glance seems to be propitious to overcome the difficulty, has severe defects. For example, in the case when S consists of a double sheet that carries only dipoles, the right-hand side of (1.3b) becomes naught and claims $D_n^{(2)} - D_n^{(1)} = 0$, which is not correct.* Furthermore, in cases of material sheets that are represented, for example, by impedance or resistive or other more general type of boundary conditions one cannot guess the right-hand side of (1.3b).

To avoid the difficulties mentioned above, we propose to add the following assumption to the Maxwell equations [5]:

Maxwell equations are valid in the whole of the four-dimensional space in the sense of distribution.

The results of this assumption (or postulate) will be seen after Chapter 2. Here we confine ourselves to make only the following remarks that are of importance.

- i. This assumption not only legitimizes the use of the Dirac delta functions to represent surface (or line or point) charges and currents (i.e., ρ and \mathbf{J}) but also claims that the field components \mathbf{E} ,

*Correct expression is $\lambda[D_n^{(2)} - D_n^{(1)}] = -\text{div}\{(1/\lambda)\rho_1\mathbf{n}\}$. Here ρ_1 stands for the density of the dipole distribution while λ is a parameter depending on the curvature of S and \mathbf{n} is the unit normal vector to S [see Section 3.2.1, formula (3.19c)].