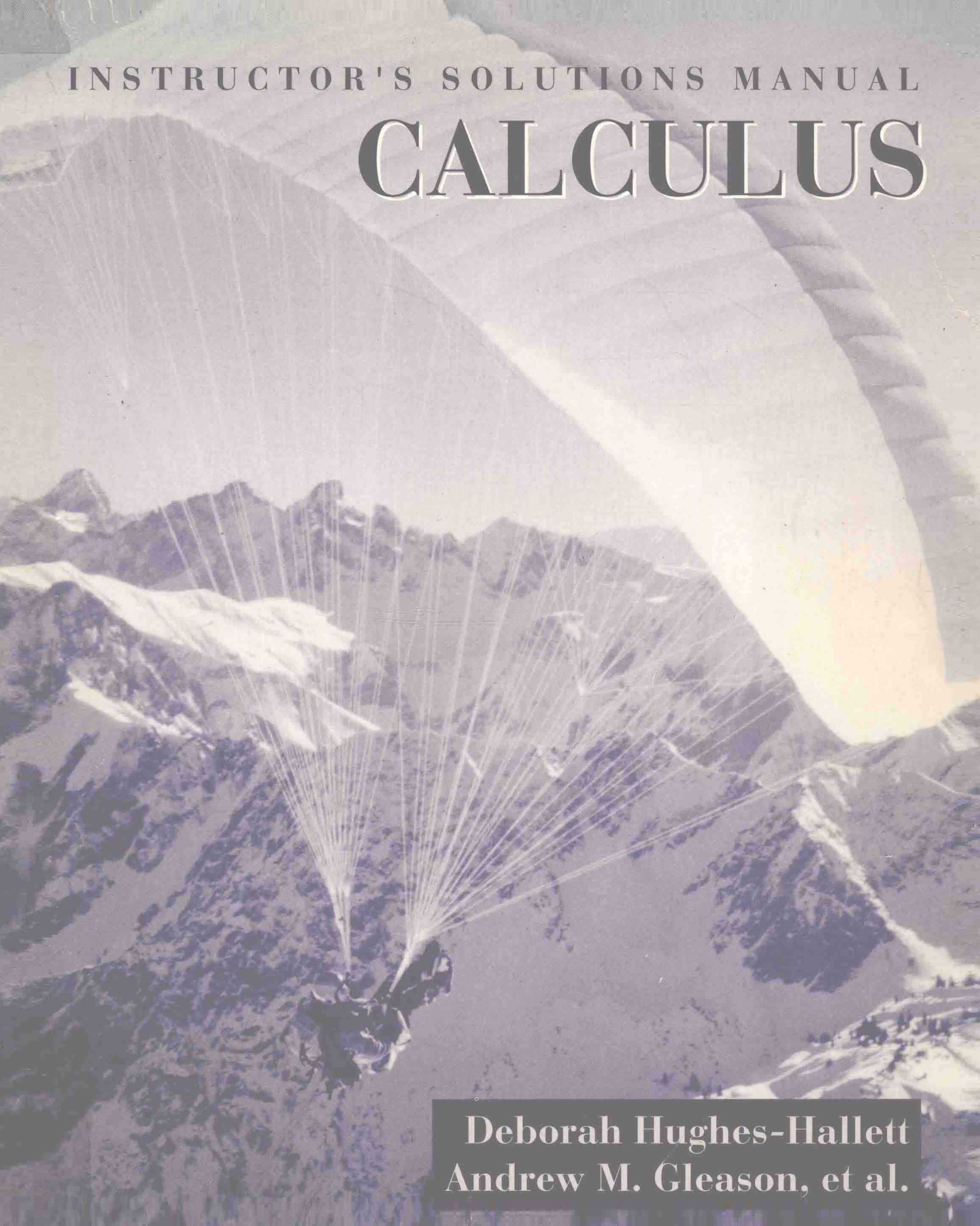
INSTRUCTOR'S SOLUTIONS MANUAL

CALCULUS



Deborah Hughes-Hallett
Andrew M. Gleason, et al.

INSTRUCTOR'S SOLUTIONS MANUAL
to accompany

CALCULUS

Deborah Hughes-Hallett
Harvard University

Andrew M. Gleason
Harvard University

et al.

Prepared By: Duff G. Campbell
Kenny Ching
Eric Connally
Adrian Iovita
Alex Kasman
Stephen A. Mallozzi
Michael Mitzenmacher

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CHAPTER ONE

1.1 SOLUTIONS

1. **(I)** The first graph does not match any of the given stories. In this picture, the person keeps going away from home, but his speed decreases as time passes. So a story for this might be: *I started walking to school at a good pace, but since I stayed up all night studying calculus, I got more and more tired the farther I walked.*
   (II) This graph matches (b), the flat tire story. Note the long period of time during which the distance from home did not change (the horizontal part).
   (III) This one matches (c), in which the person started calmly but sped up.
   (IV) This one is (a), in which the person forgot her books and had to return home.
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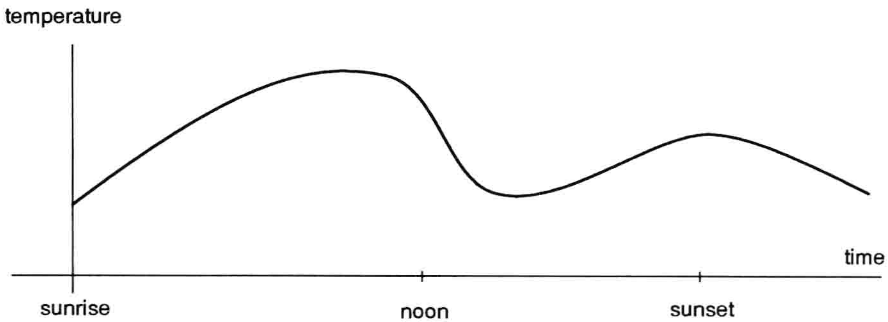

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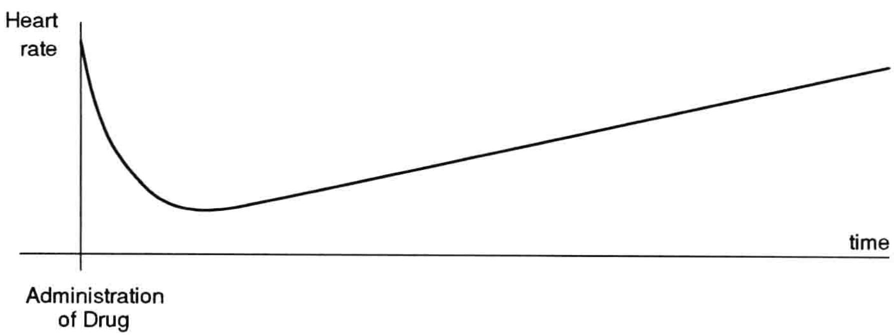

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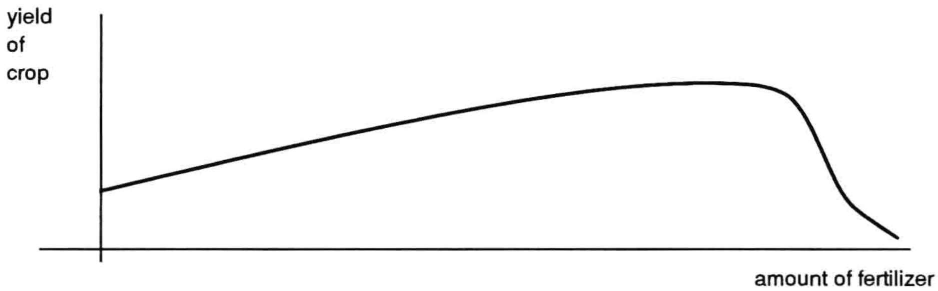

5. At first, as the number of workers increases, productivity also increases. As a result, the graph of the curve goes up initially. After a certain point the curve goes downward; in other words, as the number of workers increases beyond that point, productivity decreases. This might be due either to the inefficiency inherent in large organizations or simply to workers getting in each other's way as too many are crammed on the same line.

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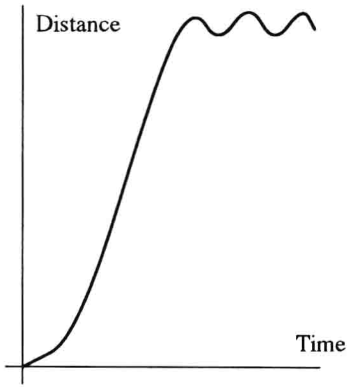

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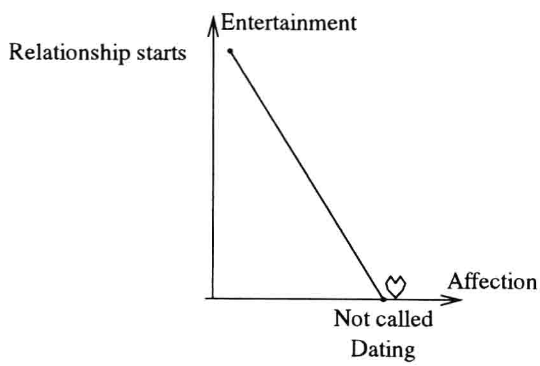


8. Generally manufacturers will produce more when prices are higher. Therefore, the first curve is a supply curve. Consumers consume less when prices are higher. Therefore, the second curve is a demand curve.

9. The price p_1 represents the maximum price any consumer would pay for the good. The quantity q_1 is the quantity of the good that could be given away if the item were free.

10. Domain: $-1 \leq x \leq 1.5$ Range: $-1.5 \leq y \leq 0$

11. (a) $-5 \leq p \leq 0, \quad 2 \leq p < 6$
(b) $r \geq 0$
(c) $0 \leq r < 2$ and $r > 5$

12. There is no real $g(y)$ when $y^2 - y = 0$ so $y = 0, 1$.
Solving

$$g(y) = \frac{1}{y^2 - y} = \frac{1}{2}$$

gives

$$\begin{aligned} 2 &= y^2 - y \\ y^2 - y - 2 &= 0 \\ (y-2)(y+1) &= 0 \end{aligned}$$

so $y = -1$ or 2.

13. The values $x \geq 2$ and $x \leq -2$ do not determine real values for f, because at those points either the denominator is zero or the square root is of a negative number.
If $f(x) = 5$ then $\frac{1}{\sqrt{4-x^2}} = 5$, or $\sqrt{4 - x^2} = \frac{1}{5}$. Solving for x, we have

$$x = \pm\sqrt{\frac{99}{25}} = \pm\frac{3}{5}\sqrt{11}.$$

14. Looking at the given data, it seems that Galileo's hypothesis was incorrect. The first table suggests that velocity is not a linear function of distance, since the increases in velocity for each foot of distance are themselves getting smaller. Moreover, the second table suggests that velocity is instead proportional to *time*, since for each second of time, the velocity increases by 32 ft/sec.

1.2 SOLUTIONS

1. Rewriting the equation as $y = -\frac{5}{2}x + 4$ shows that the slope is $-\frac{5}{2}$ and the vertical intercept is 4.

2. Slope = $\frac{6-0}{2-(-1)} = 2$ so the equation is $y - 6 = 2(x - 2)$ or $y = 2x + 2$.

3. $y - c = m(x - a)$

4. $y = 5x - 3$. Since the slope of of the line is 5, we want a line with slope $-\frac{1}{5}$ passing through the point $(2, 1)$. The equation is $(y - 1) = -\frac{1}{5}(x - 2)$, or $y = -\frac{1}{5}x + \frac{7}{5}$.

5. The line $y + 4x = 7$ has slope -4. Therefore the parallel line has slope -4 and equation $y - 5 = -4(x - 1)$ or $y = -4x + 9$. The perpendicular line has slope $\frac{-1}{(-4)} = \frac{1}{4}$ and equation $y - 5 = \frac{1}{4}(x - 1)$ or $y = 0.25x + 4.75$.

6. The intercepts appear to be (0,3) and (7.5,0), giving

$$\text{Slope} = \frac{-3}{7.5} = -\frac{6}{15} = -\frac{2}{5}.$$

The y-intercept is at (0,3), so

$$y = -\frac{2}{5}x + 3$$

is a possible equation for the line (answers may vary).

7. (a) (V)
(b) (IV)
(c) (I)
(d) (VI)
(e) (II)
(f) (III)

8. (a) is (V), because slope is negative, y intercept is 0
(b) is (VI), because slope and y intercept both positive
(c) is (I), because slope is negative, y intercept is positive
(d) is (IV), because slope is positive, y intercept is negative
(e) is (III), because slope and y intercept are both negative
(f) is (II), because slope is positive, y intercept is 0

9. (a) Finding slope (-50) and intercept gives $q = 1000 - 50p$.
(b) Solving for p gives $p = 20 - 0.02q$.

10. Given that the equation is linear, choose any two points, e.g. (5.2, 27.8) and (5.3, 29.2). Then

$$\text{Slope} = \frac{29.2 - 27.8}{5.3 - 5.2} = \frac{1.4}{0.1} = 14$$

Using the point-slope formula, with the point $(5.2, 27.8)$, we get the equation

$$y - 27.8 = 14(x - 5.2)$$

which is equivalent to

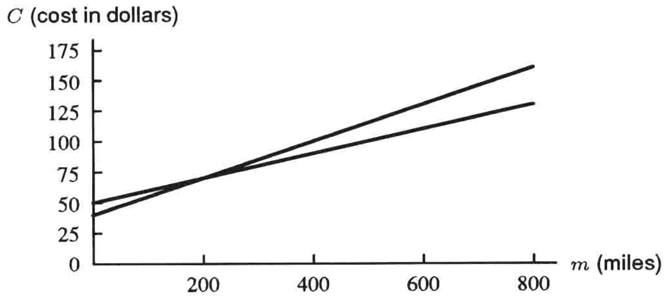
$$y = 14x - 45.$$

11. For the line $3x + 4y = -12$, the x-intercept is $(-4, 0)$ and the y-intercept is $(0, -3)$. The distance between these two points is

$$d = \sqrt{(-4 - 0)^2 + (0 - (-3))^2} = \sqrt{16 + 9} = \sqrt{25} = 5.$$

12. (a) The first company's price for a day's rental with m miles on it is $C_1(m) = 40 + 0.15m$. Its competitor's price for a day's rental with m miles on it is $C_2(m) = 50 + 0.10m$.

(b)



(c) If you are going more than 200 miles, the competitor is cheaper. If you are going less than 200 miles, the first company is cheaper.

13. (a) Given the two points $(0, 32)$ and $(100, 212)$, and assuming the graph is a line,

$$\text{Slope} = \frac{212 - 32}{100} = \frac{180}{100} = 1.8.$$

(b) The F-intercept is $(0, 32)$, so

$$^\circ\text{Fahrenheit} = 1.8(^\circ\text{Celsius}) + 32.$$

(c) If the temperature is 20°Celsius, then

$$^\circ\text{Fahrenheit} = 1.8(20) + 32 = 68^\circ\text{Fahrenheit}.$$

(d) If °Fahrenheit = °Celsius then

$$\begin{aligned} ^\circ\text{Celsius} &= 1.8^\circ\text{Celsius} + 32 \\ -32 &= 0.8^\circ\text{Celsius} \\ ^\circ\text{Celsius} &= -40^\circ = {}^\circ\text{Fahrenheit} \end{aligned}$$

14. (a) The variable costs for x acres are $\$200x$, or $0.2x$ thousand dollars. The total cost, C (again in thousands of dollars), of planting x acres is:

$$C = f(x) = 10 + 0.2x.$$

This is a linear function. See Figure 1.1. Since $C = f(x)$ increases with x, f is an increasing function of x. Look at the values of C shown in the table; you will see that each time x increases by 1, C increases by 0.2. Because C increases at a constant rate as x increases, the graph of C against x is a line.

(b)

Cost of Planting Seed

x	C
0	10
2	10.4
3	10.6
4	10.8
5	11
6	11.2

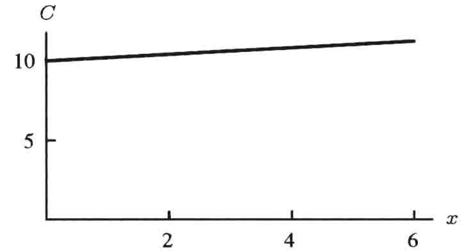


Figure 1.1

(c) What does the 0.2 tell us? It reflects the variable costs, telling us that for every additional acre planted, the costs go up by 0.2 thousand dollars. The rate at which the cost is increasing is 0.2 thousand dollars per acre. Thus the variable costs are represented by the slope of the line $f(x) = 10 + 0.2x$.

What does the 10 tell us? For $C = f(x) = 10 + 0.2x$, the intercept on the vertical axis is 10 because $C = f(0) = 10 + 0.2(0) = 10$. Since 10 is the value of C when $x = 0$, we recognize it as the initial outlay for equipment, or the fixed cost.

15.

16. (a) Let

$$I = \text{number of Indian peppers}$$
$$M = \text{number of Mexican peppers.}$$

Then (from the given information)

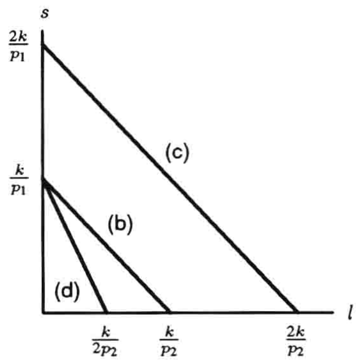
$$1{,}200I + 900M = 14{,}000$$

is the Scoville constraint.

(b) Solving for I yields

$$I = \frac{14{,}000 - 900M}{1{,}200} = \frac{35}{3} - \frac{3}{4}M.$$

17. (a) $k = p_1 s + p_2 l$ where s = # of liters of soda and l = # of liters of oil.
(b) If $s = 0$, then $l = \frac{k}{p_2}$. Similarly, if $l = 0$, then $s = \frac{k}{p_1}$. These two points give you enough information to draw a line containing the points which satisfy the equation.

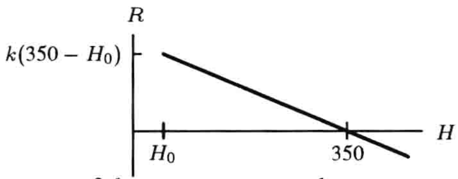


(c) If the budget is doubled, we have the constraint: $2k = p_1 s + p_2 l$. We find the intercepts as before. If $s = 0$, then $l = \frac{2k}{p_2}$; if $l = 0$, then $s = \frac{2k}{p_1}$. The intercepts are both twice what they were before.
(d) If the price of oil doubles, our constraint is $k = p_1 s + 2p_2 l$. Then, calculating the intercepts gives that the s intercept remains the same, but the l intercept gets cut in half. $s = 0$ means $l = \frac{k}{2p_2} = \frac{1}{2}\frac{k}{p_2}$. Therefore the maximum amount of oil you can buy is half of what it was previously.

18. (a) Since the population center is moving west at 50 miles per 10 years, or 5 miles per year, if we start in 1990, when the center is in De Soto, its distance d west of Steelville t years after 1990 is given by
$$d = 5t.$$
(b) It moved 700 miles over the 200 years from 1790 to 1990, so its average speed was $\frac{700}{200} = 3.5$ miles/year, somewhat slower than its present rate.
(c) According to the function in (a), after 300 years the population center would be 1500 miles west of Steelville, in Baja, California, which seems rather unlikely.

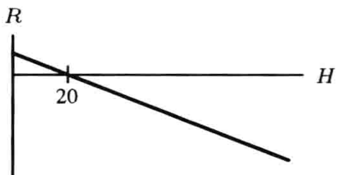
19. Given $l - l_0 = al_0(t - t_0)$ with l_0, t_0 and a all constant,
(a) We have $l = al_0(t - t_0) + l_0 = al_0 t - al_0 t_0 + l_0$, which is a linear function of t with slope al_0 and y-intercept at $(0, -al_0 t_0 + l_0)$.
(b) If $l_0 = 100$, $t_0 = 60°$F and $a = 10^{-5}$, then
$$\begin{aligned} l = 10^{-5}(100)t - 10^{-5}(100)(60) + 100 &= 10^{-3}t + 99.94 \\ &= 0.001t + 99.94 \end{aligned}$$
(c) If the slope is positive, (as in (b)), then as the temperature rises, the length of the metal increases: it expands. If the slope were negative, then the metal would contract as the temperature rises.

20. (a) $R = k(350 - H)$, where k is a positive constant.
If H is greater than 350°, the rate is negative, indicating that a very hot yam will cool down toward the temperature of the oven.
(b) Letting H_0 equal the initial temperature of the yam, the graph of R against H looks like:



Note that by the temperature of the yam, we mean the average temperature of the yam, since the yam's surface will be hotter than its center.

21. (a) $R = k(20 - H)$, where k is a positive constant. For $H > 20$, R is negative, indicating that the coffee is cooling.
(b)



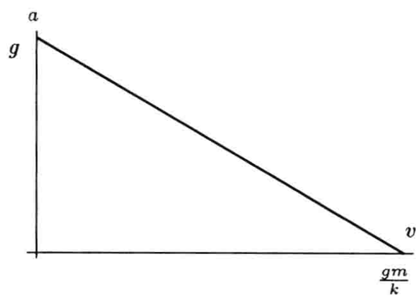
22. (a) Assembling the given information, we have

$$F = ma = F_g - F_r = (mg - kv)$$

where k is the constant that relates velocity to air resistance (which depends on the shape of the object).
(b) Solving the above equation for a, we have

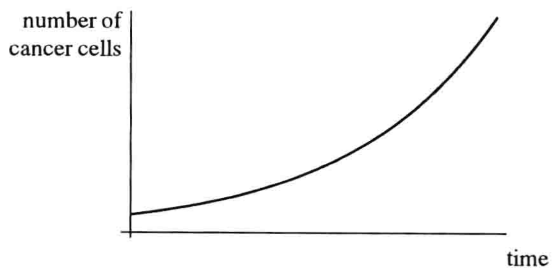
$$a = g - \frac{k}{m}v$$

(c)



1.3 SOLUTIONS

1.



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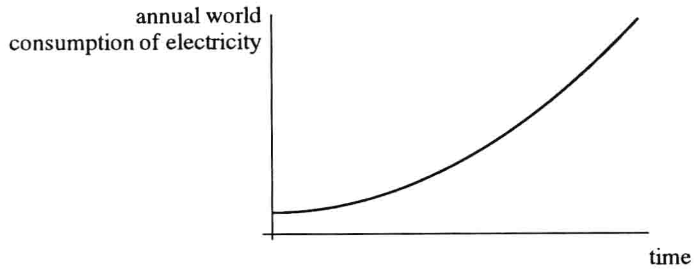

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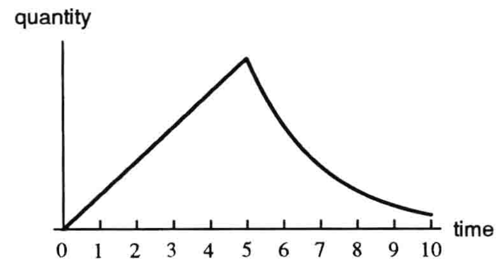

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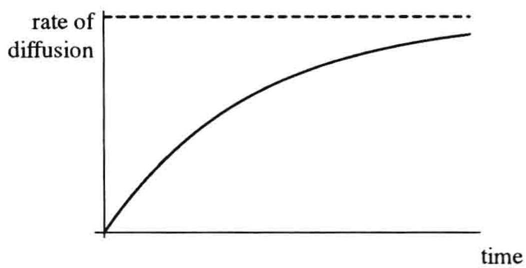

5. (a) This is the graph of a linear function, which increases at a constant rate, and thus corresponds to $k(t)$, which increases by 0.3 over each interval of 1.
   (b) This graph is concave down, so it corresponds to a function whose increases are getting smaller, as is the case with $h(t)$, whose increases are 10, 9, 8, 7, and 6.
   (c) This graph is concave up, so it corresponds to a function whose increases are getting bigger, as is the case with $g(t)$, whose increases are $1, 2, 3, 4$, and 5.
6. (a) This is a linear function, corresponding to $g(x)$, whose rate of decrease is constant.
   (b) This graph is concave down, so it corresponds to a function whose rate of decrease is increasing, like $h(x)$.
   (c) This graph is concave up, so it corresponds to a function whose rate of decrease is decreasing, like $f(x)$.

7. $f(s) = 2(1.1)^s$, $g(s) = 3(1.05)^s$, and $h(s) = (1.03)^s$.

8. The values of $f(x)$ given seem to increase by a factor of 1.4 for each increase of 1 in x, so we expect an exponential function with base 1.4. To assure that $f(0) = 4.30$, we multiply by the constant, obtaining

$$f(x) = 4.30(1.4)^x.$$

9. Each increase of 1 in t seems to cause $g(t)$ to decrease by a factor of 0.8, so we expect an exponential function with base 0.8. To make our solution agree with the data at $t = 0$, we need a coefficient of 5.50, so our completed equation is

$$g(t) = 5.50(0.8)^t.$$

10. We look for an equation of the form $y = y_0 a^x$ since the graph looks exponential. The points $(0, 3)$ and $(2, 12)$ are on the graph, so

$$3 = y_0 a^0 = y_0$$

and

$$12 = y_0 - a^2 = 3 \cdot a^2, \quad \text{giving} \quad a = \pm 2.$$

Since $a > 0$, our equation is $y = 3(2^x)$.

11. We look for an equation of the form $y = y_0 a^x$ since the graph looks exponential. The points $(1, 6)$ and $(2, 18)$ are on the graph, so

$$6 = y_0 a^1 \quad \text{and} \quad 18 = y_0 a^2$$

Therefore $\dfrac{18}{6} = \dfrac{y_0 a^2}{y_0 a} = a$, and so $6 = y_0 a = y_0 \cdot 3$, so $y_0 = 2$. Hence $y = 2(3^x)$.

12. We look for an equation of the form $y = y_0 a^x$ since the graph looks exponential. The points $(-1, 8)$ and $(1, 2)$ are on the graph, so

$$8 = y_0 a^{-1} \quad \text{and} \quad 2 = y_0 a^1$$

Therefore $\dfrac{8}{2} = \dfrac{y_0 a^{-1}}{y_0 a} = \dfrac{1}{a^2}$, giving $a = \frac{1}{2}$, and so $2 = y_0 a^1 = y_0 \cdot \frac{1}{2}$, so $y_0 = 4$.
Hence $y = 4\left(\frac{1}{2}\right)^x = 4(2^{-x})$.

13. The difference, D, between the horizontal asymptote and the graph appears to decrease exponentially, so we look for an equation of the form

$$D = D_0 a^x$$

where $D_0 = 4 =$ difference when $x = 0$. Since $D = 4 - y$, we have

$$4 - y = 4a^x \quad \text{or} \quad y = 4 - 4a^x = 4(1 - a^x)$$

The point $(1, 2)$ is on the graph, so $2 = 4(1 - a^1)$, giving $a = \frac{1}{2}$.
Therefore $y = 4(1 - (\frac{1}{2})^x) = 4(1 - 2^{-x})$.

14. (a) The formula is $Q = Q_0\left(\frac{1}{2}\right)^{(t/1620)}$.
(b) The percentage left after 500 years is

$$\frac{Q_0(\frac{1}{2})^{(500/1620)}}{Q_0}.$$

The Q_0's cancel giving

$$\left(\frac{1}{2}\right)^{(500/1620)} \approx 0.807,$$

so 80.7% is left.

15. Let Q_0 be the initial quantity absorbed in 1960. Then the quantity, Q of strontium-90 left after t years is

$$Q = Q_0\left(\frac{1}{2}\right)^{(t/29)}.$$

Since $1990 - 1960 = 30$ years elapsed, the fraction of strontium-90 left in 1990 is

$$Q = \frac{Q_0\left(\frac{1}{2}\right)^{(30/29)}}{Q_0} = \left(\frac{1}{2}\right)^{(30/29)} \approx .488 = 48.8\%.$$

16. If the pressure at sea level is P_0, the pressure P at altitude h is given by

$$P = P_0\left(1 - \frac{0.4}{100}\right)^{\frac{h}{30}},$$

since we want the pressure to be multiplied by a factor of $(1 - \frac{0.4}{100}) = 0.996$ for each 100 feet we go up to make it decrease by 0.4% over that interval. At Mexico City $h = 2237$, so the pressure is

$$P = P_0(0.996)^{\frac{7340}{100}} \approx 0.745P_0.$$

So the pressure is reduced from P_0 to approximately $0.745P_0$, a decrease of 25.5%.

17. The doubling time is approximately 2.3. For example, the population is 20,000 at time 3.7, 40,000 at time 6, and 80,000 at time 8.3.

18. The doubling time t depends only on the growth rate; it is the solution to

$$2 = (1.02)^t,$$

since 1.02^t represents the factor by which the population has grown after time t. Trial and error shows that $(1.02)^{35} \approx 1.9999$ and $(1.02)^{36} \approx 2.0399$, so that the doubling time is about 35 years.

19. The quantity Q of the substance at time t can be represented by an equation of the form

$$Q = Q_0a^t.$$

We are given $Q = 0.70Q_0$ when $t = 10$, so we have

$$\begin{aligned} 0.70Q_0 &= Q_0 a^{10} \\ 0.70 &= a^{10} \\ a &= (0.70)^{\frac{1}{10}} \\ a &\approx 0.965. \end{aligned}$$

Thus $Q = Q_0(0.70)^{\frac{1}{10}t}$, or $Q \approx Q_0(0.965)^t$.

In 50 years, $Q \approx Q_0(0.965)^{50} \approx 0.168Q - 0$, so about 17% of the original quantity is left.

To find the half life, we want to find t such that

$$\begin{aligned} 0.5Q_0 &= Q_0(0.965)^t \\ 0.5 &= (0.965)^t. \end{aligned}$$

Trying different values for t, we find

$$\begin{aligned} (0.965)^{19} &\approx 0.51 \\ (0.965)^{20} &\approx 0.49, \end{aligned}$$

so the half life is about 19.5 years.

20% will be left if

$$\begin{aligned} 0.20Q_0 &= Q_0(0.965)^t \\ 0.20 &= (0.965)^t. \end{aligned}$$

Again, trying different values for t we find

$$(0.965)^{45.2} \approx 0.20,$$

so 20% will be left after 45.2 years.

We could solve for when there is 10% remaining as above. Instead, however, we could note that since 10% is half of 20%, it should take about 19.5 years for 20% to decay to 10%. Thus the time to decay to 10% of the original amount is about $45.2 + 19.5 = 64.7$ years.

20. (a) The slope is given by

$$m = \frac{P - P_1}{t - t_1} = \frac{100 - 50}{20 - 0} = \frac{50}{20} = 2.5,$$

so the equation is

$$\begin{aligned} P - P_1 &= m(t - t_1) \\ P - 50 &= 2.5(t - 0) \\ P &= 2.5t + 50. \end{aligned}$$

(b) Given $P = P_0 a^t$ and $P = 50$ when $t = 0$,

$$50 = P_0 a^0, \text{ so } P_0 = 50.$$

Then, using $P = 100$ when $t = 20$

$$\begin{aligned} 100 &= 50a^{20} \\ 2 &= a^{20} \\ a &= (2)^{\frac{1}{20}} = 1.035. \end{aligned}$$

And so we have

$$P = 50(1.035)^t.$$

The completed table is then

TABLE 1.1 *The cost of a home*

t	a) Linear Growth Price in 1000's of $	b) Exponential Growth Price in 1000's of $
0	50	50
10	75	70.71
20	100	100
30	125	141.42
40	150	200

(c)

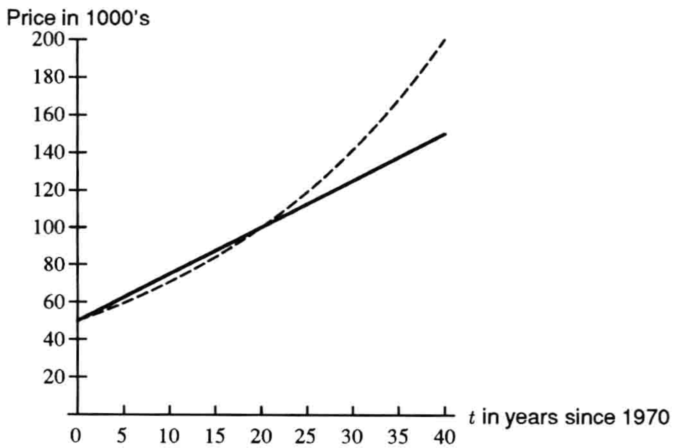

Figure 1.2