



**FUNCTIONAL APPROACH**

**TO PRECALCULUS**

FUNCTIONAL APPROACH  
TO PRECALCULUS

MUSTAFA A. MUNEM

JAMES P. YIZZE

MACOMB COUNTY COMMUNITY COLLEGE

WORTH PUBLISHERS, INC.

**FUNCTIONAL APPROACH TO PRECALCULUS**

**COPYRIGHT © 1970 BY WORTH PUBLISHERS, INC.**

**70 FIFTH AVENUE, NEW YORK, NEW YORK 10011**

**ALL RIGHTS RESERVED. NO PART OF THIS PUBLICATION MAY BE  
REPRODUCED, STORED IN A RETRIEVAL SYSTEM, OR TRANSMITTED,  
IN ANY FORM OR BY ANY MEANS, ELECTRONIC, MECHANICAL,  
PHOTOCOPYING, RECORDING, OR OTHERWISE, WITHOUT THE  
PRIOR WRITTEN PERMISSION OF THE PUBLISHERS.**

**PRINTED IN THE UNITED STATES OF AMERICA**

**LIBRARY OF CONGRESS CATALOG CARD NO. 70-115768**

**THIRD PRINTING AUGUST 1971**

**DESIGN BY MALCOLM GREAR DESIGNERS, INC.**

## **FUNCTIONAL APPROACH TO PRECALCULUS**

## PREFACE

**PURPOSE:** This text provides the preparation necessary for students who intend to take calculus or other specialized freshman-sophomore courses in college mathematics. It also gives students in general education an opportunity to fulfill their desire for a mature investigation and understanding of that level of mathematics usually referred to as “precalculus” mathematics.

**PREREQUISITES:** It is assumed that the students who use this text have had the equivalent of at least one year of plane geometry as taught in high schools and that they have the manipulatory skills which are usually acquired in one and a half years of high school algebra.

**GOALS AND EXPOSITION:** The text was written with three goals in mind: first, that the student be able to learn from the text itself; second, that he be led to reason carefully and write precisely; and third, that he gain the ability to apply to specific situations the ideas he has learned. To accomplish these goals, the exposition has been interspersed with many examples. Geometric interpretations supplement explanations whenever possible. Definitions and theorems have been carefully stated, and there is a reasonable balance between theory on one hand, and technique, drill, and application, on the other. Review problem sets at the end of each chapter will help students to understand the material covered in each chapter.

Our primary goal, that the presentation be clear and accessible to students, has led us to write some passages in ways that will be (and have been!) questioned by some mathematicians. Aware of these dilemmas we have nonetheless chosen to be guided by what our experience in the classroom has taught us is most appropriate for students.

**CONTENT:** Although the recommendations for Math O and Math A made by the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America have greatly influenced the content of this text, they have been tempered by the actual experience gained in two years of class testing preliminary editions of the book at Macomb County Community College. The function concept serves as the central theme.

Chapter 1 sets forth some important preliminary material dealing with real number sets. Set interval notation is introduced here and used throughout the text.

ACKNOWLEDGEMENTS: This text owes a great deal to the special assistance of Professors Jerald T. Ball of Chabot College, Douglas W. Hall of Michigan State University, Frank Prokop of Bradley University, David J. Foulis of the University of Massachusetts, and Sister M. Ferrer McFarland of California State College at Hayward. In addition the following teachers provided advice and criticisms for which we are indebted: Thomas F. Banchoff of Brown University, Karl W. Folley of the University of Detroit, Robert J. Kosanovich of Ferris State College, Richard T. Kuechle of Foothill College, Donald J. Lewis of the University of Michigan, James L. Murphy of Michigan State University, Joseph Murray of the State University of New York at Farmingdale, Henry F. Navarro of Henry Ford Community College, Anthony J. Nespole of Queensborough Community College, Joseph H. Oppenheim of San Francisco State College, David L. Outcalt of the University of California at Santa Barbara, George C. Ragland of Florissant Valley Community College, Franz Schnitzer of Wayne State University, and Norman Wheeler of Schoolcraft College.

Mr. Robert C. Andrews of Worth Publishers, who coordinated this project, merits special thanks. Finally, we thank our colleagues at Macomb, especially Professors William Tschirhart and George Gorte, for their helpful suggestions and criticisms.

Mustafa A. Munem  
James P. Yizze

*Warren, Michigan*  
*April, 1970*

# CONTENTS

<b>CHAPTER 1</b>	<b>Sets and Numbers</b>	<b>1</b>
Section 1	Sets	3
Section 2	Real Numbers	14
Section 3	Order	24
Section 4	Absolute Value	38
Section 5	Cartesian Coordinate System and Distance Formula	50
 <b>CHAPTER 2</b>	 <b>Relations and Functions</b>	 <b>65</b>
Section 1	Relations	67
Section 2	Functions	74
Section 3	Symmetry	89
Section 4	Types of Functions	95
Section 5	Composite Functions	104
Section 6	Inverse Functions	109
 <b>CHAPTER 3</b>	 <b>Polynomial Functions</b>	 <b>121</b>
Section 1	Introduction	123
Section 2	Linear Functions	124
Section 3	Systems of Linear Equations	139
Section 4	Determinants	152
Section 5	Quadratic Functions	163
Section 6	Polynomial Functions of Degree Greater Than 2	178

<b>CHAPTER 4</b>	<b>Exponential and Logarithmic Functions</b>	193
Section 1	Introduction	195
Section 2	Properties of Exponents	195
Section 3	Exponential Functions and Their Properties	202
Section 4	Logarithmic Functions and Their Properties	205
Section 5	Properties of Logarithms	209
Section 6	Computation of Logarithms	212
Section 7	Mathematical Induction	221
Section 8	Finite Sums and Series	231
<b>CHAPTER 5</b>	<b>Circular Functions</b>	241
Section 1	Introduction	243
Section 2	The Wrapping Function — a Periodic Function	244
Section 3	Circular Functions — Sine and Cosine	255
Section 4	Evaluation of Sine and Cosine	263
Section 5	Graphs of the Sine and Cosine	272
Section 6	Inverses of the Sine and Cosine	283
Section 7	Other Circular Functions	289
<b>CHAPTER 6</b>	<b>Trigonometric Functions</b>	307
Section 1	Introduction	309
Section 2	Angles	309
Section 3	Trigonometric and Circular Functions	322
Section 4	Trigonometric and Circular Function Identities	328
Section 5	Trigonometric Equations	337
Section 6	Triangle Trigonometry	342



<b>CHAPTER 7</b>	<b>Vectors in the Plane</b>	359
Section 1	Introduction	361
Section 2	Geometric Approach to Vectors in the Plane	361
Section 3	Analytic Representation of Vectors in the Plane	368
Section 4	Inner Product	377
Section 5	Applications	382
<b>CHAPTER 8</b>	<b>Complex Numbers and Theory of Equations</b>	397
Section 1	Introduction	399
Section 2	Complex Numbers	399
Section 3	Geometric Representation of Complex Numbers	406
Section 4	Polar Coordinates	409
Section 5	Roots of Complex Numbers	426
Section 6	Complex Zeros of Polynomial Functions	431
<b>CHAPTER 9</b>	<b>Analytic Geometry</b>	437
Section 1	Introduction	439
Section 2	Circle	440
Section 3	Translations	445
Section 4	Ellipse	451
Section 5	Hyperbola	463
Section 6	Parabola	471
Section 7	Conics	480

<b>APPENDIXES</b>	489
Appendix A Tables	491
Appendix B Field Axioms for Real Numbers	507
Appendix C Trigonometric and Circular Identities	509
Appendix D Rational Functions	511
 <b>ANSWERS TO SELECTED PROBLEMS</b>	 519
 <b>INDEX</b>	 553

## CHAPTER 1

# Sets and Numbers

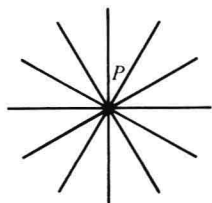


# 1 SETS AND NUMBERS

## 1 Sets

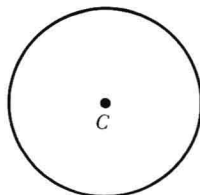
The primary objective in this section is to present enough about set theory so that the language of sets can be used later to describe mathematical concepts. *Sets* are collections of objects. For example, we can speak of the set of students in a particular course or the set of automobiles in the parking lot or the set of all books in the school library or the set of all letters in the word Florida. In geometry, we speak of a set of lines passing through a fixed point  $P$  in the plane (Figure 1) or we may refer to the set of all points that are equidistant from a fixed point  $C$  (Figure 2) or to the set of points of intersection of two circles in a plane (Figure 3). We also speak of sets of numbers, such as the set of all count-

Figure 1



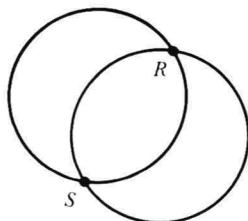
A set of  
lines passing  
through  $P$

Figure 2



Set of all points  
equidistant from  
the point  $C$

Figure 3



$R$  and  $S$  are the points  
of intersection of  
the two circles

ing numbers (1,2,3, . . . ,etc.) or the set of prime numbers greater than 2 and less than 75.

The set which has no members is called the *null set* or *empty set* and is denoted by  $\emptyset$  or by  $\{ \}$ . For example, the set of all women presidents in the United States is the empty set, since no woman has been elected to the presidency of the United States. It is important to keep in mind that 0 and  $\emptyset$  are not the same.

A set is said to be *finite* if it is possible to list or enumerate *all* the members of the set; a set which is neither finite nor empty is an *infinite* set. For example, if  $A$  is the set of all twenty students in a particular class,  $A$  is a finite set since its elements can be enumerated. If we use " $N(A)$ " to denote "number of elements in  $A$ ," then  $N(A) = 20$ . On the other hand, if  $C$  is the set of all counting numbers,  $C$  is an infinite set since enumeration is impossible in this case.

Set descriptions are usually included between braces. Finite sets can be described by enumeration. For example,  $A = \{a,b,c,d\}$  denotes that  $A$  is a finite set containing elements  $a$ ,  $b$ ,  $c$ , and  $d$  and no others. We use the notation " $a \in A$ " to indicate that " $a$  is an element of set  $A$ ."

It is important to realize that  $\emptyset$  is different from  $\{0\}$ , since  $\{0\}$  is a set with one element, 0, whereas  $\emptyset$  is a set that contains no elements.

Besides enumeration, another set description, *set builder notation*, takes the form  $A = \{x|x \text{ has property } P\}$  which is read " $A$  is the set of all elements  $x$  such that  $x$  has property  $P$ ." For example,  $E = \{x|x \text{ is an even counting number}\}$  is read " $E$  is the set of all  $x$  such that  $x$  is an even counting number." Notice that in this case,  $2 \in E$ ,  $4 \in E$ ,  $6 \in E$ , etc. Since  $E$  is an infinite set, it is impossible to enumerate all the elements of  $E$ ; however, we can use the fact that the members of  $E$  form a generally known pattern to write  $E$  as  $E = \{2,4,6, \dots\}$ , where the three dots mean the same as "etc." We use the symbol  $\notin$  to mean "is not a member of"; hence,  $1 \notin E$ ,  $3 \notin E$ ,  $5 \notin E$ .

### EXAMPLE

Use set notation to describe each of the following sets.

- $A$ , the set of all counting numbers less than 7
- $B$ , the set of all counting numbers greater than 3
- $C$ , the set of all students less than 2 inches tall

### SOLUTION

- $A = \{x|x \text{ is a counting number less than } 7\}$   
or, equivalently,  $A = \{1,2,3,4,5,6\}$   
Note that  $7 \notin A$ .
- $B = \{x|x \text{ is a counting number greater than } 3\}$ .  
 $B$  cannot be described by enumeration because  $B$  is an infinite set; however, the known pattern of the elements of  $B$  suggests that  $B$  can be written as  $B = \{4,5,6,7, \dots\}$ .
- Since  $C$  has no members,  $C = \emptyset$ .

## 1.1 Set Relations

Suppose that  $F$  is the set of all Ford automobiles and that  $M$  is the set of all motor vehicles. Clearly, all the members of  $F$  are also found in  $M$ . We say that  $F$  is a subset of  $M$  or, symbolically,  $F \subseteq M$ , which is read “ $F$  is contained in  $M$ .” The set of all girls in a biology class is a subset of the set of all students in the class. In general, set  $A$  is a *subset* of set  $B$ , written  $A \subseteq B$ , if every element of  $A$  is an element of  $B$ . *The empty set is considered to be a subset of every set.*

### EXAMPLES

- 1 Let  $A = \{1, 2, 3, 4, 5, 6\}$  and  $B = \{2, 5, 3\}$ ; then  $B \subseteq A$ .
- 2 If  $A = \{1, 2, 3\}$  and  $B = \{x | x \text{ is a counting number less than } 4\}$ , then  $A \subseteq B$  and  $B \subseteq A$ .

Example 2 motivates the definition of *equality of sets*; for, if  $A \subseteq B$  and  $B \subseteq A$ , we consider  $A$  and  $B$  to be different names for the same sets and we write  $A = B$ .

In Example 1, we have  $B \subseteq A$ , but  $B \neq A$ .  $B$  is an example of a proper subset of  $A$ . In general,  $B$  is said to be a *proper subset* of a set  $A$ , written  $B \subset A$  (notice that the horizontal bar is left off), if all members of  $B$  are in  $A$  and  $A$  has at least one member not in  $B$ , that is,  $B \subseteq A$  but  $B \neq A$ .

### EXAMPLES

- 1 If  $A = \{1, 2, 3\}$ ;  $B = \{2, 3, 1, 7, 9\}$ ; and  $C = \{2, 3, 1\}$ , then  $A \subseteq B$ ,  $C \subseteq B$ ,  $A \subseteq C$ , and  $C \subseteq A$ . More precisely,  $A \subset B$ ,  $C \subset B$ , and  $A = C$ .
- 2 List all subsets of  $\{a, b, c\}$ .

SOLUTION.  $\{a\}$ ,  $\{b\}$ ,  $\{c\}$ ,  $\{a, b\}$ ,  $\{a, c\}$ ,  $\{b, c\}$ ,  $\{a, b, c\}$  and  $\emptyset$  are the subsets of  $\{a, b, c\}$ . Note that all the subsets, with the exception of  $\{a, b, c\}$  itself, are proper subsets.

- 3 If  $A = \{2, 3, 4\}$ ,  $B = \{1, 2, 3, 4, 7, 8\}$ , and  $C = \{7, 8\}$ , then  $A \subset B$  and  $C \subset B$ .

In Example 3, we see that  $A$  and  $C$  have no members in common. The set of all girls taking biology has no member in common with the set of all boys taking the same class. Such sets are called disjoint sets. In general, two sets which have no members in common are *disjoint*. For example, the sets  $\{1, 2, 3\}$  and  $\{4, 8, 10\}$  are disjoint sets.

Suppose that  $A = \{1, 2, 3\}$  and  $B = \{2, 8, 9\}$ . Clearly,  $A \not\subseteq B$  and  $B \not\subseteq A$ . (Why?) Also,  $A$  and  $B$  are *not* disjoint because  $2 \in A$  and  $2 \in B$ . We say that  $A$  and  $B$  are overlapping. In general, sets  $A$  and  $B$  *overlap*

if there is at least one member common to  $A$  and  $B$  and if each set contains at least one member not found in the other. For example, if  $A = \{2, 3, 5, 9\}$  and  $B = \{3, 5, 10, 11, 12\}$ ,  $A$  and  $B$  overlap because  $3 \in A$  and  $3 \in B$ ,  $2 \in A$  and  $2 \notin B$ , and  $11 \in B$  but  $11 \notin A$ .

When the selection of elements of subsets is limited to some fixed set, the limiting set is called a *universal set* or a *universe*. A universal set represents the complete set or the largest set from which all other sets in that same discussion are formed. The choice of the universal set is dependent upon the problem being considered. For example, in one case it may be the set of all people in the United States, and in another, it may be the set of all people in Michigan.

### EXAMPLE

Describe set  $A$  where  $A = \{x | x \text{ is a number greater than 2 and } x \text{ is a member of universal set } U\}$ .

- a)  $U = \{1, 2, 3, \frac{4}{3}, \frac{1}{8}\}$
- b)  $U$  is the set of all counting numbers.
- c)  $U = \{0, 1, 2\}$

### SOLUTION

- a)  $A = \{3\}$
- b)  $A = \{x | x \text{ is a counting number greater than 2}\}$   
or, equivalently,  
 $A = \{3, 4, 5, \dots\}$
- c)  $A = \emptyset$

Subsets can be represented pictorially by drawings called *Venn diagrams*. These diagrams often help in understanding set concepts. If we let  $U$  be the universal set, an arbitrary set  $A \subseteq U$  can be represented as another closed region within the closed region representing  $U$  (Figure 4). Each of the four set relations discussed above can be represented by one of four Venn diagrams (Figure 5a, b, c, and d).

Figure 4

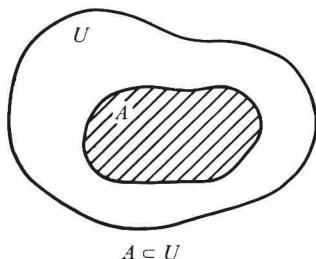
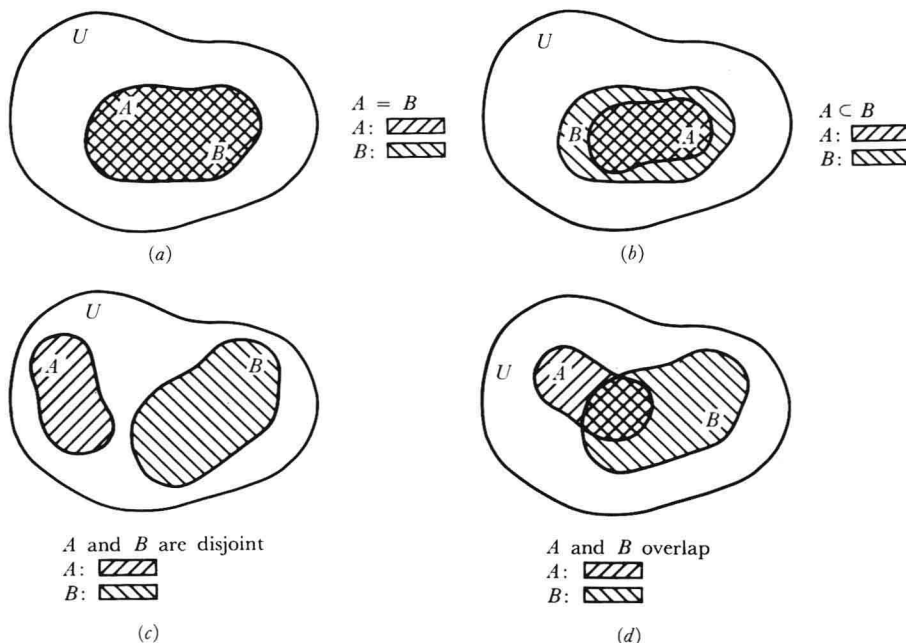




Figure 5

**EXAMPLE**

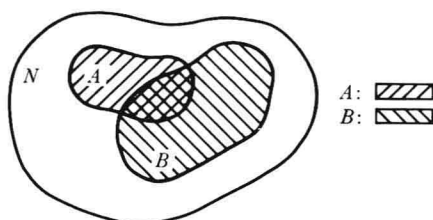
Let  $N$  be the set of counting numbers and assume that

$A = \{x | x = 3n, n \in N\}$  and  $B = \{y | y = 4m, m \in N\}$ .

Use a Venn diagram to illustrate the set relationship between  $A$  and  $B$ .

**SOLUTION.**  $A = \{3, 6, 9, 12, \dots\}$  and  $B = \{4, 8, 12, 16, \dots\}$  are infinite sets.  $3 \in A$  but  $3 \notin B$ ;  $4 \in B$  but  $4 \notin A$ ; however,  $12 \in A$  and  $12 \in B$ . Hence,  $A$  and  $B$  are overlapping subsets of  $N$  (Figure 6).

Figure 6



## 1.2 Set Operations

Consider a universal set  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ . From  $U$ , we can form  $A = \{1, 2, 3, 4\}$  and  $B = \{1, 3, 7\}$ . How can sets  $A$  and  $B$  be used to form other sets? One way is simply to combine all the elements of  $A$  and  $B$