

PRINCIPLES OF EXPERIMENTATION AND MEASUREMENT

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AND MEASUREMENT***

To Doris

Preface

This book was written as an introduction to the process of measurement and experimentation. It is intended mainly for engineers, but the draft material has been used by students in both the physical and social sciences as an introduction to the field. The arrangement of material is designed to be useful to the student as he actually performs an experiment. Thus, the planning of an experiment is discussed in Chapter 2, the actual performance in Chapter 3, the treatment of results in Chapters 4 and 5, and the reporting in Chapter 6.

At my university the material of this book has been used for several years in a one-semester lecture and project course. The project has been emphasized in this course and this accounts for the small emphasis placed on exercises in this text (except for the statistics section). A series of example projects attempted by students is listed in Appendix III. The contents of this book have been determined in large measure by the students' need to know the material in order to properly complete their projects.

An attempt has been made to make the material reasonably self-sufficient in order that a student working on his own may be able to find the book of use.

The first acknowledgement must go to my students over the past few years, the answers to whose questions form the essence of this book. Thank you to Bruce Hutchinson whose notes formed the basis for parts of Chapter 4, and to Ben van der Hoff whose notes were the basis for Sections 5.7 and 6.11. My thanks also to Tim Topper and John Hanson whose comments have been so helpful and to Bobbie Taylor who typed the many drafts.

Gordon M. Bragg

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The Place of Experimentation and Measurement in Science and Technology

The performing of experiments and measurements in an efficient and useful way is the subject of this book. The methods described are quite general and are used in all fields of science and technical endeavor.

1.1 The Obtaining of Information

In the fields of scientific and engineering study and research there are two main ways to obtain information: analysis and experiment. Analysis is the use of agreed upon theories and mathematical formulas to predict and analyze physical situations. In experimentation we return to the actual phenomena under study and actually measure what happens.

The scientific and technical advances over the past two centuries have provided for us a huge quantity of theory and analysis. For this reason, much of undergraduate education in the sciences and technologies is devoted to presenting and explaining this analysis. However, in spite of its quantity, this analysis does not explain (at least in sufficient detail) many phenomena of interest to us. We must still have recourse to experiment and measurement even in well-understood subjects. It is the purpose of this book to introduce to you the professional practice of experimentation and measurement.

1.2 The Professional Approach to Measurement

One of the fundamental aspects of professional practice is that the problems are not well formulated. Consider law and medicine. Lawyers spend considerable

proportions of their time on deciding the point of law which is in question for a particular case or client. The client is not in a position to do this himself. If he were, in many cases he could decide for himself whether his activities were within the law, whether he should prosecute, etc. In medicine, diagnosis is a fundamental aspect of the treatment. Once the diagnosis is made, medical texts make the course of treatment clear in many cases. The problem in engineering and the sciences is similar. Definition of the problem is a most important aspect of the profession.

Difficulties arise when these professional aspects are taught in an academic environment. The techniques required for doing this work are not *hard*. That is, numbers, mathematics, lists of procedures, etc., are not sufficient knowledge for a competent professional. The medical and legal professions have known this for centuries and have, respectively, developed the clinical method and case study methods of teaching these techniques. In both cases, examples are worked or treated under the guidance of an experienced professional. In experimentation the obvious analogy to these methods is the project method. This book is designed to aid you in accomplishing projects which involve measuring some quantity.

Why measurement? In the past decade "science" has become a major (and occasionally the only) component of undergraduate curricula in the professions. This has been especially true of engineering. This type of curriculum emphasizes hard information such as applied mathematics, physics, and chemistry as a necessary ingredient in an engineering curriculum. In the past few years, however, it has become clear that when carried to extremes, this emphasis trains scientists and not engineers. This book is an attempt to redress the balance in two ways. We shall attempt to present the *soft* side of the professional approach, and we shall apply it to measurement. It is considerably easier to obtain information analytically compared to experimentally if the analysis exists and is accurate. Therefore, much of engineering education consists of learning this analytical material. However, all analysis must be compared with the real world by tests. These tests involve measurements, which are another path to physical knowledge.

In summary then: *We shall study the professional approach to obtaining measurements with the objective of learning (1) how measurements are obtained in realistic situations, (2) how an organized professional approach can aid us in this as well as other problems, and (3) that measurement is a respectable way of obtaining physical knowledge comparable with analysis.*

In the past few years there has been considerable research effort devoted to quantifying and analyzing the problem-solving process. This work includes the studies of systems analysis, operations research, linear programming, statistical design and analysis of experiments, control theory, and several other fields. These fields are all comprehensive subjects in themselves. Part of our objective in this book will be to introduce some of these subjects very briefly and to outline a few of the things which these tools can accomplish.

1.3 The Scientific Method

Our factual knowledge of the physical world is based upon science. How is this knowledge obtained? The “classical” scientific method assumes three stages:

1. A *hypothesis* attempting to explain a phenomenon is proposed; for example, that force is proportional to the product of mass and acceleration. Force, mass, and acceleration are also defined. All consequences of the hypothesis must be logical.
2. An *experiment* or measurement is made in the environment where the hypothesis is assumed to be true. In the example, force, mass, and acceleration are measured over a range of each.
3. The experiment and the hypothesis are compared. If the hypothesis seems to be accurate and acceptable to the community of people most competent to judge both experiment and hypothesis, then the hypothesis becomes a *theory*.

This simple concept of the scientific method has been much battered about by historians, philosophers, and scientists, particularly in the last 20 years. The actual process of an investigation can be much different from this in many cases. For example, it is common for hypotheses to be proposed on the basis of experiments rather than vice versa. It is common to assume hypotheses which are known not to be strictly accurate in order to explain phenomena. It is also common for scientists to speak of *models* other than theories. A model can be a physical description of a phenomenon which is adaptable to mathematical analysis. The mathematical model then is proposed not so much as an explanation of nature but rather as something which it is hoped will behave in a way similar to the phenomena.

Our interest in this procedure stems from our involvement in the experimental part of the process. The pure scientist performs experiments to *discover* basic behavior and to *compare* his results with models and theories.

1.4 The Engineering Approach to Measurement

The essential difference between engineering and science may be described by saying that the objectives of “pure” engineering are creative while those of “pure” science are analytical. Both require information about the physical environment and also, in modern terms, about the social and economic environment. Because of this, the measurement and analysis techniques are common to both, with engineering more dependent on science than vice versa. In addition, the detailed application of measurements is rather broader in the engineering case. Consider the following examples of measurements which would have little usefulness or relevance to scientists:

1. Measurements of factory production rates on various days.
2. Measurements of radio reception at various distances from the aerial.
3. Measurements of movement of house foundations due to frost.
4. Measurements of variations of dimensions in manufactured products due to machine tool wear.

It is obvious that the objectives of engineering measurement are intimately tied to the objectives of the engineering project. This importance of defining the objective will be referred to again in Chapter 2. It can be seen that while we shall use the information and the tools of science, we shall use them to quite different ends.

1.5 The Variability of Measurements

Measurements are inherently variable. No measurement can be repeated exactly. In many simple measurements, such as measuring the dimensions of a room, the precision is not sufficient to detect these variations. In other measurements, the inherent variability is obvious. The daily production of a large factory will vary from day to day due to hundreds of different factors. Repeated precise measurements of the same object with the same instrument will produce variations. The reading of a micrometer will vary, depending on such quantities as

1. Operator changes.
2. Room temperature.
3. Dirt on the workpiece measured.
4. Tension on the spindle.
5. Zeroing error.
6. Misreading of the dial.
7. Interpolation errors.

This variability is fundamental to all measurement systems.

In atomic physics, this variability has a theoretical basis. Heisenberg's uncertainty principle states that "The product of uncertainty in position and uncertainty in momentum may not be less than a constant."* This means that the more accurately we know the position of a small particle, the less accurately we know the momentum of that particle and vice versa. Intuitively, this may be explained by the fact that if we wish to know the position of a particle, it is necessary to slow it down, thus changing the momentum. If we wish to know the momentum, we cannot slow it down, and thus we are uncertain about the particle position. To simply observe the particle will not let us out of this impasse since observation implies light, and light involves photon bombardment of the particle.

The problem in macroscopic measurements arises from a different source and is not so generally based on Heisenberg's principle. The macroscopic problem arises from two facts:

*The constant is $h/4\pi$, where h is Planck's constant.

1. It is impossible to measure something without using the phenomenon to influence the measuring instrument. This implies that the measurement changes the phenomenon (similar to Heisenberg's principle).
2. It is impossible to remove totally the influence of variables which influence the process but which are not wanted in the measurement.

An example of the first would be the placing of a large thermometer into a cup of coffee. The thermometer will cool the coffee. The second problem occurs because the coffee is cooling down in time, because the coffee has convection currents in it which cause and are the result of temperature differences, because there is a finite error in the calibration of the thermometer, etc.

In simple experiments the variability is often smaller than any necessary accuracy required. In difficult measurements it is possible for variations in results to be many times larger than the measurement required. In astronomical radio telescopes it is not uncommon to receive signals of importance which are many times less strong than the associated "static."

1.6 Exercises

- 1.1. Describe how the measuring process affects the thing measured in the following cases:
 - (a) Measurement of light level in a room with a light meter.
 - (b) Measurement of the thickness of a human hair with a micrometer.
 - (c) Measurement of an assembly line worker's output by an observer.
 - (d) Determination of tooth decay by X-ray.
 - (e) Determination of the breaking strength of glass by bending glass rods.
- 1.2 Report on the process of discovery of any of the following:
 - (a) Newton's laws.
 - (b) The transistor.
 - (c) The special theory of relativity.
 - (d) The laser.
 - (e) X-rays.

1.7 Suggestions for Further Reading

Arons, A. B., and Bork, A. M., *Science and Ideas, Selected Readings*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1969.

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Koestler, A., *The Sleepwalkers*, Hutchinson & Co. (Publishers) Ltd., London, 1959.

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2

Defining the Problem

The most important aspect of any measurement or experiment is the problem definition, because a proper definition will indirectly include the complete statement of the process to be followed. Four examples of this are discussed below.

2.1 Examples of Problem Definition

Example 1

It is required to find the height of waves in Lake Huron. The wave heights in a situation such as this range from less than 1/10 in. to several feet. A measuring instrument precise enough to measure wave heights of 1/10 in. (say to 10%) would require a measurement precision of 1/100 in. An instrument of this sophistication is unlikely to be strong enough to resist waves several feet high. The large waves will occur only during storms and probably will be highest in the winter, a period when it will be very difficult to service the instruments and record from them. In addition the large waves will be very infrequent. How long a sample is required and under what range of conditions? Obviously a measurement process which enables all these conditions to be measured would be extremely expensive.

If we consider the reason for the measurements, we can eliminate those which are not required. If the wave heights are required for ship design, then the frequency and magnitude of the largest waves will be important. In fact, in the extreme case a new measurement may not be required at all. A search of the log books of ships in Lake Huron during a very bad storm sometime in the past 10 or 20 years might well turn up an estimate by the ship's captain of the highest wave he had seen on the lake in his experience. This could well be sufficient. Modern commercial ships are seldom less than 200 ft long. Any wave less than, say, 2 ft