LINEAR MATHEMATICS

A PRACTICAL APPROACH

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LINEAR MATHEMATICS: A PRACTICAL APPROACH

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To my spouse, Frederick Chichester, whose versatile skills provided me with the basic ideas of Section 6.4, with numerous delicious meals, and with continuous emotional support

and to my children, Lori and Edward Kenschaft, who have encouraged my career by proofreading parts of this book and making up some exercises, by helping (sometimes cheerfully) with the housework, and by patience through many long-distance telephone calls

Preface

Mathematics is both one of the great achievements of human creativity and a pervasive practical skill in our culture, vital to higher education in almost every discipline. This text is designed to help nontechnical majors learn enough challenging mathematics to experience both the beauty of abstract patterns and the excitement of discovering how they can model the "real" world.

Carefully chosen examples and exercises are the basis of the mathematical presentation. Short examples showing how the mathematics can be applied to the real world occur in most sections of the book; the numbers are kept small in these examples so that a student can follow them and do the corresponding exercises without groping through a maze of arithmetic. A few longer examples with realistic numbers are also included; these can be omitted or used as an entire lesson's discussion. The important topic of input–output analysis appears several times in the linear algebra half of the book to help students develop a genuine "feel" for this basic application.

Each section of the book corresponds to a day's lesson and is accompanied by two or three sets of problems. Either exercise set A or set B provides a complete homework assignment: together, the sets include plenty of problems for review and classroom demonstration. Set C provides supplementary problems, usually more advanced than those in sets A and B, but sometimes displaying different applications. At the end of each chapter is a sample test. The answers to all sample tests and to exercise sets A and C are included since I believe that instant feedback spurs students to continue the essential practice that exercises provide. Flexibility is added by including the answers to the B exercises only in the Instructor's Manual. These can be easily reproduced by teachers who want to share them with their students.

The only prerequisite for the course is two years of high school algebra—or one good year. No knowledge of geometry or calculus is needed to study this book. Although topics not usually covered in elementary algebra appear from the first sections to stimulate the students' interest, there is a considerable amount of review material included near the beginning of the book, since most students appreciate such help when starting a new math course. More thorough reviews of the elementary topics essential for reading this book are found in the appendixes, along with exercises and answers.

Covering one section a day with time out for reviews and tests, I can complete about 30 of the 44 sections of this text in a three-credit, one-semester course at Montclair State College. (Starred sections and chapters are optional; later material is not dependent on them.)

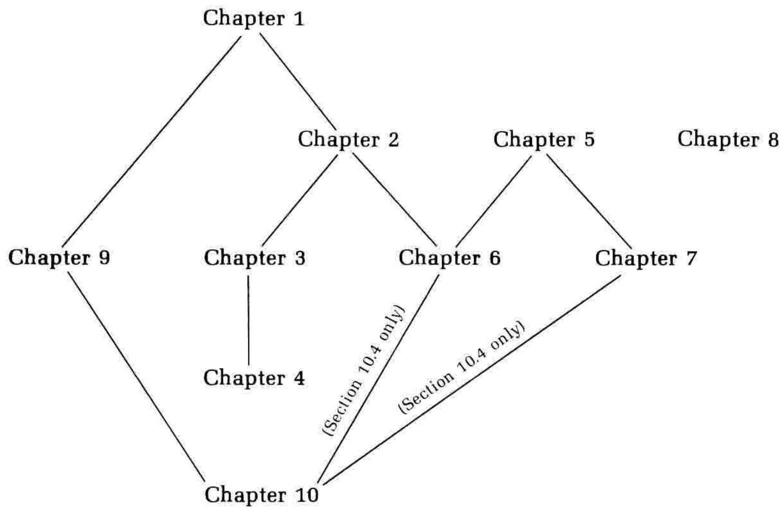
There are many ways that sections can be omitted to adapt the book to a semester course. I would suggest that the following sections are essential for an introductory course in linear algebra and linear programming: 1.1, 1.2, 1.3, 1.4, 1.5, 2.1, 2.2, 2.4, 3.1, 3.2, 3.4, 5.1, 5.2, 5.4, 6.1, and 6.2.

A fairly traditional course in linear algebra interwoven with modern applications can proceed directly through the book until Section 6.2, omitting Sections 1.6 and 2.6.

A linear mathematics course similar to those called "Finite Mathematics" can be given by covering Sections 1.1, 1.2, 1.3, 1.4, 1.5, 2.1, 2.2, 2.3, 2.4, 5.1, 5.2, 5.3, 5.4, 6.1, 6.2, 9.1, 9.2, and 9.3, and Chapter 10. If ample time is desired for probability, Chapter 9 can be inserted any time after Chapter 1.

A liberal arts survey course on the uses of mathematics in the contemporary world can be based on Chapters 1, 2, 5, 8, and 9. It is also possible to cover the first four sections of Chapter 2 before Chapter 1 if a professor prefers to introduce solving simultaneous equations before presenting matrices.

Most of these approaches have already been class-tested using earlier editions of the book; this diagram shows which chapters are required for those following and may be helpful to others planning their own course.



If the appendixes are to be used as lessons, Appendix 3 should be inserted before Section 1.5, Appendix 1 before Chapter 2, and Appendix 2 before Chapter 5.

Many thanks are hereby relayed to the students and to my colleagues at Montclair State College who have given a steady stream of suggestions and corrections, semester after semester, as this book went through its four preliminary versions. I have been repeatedly touched by the friendliness and

openness with which these comments have been generously offered, always with an eye to improving the book's effectiveness. I am indeed fortunate to teach and write in such a supportive atmosphere.

I am grateful to Kenneth Bergstresser of Washington State University and to David A. Cox of Douglass College, Rutgers University, for offering suggestions after preliminary editions of this book were used in their classes. Others who have read large portions of the text and wrote detailed, painstaking comments include Robert Bixby, Northwestern University; Robert Canavan, Monmouth College; Robert Hughes, Boise State University; Kenneth Kalmanson, Montclair State College; Joseph Rosenstein, Rutgers University; M. A. Shiro, Bloomfield College: Neil Weiss, Arizona State University; and Frank Young, Knox College. Kenneth Bergstresser and Kusum Jain proofread the typeset pages and recalculated the answers to all the problems. To all of them I want to express deep appreciation.

I also wish to thank Peter Casazza, University of Alabama, Huntsville; Charles Jackson, Jamestown Community College; Sister Janice Marie, Our Lady of Angels College; Thomas Kearns, Northern Kentucky University; Calvin King, Tennessee State University; Russell Lee, Allan Hancock College; John Mack, University of Kentucky; Conrad McKnight, University of Southwestern Louisiana; James Osborn, Georgia Institute of Technology; Richard Panicucci, Fairleigh Dickinson University, Teaneck, New Jersey; Peter Rice, University of Georgia; James Russo, Roger Williams College: Roger Turcotte, Collège Brébeuf, Montréal; and Wayne Wallace, University of Wisconsin, Oshkosh, for their valuable suggestions for improving the manuscript.

Finally I want to thank the people at Worth Publishers, who not only recruited the impressive array of reviewers listed above, but have thoughtfully helped in every stage of the book's planning and production. Although there are a few names I sorely want to mention, I know that if I begin, I will not be able to find a sensible stopping place until I have mentioned almost everyone in the company.

Like a child, a book grows with the input of so many people that the only time a parent feels completely responsible for it is when it errs. Yet as this book-child of mine approaches adulthood, I find myself hoping that it conveys to its readers the excitement of discovery that I myself have felt while learning about the pervasive power of mathematics to improve our understanding of many other fields.

January, 1978 Patricia Clark Kenschaft
Upper Montclair, N.J.

Applications

Linear Mathematics: A Practical Approach

Contents

Chapter 1

MATRICES: BASIC SKILLS AND APPLICATIONS

1.1 Definitions, Addition, Scalar Multiplication, and Notation

Arithmetic is the study of numbers. Geometry is the study of shapes. Algebra is the study of equations. And linear algebra, the subject matter of the first four chapters of this book, is the study of matrices.

1.1.1 Definition

A matrix (plural: matrices) is a rectangular array of numbers.

1.1.2 Example

$$\begin{bmatrix} 1.22 & 1.64 & 1.97 \\ 0.90 & 1.56 & 1.89 \\ 0.45 & 0.88 & 1.08 \\ 0.21 & 0.21 & 0.30 \end{bmatrix}$$

is a matrix. You have already seen matrices in many contexts. The matrix above, for example, appears in a table of the cost of mailing books:

	8 oz	1 lb	2 lb
Airmail	1.22	1.64	1.97
First class	0.90	1.56	1.89
Printed matter	0.45	0.88	1.08
Book rate	0.21	0.21	0.30

The first chapter of this text reminds you of some facts you already know about matrices, introduces some unfamiliar applications, and explores some properties of matrices that mathematicians use. In many ways matrices are like numbers; they can be added and multiplied, for example. And they also have novel useful properties, as we shall see. The next three chapters explore these properties further.

In Chapters 5 through 8 of this book rectangular arrays of numbers will be used in a somewhat different subject, linear programming, which uses some of the same skills that you will develop in linear