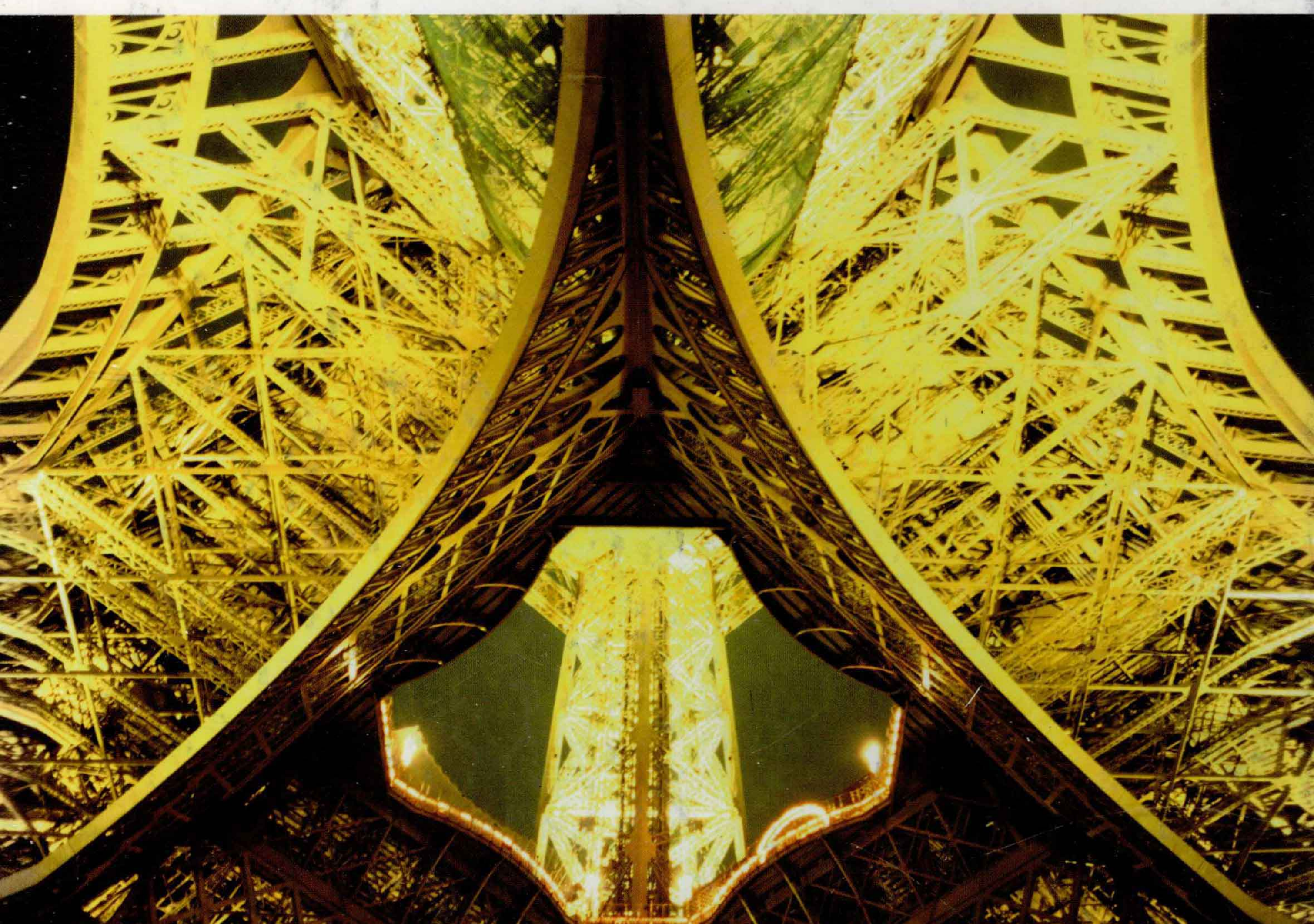
Advanced Engineering Mathematics 5th Edition



Peter V. O'Neil

ADVANCED ENGINEERING MATHEMATICS

5th Edition

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Preface

This Fifth Edition of *Advanced Engineering Mathematics* has two primary objectives.

The first is to make available much of the post-calculus mathematics needed and used by today's scientists, engineers, and applied mathematicians, in a setting that is helpful to both students and faculty. Throughout, it is recognized that mathematics provides powerful ways of modeling physical processes, but that these can lead to false conclusions if misunderstood or misapplied. This warrants careful attention to details in making correct statements of theorems and methods and in analyzing results.

The second objective is to engage theory with computational facility. Scientists, engineers, mathematicians, economists, ecologists, and other professionals often need to calculate things, moving from theory to practice. This involves acquiring skills in manipulating series, integrals, transforms, conformal mappings, and other standard objects used in mathematical modeling, as well as in carrying out calculations to reach reliable conclusions. Successful applications of such skills can be seen in major projects all over the world—the space shuttle, the Golden Gate Bridge, Kuala Lumpur's Petronas Towers, the Odeillo Solar Furnace in southern France, the English Channel tunnel joining the United Kingdom and France, the Ganter Bridge in Switzerland, and many others.

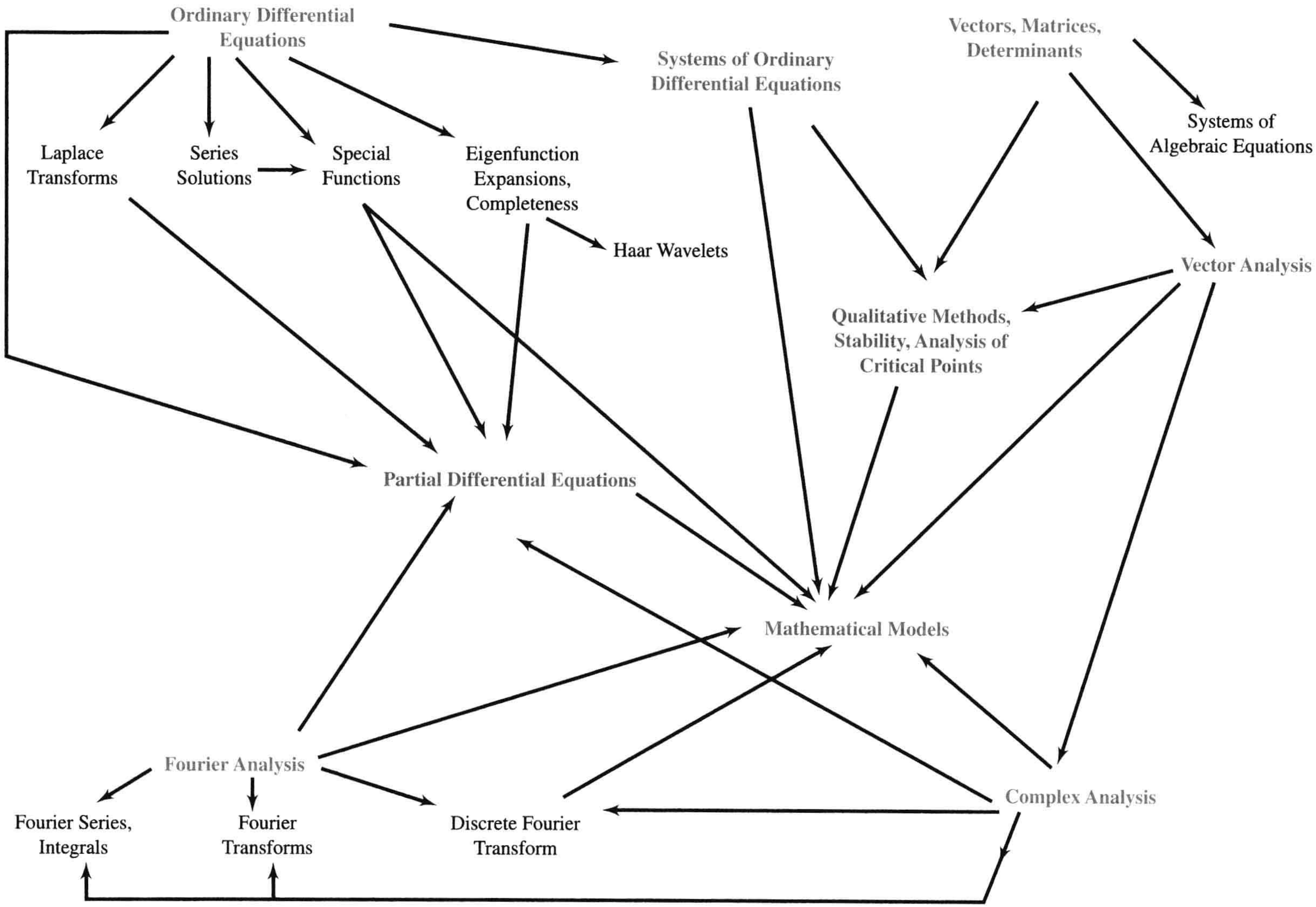
To meet these objectives, the following changes have been made in this edition.

- The wide availability of powerful and convenient computer software is an invitation to probe the relationship between the mathematics and real-world conclusions, connecting theory with models and the phenomena they describe. Making use of this capability should be an important part of a student's experience. Throughout the text, the student is asked to experiment with computations. These include generating direction fields and phase portraits, seeing the convergence of Fourier series and eigenfunction expansions through graphs of partial sums, observing the effects of filters on signals, seeing how various parameters and forcing terms influence solutions of wave and heat equations, constructing waves as sums of forward and backward waves, and using the discrete Fourier transform to approximate Fourier transforms and to sample Fourier series.

- Partial differential equations are given a reorganized and more detailed treatment. Chapter 16 covers the wave equation, first developing Fourier series solutions on a bounded interval, then Fourier integral and transform techniques for problems on the line and half-line. Characteristics are used to solve the wave equations with a forcing term, and transformations are used to deal with nonhomogeneous boundary and initial conditions. Chapter 17 follows a similar program for the heat equation, considering first solutions on a bounded interval, then on the line and half-line. New material on the nonhomogeneous heat equation is included. Finally, a chapter on the Dirichlet and Neumann problems has been added.

- The importance of the Fourier transform in modern science and engineering has been acknowledged by extending its discussion to include windowing, filtering, and use of the N-point discrete Fourier transform to sample Fourier series and approximate Fourier transforms.

Organizational Overview



- Special functions are given a more thorough, unified treatment. Legendre polynomials, Bessel functions, and then other orthogonal polynomials are placed in the context of Sturm–Liouville theory and general eigenfunction expansions, with specific examples. These are followed by mean square convergence, the significance of completeness of the eigenfunctions, and the relationship between completeness and Parseval's theorem. Haar wavelets are discussed in the context of completeness and orthogonal expansions.

- The treatment of systems of linear, ordinary differential equations has additional material on the case of repeated eigenvalues of the coefficient matrix.

- The discussion of the qualitative behavior of nonlinear systems has more details on phase portraits and the classification of critical points and stability, including Lyapunov's criteria.

- The sections on determinants have been reorganized to provide greater clarity in understanding properties of determinants.

- The section containing answers to odd-numbered problems has been expanded, including details for some of the more difficult problems, as well as more illustrations.

- Behind the mathematics we usually find interesting people and events. Some of their stories are told in Chapters 26 and 27, but historical perspectives are also included throughout the text. For example, Section 14.9.1, on the fast Fourier transform, begins with a review of the personal interactions that led to the publication of the famous Cooley–Tukey paper, the first detailed description of the algorithm.

The chart opposite offers a complete organizational overview.

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Dr. Thomas O'Neil of the California Polytechnic State University contributed material to previous editions, and much of this is continued in this edition. Dr. Fred Martens of the University of Alabama has helped with error checking in both text and problems and is the author of the solutions manual accompanying this edition. Finally, I want to acknowledge Rich Jones, who had the vision for the first edition of this text many years ago.

I would also like to acknowledge my debt to the reviewers, whose suggestions for improvements and clarifications are much appreciated:

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PART 1

Ordinary Differential Equations

CHAPTER 1
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Series Solutions

A *differential equation* is an equation that contains one or more derivatives. For example,

$$y''(x) + y(x) = 4\ \sin(3x)$$

and

$$\frac{d^4w}{dt^4} - (w(t))^2 = e^{-t}$$

are differential equations. These are *ordinary* differential equations because they involve only total derivatives, rather than partial derivatives.

Differential equations are interesting and important because they express relationships involving rates of change. Such relationships form the basis for developing ideas and studying phenomena in the sciences, engineering, economics, and, increasingly, in other areas, such as the business world and the stock market. We will see examples of applications as we learn more about differential equations.

The *order* of a differential equation is the order of its highest derivative. The first example given above is of second order, while the second is of fourth order. The equation

$$xy' - y^2 = e^x$$

is of first order.

A *solution* of a differential equation is any function that satisfies it. A solution may be defined on the entire real line, or on only part of it, often an interval. For example,

$$y = \sin(2x)$$

is a solution of

$$y'' + 4y = 0,$$

because, by direct differentiation,

$$y'' + 4y = -4\sin(2x) + 4\sin(2x) = 0.$$

This solution is defined for all x (that is, on the whole real line).

By contrast,

$$y = x\ln(x) - x$$

is a solution of

$$y' = \frac{y}{x} + 1,$$

but this solution is defined only for $x > 0$. Indeed, the coefficient $1/x$ of y in this equation means that $x = 0$ is disallowed from the start.

We now begin a systematic development of ordinary differential equations, starting with the first-order case.

CHAPTER 1

First-Order Differential Equations

1.1 Preliminary Concepts

Before developing techniques for solving various kinds of differential equations, we will develop some terminology and geometric insight.

1.1.1 General and Particular Solutions

A first-order differential equation is any equation involving a first derivative, but no higher derivative. In its most general form, it has the appearance

$$F(x, y, y') = 0, \tag{1.1}$$

in which $y(x)$ is the function of interest and x is the independent variable. Examples are

$$y' - y^2 - e^y = 0,$$
$$y' - 2 = 0,$$

and

$$y' - \cos(x) = 0.$$

Note that y' must be present for an equation to qualify as a first-order differential equation, but x and/or y need not occur explicitly.

A *solution* of equation (1.1) on an interval I is a function φ that satisfies the equation for all x in I. That is,

$$F(x, \varphi(x), \varphi'(x)) = 0 \quad \text{for all } x \text{ in } I.$$

For example,

$$\varphi(x) = 2 + ke^{-x}$$

is a solution of

$$y' + y = 2$$

for all real x and for any number k. Here I can be chosen as the entire real line. And

$$\varphi(x) = x\ln(x) + cx$$

is a solution of

$$y' = \frac{y}{x} + 1$$

for all $x > 0$ and for any number c.

In both of these examples, the solution contained an arbitrary constant. This is a symbol independent of x and y that can be assigned any numerical value. Such a solution is called the *general solution* of the differential equation. Thus,

$$\varphi(x) = 2 + ke^{-x}$$

is the general solution of $y' + y = 2$.

Each choice of the constant in the general solution yields a *particular solution*. For example,

$$f(x) = 2 + e^{-x}, \quad g(x) = 2 - e^{-x},$$

and

$$h(x) = 2 - \sqrt{53}e^{-x}$$

are all particular solutions of $y' + y = 2$, obtained by choosing, respectively, $k = 1, -1$, and $-\sqrt{53}$ in the general solution.

1.1.2 Implicitly Defined Solutions

Sometimes we can write a solution explicitly giving y as a function of x. For example,

$$y = ke^{-x}$$

is the general solution of

$$y' = -y,$$

as can be verified by substitution. This general solution is explicit, with y isolated on one side of an equation and a function of x on the other.

By contrast, consider

$$y' = -\frac{2xy^3 + 2}{3x^2y^2 + 8e^{4y}}.$$

We claim that the general solution is the function $y(x)$ implicitly defined by the equation

$$x^2y^3 + 2x + 2e^{4y} = k, \tag{1.2}$$

in which k can be any number. To verify this, implicitly differentiate equation (1.2) with respect to x, remembering that y is a function of x. We obtain

$$2xy^3 + 3x^2y^2y' + 2 + 8e^{4y}y' = 0,$$

and solving for y' yields the differential equation.

In this example we are unable to solve equation (1.2) explicitly for y as a function of x, isolating y on one side. Equation (1.2), implicitly defining the general solution, was obtained by a technique we will develop shortly, but this technique cannot guarantee an explicit solution.

1.1.3 Integral Curves

A graph of a solution of a first-order differential equation is called an *integral curve* of the equation. If we know the general solution, we obtain an infinite family of integral curves, one for each choice of the arbitrary constant.

EXAMPLE 1.1

We have seen that the general solution of

$$y' + y = 2$$

is

$$y = 2 + ke^{-x}$$

for all x. The integral curves of $y' + y = 2$ are graphs of $y = 2 + ke^{-x}$ for different choices of k. Some of these are shown in Figure 1.1. ■

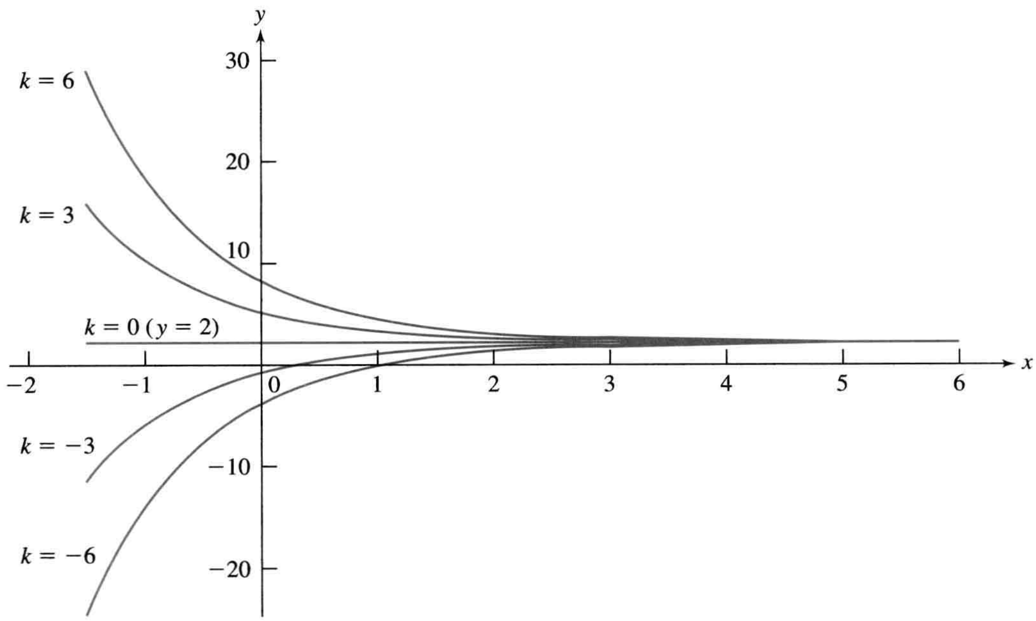

FIGURE 1.1 *Integral curves of $y' + y = 2$ for $k = 0, 3, -3, 6$, and -6.*

EXAMPLE 1.2

It is routine to verify that the general solution of

$$y' + \frac{y}{x} = e^x$$

is

$$y = \frac{1}{x}\left(xe^x - e^x + c\right)$$