Ideas An Introduction Second Edition

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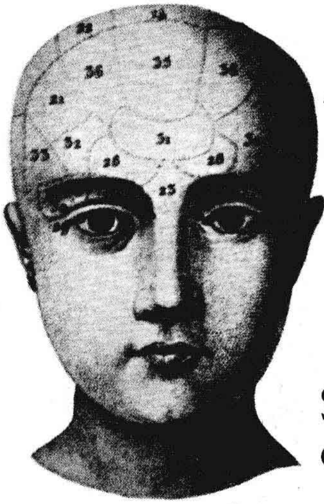

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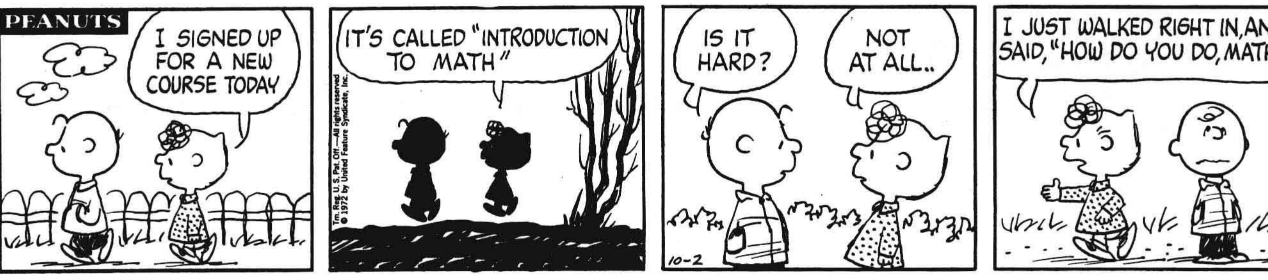

PEANUTS
I SIGNED UP FOR A NEW COURSE TODAY
IT'S CALLED "INTRODUCTION TO MATH"
IS IT HARD?
NOT AT ALL..
I JUST WALKED RIGHT IN, AN
SAID, "HOW DO YOU DO, MAT

Preface

The first edition of this text was very flexible—as shown by the large variety of courses for which it was used. Schools used the text variously in courses introducing the main ideas of contemporary mathematics to the non-physical science student, courses designed for prospective and in-service elementary school teachers, and courses for social science and business administration majors.

This second edition is even more flexible. This book is designed so that a good solid course can be given to students who have no previous mathematics past arithmetic, although a course in Beginning Algebra helps to insure success with the types of reasoning utilized.

We have made several substantial changes in this second edition. Probably the most obvious change is the new design and format. The new design was carefully thought out so that the text will be easier to read. At the same time, we have tried to make the format more open and appealing, so that the student will want to read the text. All mathematical figures have been redrawn, and a second color is often used for distinction and emphasis. Much illustrative material has been included from several broad areas of mathematical culture. Mathematics is fascinating, lively, and exciting, and we hope that the combination of text and illustrations will help convince the student of this.

Content has been revised in line with the many suggestions we received from users of the first edition. Chapter 2, sets, has a new discussion on sets of numbers. Chapter 3, mathematical systems, has been rearranged somewhat. Chapter 4, logic, has a new section on switching circuits. Chapter 5, now titled "Relations and Functions," includes a complete treatment of these important topics. Chapter 6, probability, contains an improved section dealing with simultaneous and successive events, stressing the appropriate use of additive and multiplicative principles in computing probabilities. Chapter 7, statistics, includes some new material on the normal approximation to the binomial distribution, which is used to clarify and reinforce the section on hypothesis testing. Chapter 8, matrices, has been extensively simplified and rewritten. Chapter 9, computers, is now oriented towards FORTRAN with a good discussion of BASIC. Chapter 10 is completely new in the text—it offers an introduction to transformational geometry, which is a fascinating new way to learn the ideas of Euclidean geometry.

Too many introductions to transformational geometry present it only at an abstract and advanced level; we feel the approach here is accessible to students who have had only a brief introduction to high school geometry.

We feel that the exercises in a mathematics textbook are just as important as the discussions. Several hundred of the problems of the first edition have been changed and improved for this second edition. The problems in most sets range from those that reinforce the new definitions and concepts to some that are fairly challenging and thought-provoking. We present many worked examples which illustrate and clarify the points discussed. These examples are set off in the text so that they can be readily identified.

One thing that has not been changed for this edition is the great flexibility of the text for classroom use. We provide a large selection of topics that give the instructor considerable latitude in structuring courses. This text can be used selectively for quite a variety of semester or quarter courses, and the entire text can be used for a full year course. A typical course for liberal arts students would include all or most of Chapters 1, 2, 3, 4, 6, and 7. A Course for elementary school teachers could emphasize Chapters 1, 2, 3, 4, 5, and 10; while social science and business majors would benefit from a study of Chapters 2, 5, 6, 7, 8, and 9.

We have been extremely gratified by the success of the first edition of this book. We tried to write a text that would attract students and make them want to study mathematics. Judging from the response we have received from instructors and students alike, we have succeeded. The users of this text have repeatedly noted the strong points of the first edition. They especially like the many applications presented; there are even more in this edition. Also frequently mentioned were the multitude of problems, the clear explanations, and the large variety of interesting topics presented.

Many instructors and students have helped us prepare this revision. We received considerable help from Chris Siragusa, American River College, Margaret Lial, also of American River College, made many useful suggestions, especially in the chapter on matrices. Many of the preliminary sketches for figures were drawn by Mary Miller.

Charles D. Miller
Vern E. Heeren

Finnegans Wake (1939) by James Joyce, from Part II

Problem ye ferst, construct ann aquilittoral dryankle Probe loom! With his primal handstoe in his sole salivarium. Concoct an equoangular trillitter.[1] On the name of the tizzer and off the tongs and off the mythametical tripods. Beatsoon.

The boss's bess bass is the browd of Mullingar.

INGENIOUS LABOUR-TENACITY AS BETWEEN INGENUOUS AND LIBERTINE.

Can you nei do her, numb? asks Dolph,[2] suspecting the answer know. Oikkont, ken you, ninny? asks Kev,[3] expecting the answer guess.[4] Nor was the noer long disappointed for easiest of kisshams, he was made vicewise. Oc, tell it to oui, do, Sem! Well, 'tis oil thusly. First mull a mugfull of mud, son.[5] Oglores, the virtuoser prays, olorum! What the D.V. would I do that for? That's a goosey's ganswer you're for giving me, he is told, what the Deva would you do that for?[6] Now, sknow royol road to Puddlin, take your mut for a first beginning, big to bog, back to bach. Anny liffle mud which cometh out of Mam will doob, I guess. A.I. *Amnium instar*. And to find a locus for an alp get a howlth on her bayrings as a prisme O and for a second O unbox your compasses. I cain but are you able? Amicably nod. Gu it! So let's seth off betwain us. Prompty? Mux your pistany at a point of the coastmap to be called *a* but pronounced olfa. There's the isle of Mun, ah! O! Tis just. *Bene*! Now, whole in applepine odrer.[7]

PROPE AND PROCUL IN THE CONVERGENCE OF THEIR CONTRAPULSIVENESS.

The aliments of jumeantry.

Wolsherwomens at their weirdst.

[1] As Rhombulus and Rhebus went building rhomes one day.
[2] The trouveller.
[3] Of the disorded visage.
[4] Singlebarrelled names for doubleparalleled twixtytwins.
[5] Like pudging a spoon fist of sugans into a sotspot of choucolout.
[6] Will you walk into my wavetrap? said the spiter to the shy.
[7] If we each could always do all we ever did.

Chapter 1 Numeration Methods

Contents

Chapter 4

Symbolic Logic

Chapter 5

Relations and Functions

Chapter 6

Probability

Chapter 10

Transformational Geometry

The idea of number is the oldest and most central of all mathematical concepts. It must have been very early in the evolution of man that the need first arose for a method of dealing with and using numbers. Casual observation as well as systematic study by psychologists and other scientists reveal that many animals have the ability to distinguish between different numbers of objects. In lower forms of animal life, however, this innate ability is apparently limited to numbers rarely greater than three or four. Indeed, this limitation is noted even in some existing primitive human societies, where the language contains no words for numerical designation other than perhaps *one, two, three,* and *many*.

It is a tribute to man's unique intellectual capacity that he alone has been able to overcome such conceptual limitations. Early man managed to effectively extend his meager number perception by using the process of *matching* to aid in counting objects. For example, a collection of pebbles might have been matched, one for one, with a group of animals and then, by referring to the pebbles, one could recall and communicate the *number* of animals in the group. All such numbers, including one, two, three, four, etc. are called **counting numbers.** A counting number most often indicates "how many" objects are contained in a particular collection, in which case we refer to it as the **cardinal number** of that collection. If there were twelve animals in the group mentioned above, we would say that the group's cardinal number (or its cardinality) is twelve. In Chapter 2 we will discuss cardinal numbers of sets in general.

All civilizations throughout history have developed, at least to some extent, ways of using the idea of a cardinal number. Furthermore, it seems that a second numerical concept has almost always accompanied that of cardinal number, namely that of **ordinal number.** As the name implies, this type of number refers to the order, or arrangement, of objects in a collection rather than to the number of objects in the collection. For example, suppose there are eight teams in a certain football league. At the end of the season, these teams

are ranked according to the ordinal numbers *first, second, third,* etc. As a further illustration, consider the days of the week. Tuesday is the *third* (ordinal number) day of the week, Thursday is the *fifth* (ordinal); whereas the number of days whose names begin with the letter T is *two* (cardinal).

Cardinal and ordinal numbers can usually be distinguished by the form of their names in the language. For example, first, second, third, ninth, and forty-sixth indicate order as opposed to one, two, three, nine, and forty-six which indicate "how many." In many non-English languages, cardinal and ordinal numbers are commonly indicated with different forms of the same base word. Nevertheless, one should take care not to depend entirely upon this fact to identify cardinal and ordinal numbers. If we say that our football team is number *one* in the league or that this is paragraph *four* of this chapter, we are really using ordinal numbers even though we use the terms *one* and *four* rather than *first* and *fourth*.

Let's return to the development of counting numbers. As we will see, necessity motivated the succeeding steps. After primitive man had learned the process of matching pebbles with animals, he discovered that he must devise a more efficient method to record large numbers. Suppose a primitive hunter encounters a herd of two hundred forty buffalo. Realizing that a definite count of heads (or tails) would be advantageous in convincing his fellow tribesmen of the herd's size, he begins gathering pebbles. In a short time though, not only is the collection becoming too heavy to carry, but the hunter starts losing count of the buffalos while in the process of searching for pebbles. The most probable solution is to find one long stick and one sharp stone and begin making a series of scratches on the stick, one for each buffalo. Only two easy-to-handle objects are required, and the scratches can be easily made without losing count, and the result, a sort of "tally stick," is easy for the hunter to carry back and show his friends.

With this development, man has progressed to the stage of *symbolizing* numbers for the sake of efficiency. Although this is still a process of matching (marks with buffalos), the final result (a series of marks) is now a more symbolic and less tangible representation of a number. This process of counting by making a mark for each object in the collection is commonly called **tallying.** For smaller numbers the result might be something like |||| or |||||||.

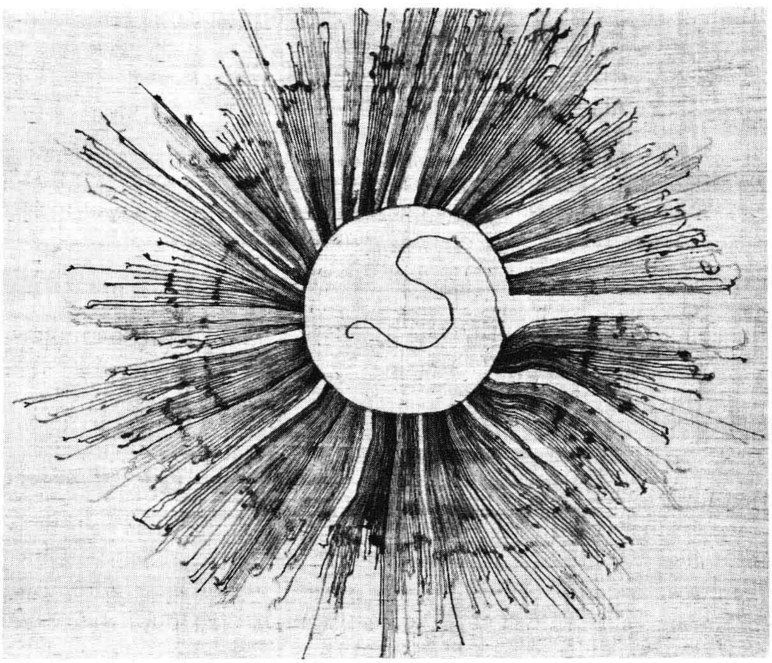
Notice that |||| is a *symbol* which designates the number four. Likewise, ||||||| denotes seven. The symbol |||| is not a number; it only denotes or represents a number. Today we commonly indicate that same number with the symbol 4. Many other symbols have been used by various people to denote this same number, for example, IV, ····, and ⊟. Any symbol that denotes a number is called a **numeral.** Hence there are many different numerals for the number four. The number itself is only an idea conceived in the minds of men. It cannot be seen, touched, or felt. The thing with which we deal directly is the numeral—a written symbol for the number.

In most cases, we need not be unduly concerned with the theoretical distinction between numbers and numerals. In fact, most often we use and speak of numerals as if they were actually numbers. Since we are accustomed to the modern Hindu-Arabic numerals 1, 2, 3, 4, 5, etc., we henceforth use them freely when referring to numbers. We should thoroughly understand, however, that these numerals provide just one of many ways of symbolizing counting numbers. Ancient Greek, Roman, Babylonian, Chinese, Egyptian, and Mayan methods of numeration, as well as others, are equally valid representations of the exact same concept, that of cardinal number.

Number	Numeral
one	|
two	||
three	|||
four	||||
five	|||||
six	||||||
etc.	etc.

Figure 1.1

Let's again consider the hypothetical method of numeration being developed. What we have so far is a set of numerals for representing cardinal numbers according to the scheme in Figure 1.1. This method would enable us to symbolize the cardinal number of any conceivable herd of dinosaurs (or of any other collection of objects, for that matter). Obviously though, the numerals for large numbers (such as two hundred forty) would be rather awkward. Even a fairly small number like twenty-five would be written |||||||||||||||| |||||||||. Besides being troublesome to write, it is rather difficult to interpret. Clearly, there must be a better way. The first section of this chapter presents a series of modifications of this method which result in increasingly more efficient approaches to the symbolizing of numbers.



Tally stick (far left) from England, about 1400, issued as a tax receipt. Each notch stands for 1 pound sterling. Left, knotted cords forming a "quipu," used by Peruvian Indians for census. Larger knots are multiples of smaller; cord color indicates male or female.

1.1 THREE METHODS OF NUMERATION

The single idea that enables us to improve the tallying notation is that of **grouping.** Instead of merely displaying one slash for each item in a set, we devise a system of numeration symbols which combines entire groups of slashes and replaces them with other kinds of symbols.

A simple grouping method

How many squares are in the collection shown in Figure 1.2? Using the tallying method of numeration, we would answer that there are ||||||||||||||||||| squares. To utilize the concept of grouping, and thus save space in our notation, we will introduce a new symbol, Λ, for the number five and divide the set of squares into groups of five,

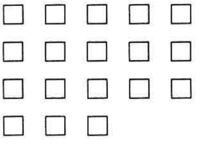


Figure 1.2

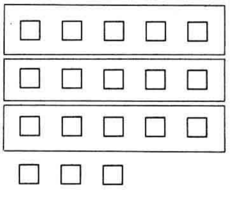


Figure 1.3

as in Figure 1.3. There are three groups of five with three squares left over: ΛΛΛ|||. This technique is clearly a space-saver among other things. The number 14 can now be written ΛΛ|||| instead of ||||||||||||||; 23 becomes ΛΛΛΛ|||. Notice that if we had chosen to make use of groups of seven instead of five, the number 23 would be written ΛΛΛ|| (three groups of seven with two left over); and for groupings of nine, we would write 23 as ΛΛ||||| (two groups of nine and five one's). The size of the groups is arbitrary. In each case the result is a more efficient numeration method than was the tallying technique. This **simple grouping method** constitutes the first step in the search for ease and efficiency. The size of the basic groups is commonly referred to as the **base.** Hence if we decide on groups of five, we call the system "base 5." If the group size is seven, we are using a "base 7" method.

Let's examine the base 5 simple grouping method a little further. Numerals for a few selected numbers are given in Figure 1.4; hopefully, the reader can supply the appropriate numeral for any number not in the table.

Simple Grouping—Base 5

Number	Numeral
1	|
2	||
3	|||
4	||||
5	Λ
6	Λ|
10	ΛΛ
15	ΛΛΛ
20	ΛΛΛΛ
23	ΛΛΛΛ|||

Figure 1.4

Suppose we wanted to write a base 5 numeral for the number 67. Would it be thirteen groups of five and two one's like this?

ΛΛΛΛΛΛΛΛΛΛΛΛΛ||

Again, it seems as if the grouping technique should enable us to simplify this long string of symbols. The introduction of the symbol Λ meant that the symbol | would never appear more than four times for a given number. Now let's introduce the new symbol N, which represents a group of five Λ's, that is, a group of twenty-five. This means the symbol Λ will never be needed more than four times either. (See Figure 1.5.)

Simple Grouping—Base 5

Number	Numeral
1	|
2	||
5	Λ
6	Λ|
10	ΛΛ
15	ΛΛΛ
20	ΛΛΛΛ
24	ΛΛΛΛ||||
25	N
26	N|
30	NΛ
31	NΛ|
35	NΛΛ
50	NN
51	NN|
55	NNΛ

Figure 1.5

Since our base is 5, the three symbols |, Λ, and N are sufficient up to the number NNNNΛΛΛΛ|||| (that is, 124). We then introduce a new symbol, M, for five N's. Hence 125 is written M, 126 is M|, 130 is MΛ, 150 is MN, 155 is MNΛ, and so forth. The next symbol, W, is necessary to represent the number 625 (that is, five

M's). Every number has a numeral in which no given symbol is used more than four times. The seven symbols shown in Figure 1.6 are sufficient for representing every cardinal number up to 78,124. What would be needed to represent 78,125?

Number		Symbol
1		|
5		Λ
25	(=5 · 5)*	N
125	(=5 · 5 · 5)	M
625	(=5 · 5 · 5 · 5)	MV
3125	(=5 · 5 · 5 · 5 · 5)	MM
15,625	(=5 · 5 · 5 · 5 · 5 · 5)	MW

*Note: the dots here indicate multiplication.

Figure 1.6

EXAMPLE 1

What counting number is denoted by MV MV M N N N Λ||||?

Solution

We see that MV is used twice, which gives 2 · 625 = 1250. The one M gives 125; the three N's give 3 · 25, or 75; the Λ gives 5; and the four |'s give 4 · 1 or 4. Adding the contributions from the five different symbols used, we have 1250 + 125 + 75 + 5 + 4 = 1459. Hence the base 5 simple grouping numeral MV MV M N N N Λ|||| denotes the number 1459. This computation can be illustrated as follows:

MV MV:	2 · 625 =	1250
M:	1 · 125 =	125
N N N:	3 · 25 =	75
Λ:	1 · 5 =	5
||||:	4 · 1 =	4
		1459

North American Indian number systems were mostly base 10 or base 5. Above, survivor of the Red Lake Chippewas (detail). Below, chronicles of a Sioux warrior painted on his buffalo robe, mid 19th century.