

# Mechanics of Materials

Fifth  
Edition

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WILLIAM F. RILEY  
LEROY D. STURGES  
DON H. MORRIS

# *Mechanics of Materials*

## *Fifth Edition*

**WILLIAM F. RILEY**

*Distinguished Professor Emeritus  
Iowa State University*

**LEROY D. STURGES**

*Associate Professor  
Aerospace Engineering and Engineering Mechanics  
Iowa State University*

**DON H. MORRIS**

*Professor  
Engineering Science and Mechanics  
Virginia Polytechnic Institute and State University*



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# AVERAGE PROPERTIES OF SELECTED ENGINEERING MATERIALS (ENGLISH SYSTEM OF UNITS)

Exact values may vary widely with changes in composition, heat treatment, and mechanical working. More precise information can be obtained from manufacturers.

Materials	Spec- ific Weight (lb/in. <sup>3</sup> )	Elastic Strength <sup>a</sup>			Ultimate Strength			Endur- ance Limit <sup>c</sup> (ksi)	Modu- lus of		Percent Elonga- tion in 2 in.	Coeffi- cient of Thermal Expan- sion (10 <sup>-6</sup> /°F)
		Ten- sion (ksi)	Comp. (ksi)	Shear (ksi)	Ten- sion (ksi)	Comp. (ksi)	Shear (ksi)		Elas- ticity (1000 ksi)	Rigid- ity (1000 ksi)		
Ferrous metals												
Wrought iron	0.278	30	<i>b</i>	<i>b</i>	48	<i>b</i>	25	23	28		30 <sup>d</sup>	6.7
Structural steel	0.284	36	<i>b</i>	<i>b</i>	66	<i>b</i>	<i>b</i>	28	29	11.0	28 <sup>d</sup>	6.6
Steel, 0.2% C hardened	0.284	62	<i>b</i>	<i>b</i>	90	<i>b</i>	<i>b</i>		30	11.6	22	6.6
Steel, 0.4% C hot-rolled	0.284	53	<i>b</i>	<i>b</i>	84	<i>b</i>	<i>b</i>	38	30	11.6	29	
Steel, 0.8% C hot-rolled	0.284	76	<i>b</i>	<i>b</i>	122	<i>b</i>	<i>b</i>		30	11.6	8	
Cast iron—gray	0.260				25	100		12	15		0.5	6.7
Cast iron—malleable	0.266	32	<i>b</i>	<i>b</i>	50	<i>b</i>	<i>b</i>		25		20	6.6
Cast iron—nodular	0.266	70			100				25		4	6.6
Stainless steel (18-8) annealed	0.286	36	<i>b</i>	<i>b</i>	85	<i>b</i>	<i>b</i>	40	28	12.5	55	9.6
Stainless steel (18-8) cold-rolled	0.286	165	<i>b</i>	<i>b</i>	190	<i>b</i>	<i>b</i>	90	28	12.5	8	9.6
Steel, SAE 4340, heat-treated	0.283	132	145		150	<i>b</i>	95	76	29	11.0	19	
Nonferrous metal alloys												
Aluminum, cast, 195-T6	0.100	24	25		36		30	7	10.3	3.8	5	
Aluminum, wrought, 2014-T4	0.101	41	41	24	62	<i>b</i>	38	18	10.6	4.0	20	12.5
Aluminum, wrought, 2024-T4	0.100	48	48	28	68	<i>b</i>	41	18	10.6	4.0	19	12.5
Aluminum, wrought, 6061-T6	0.098	40	40	26	45	<i>b</i>	30	13.5	10.0	3.8	17	12.5

Magnesium, extrusion, AZ80X	0.066	35	26	49	<sup>b</sup>	21	19	6.5	2.4	12	14.4
Magnesium, sand cast, AZ63-HT	0.066	14	14	40	<sup>b</sup>	19	14	6.5	2.4	12	14.4
Monel, wrought, hot-rolled	0.319	50	<sup>b</sup>	90	<sup>b</sup>		40	26	9.5	35	7.8
Red brass, cold-rolled	0.316	60	<sup>b</sup>	75	<sup>b</sup>			15	5.6	4	9.8
Red brass, annealed	0.316	15	<sup>b</sup>	40	<sup>b</sup>			15	5.6	50	9.8
Bronze, cold-rolled	0.320	75	<sup>b</sup>	100	<sup>b</sup>			15	6.5	3	9.4
Bronze, annealed	0.320	20	<sup>b</sup>	50	<sup>b</sup>			15	6.5	50	9.4
Titanium alloy, annealed	0.167	135	<sup>b</sup>	155	<sup>b</sup>			14	5.3	13	
Invar, annealed	0.292	42	<sup>b</sup>	70	<sup>b</sup>			21	8.1	41	0.6
Nonmetallic materials											
Douglas fir, green <sup>c</sup>	0.022	4.8	3.4		3.9	0.9		1.6			
Douglas fir, air dry <sup>c</sup>	0.020	8.1	6.4		7.4	1.1		1.9			
Red oak, green <sup>c</sup>	0.037	4.4	2.6		3.5	1.2		1.4			1.9
Red oak, air dry <sup>c</sup>	0.025	8.4	4.6		6.9	1.8		1.8			
Concrete, medium strength	0.087		1.2		3.0			3.0			6.0
Concrete, fairly high strength	0.087		2.0		5.0			4.5			6.0

<sup>a</sup> Elastic strength may be represented by proportional limit, yield point, or yield strength at a specified offset (usually 0.2 percent for ductile metals).

<sup>b</sup> For ductile metals (those with an appreciable ultimate elongation), it is customary to assume the properties in compression have the same values as those in tension.

<sup>c</sup> Rotating beam.

<sup>d</sup> Elongation in 8 in.

<sup>e</sup> All timber properties are parallel to the grain.

# AVERAGE PROPERTIES OF SELECTED ENGINEERING MATERIALS (INTERNATIONAL SYSTEM OF UNITS)

Exact values may vary widely with changes in composition, heat treatment, and mechanical working. More precise information can be obtained from manufacturers.

Materials	Elastic Strength <sup>a</sup>			Ultimate Strength		Endurance Limit <sup>c</sup> (MPa)	Modulus of Elasticity (GPa)	Modulus of Rigidity (GPa)	Percent Elongation in 50 mm	Coefficient of Thermal Expansion (10 <sup>-6</sup> /°C)	
	Density (Mg/m <sup>3</sup> )	Tension (MPa)	Comp. (MPa)	Shear (MPa)	Tension (MPa)						Comp. (MPa)
Ferrous metals											
Wrought iron	7.70	210	<i>b</i>		330	<i>b</i>	160	190		30 <sup>d</sup>	12.1
Structural steel	7.87	250	<i>b</i>		450	<i>b</i>	190	200	76	28 <sup>d</sup>	11.9
Steel, 0.2% C hardened	7.87	430	<i>b</i>		620	<i>b</i>		210	80	22	11.9
Steel, 0.4% C hot-rolled	7.87	360	<i>b</i>		580	<i>b</i>	260	210	80	29	
Steel, 0.8% C hot-rolled	7.87	520	<i>b</i>		840	<i>b</i>		210	80	8	
Cast iron—gray	7.20				170	690	80	100		0.5	12.1
Cast iron—malleable	7.37	220	<i>b</i>		340	<i>b</i>		170		20	11.9
Cast iron—nodular	7.37	480			690			170		4	11.9
Stainless steel (18-8) annealed	7.92	250	<i>b</i>		590	<i>b</i>	270	190	86	55	17.3
Stainless steel (18-8) cold-rolled	7.92	1140	<i>b</i>		1310	<i>b</i>	620	190	86	8	17.3
Steel, SAE 4340, heat-treated	7.84	910	1000		1030	<i>b</i>	520	200	76	19	
Nonferrous metal alloys											
Aluminum, cast 195-T6	2.77	160	170		250		50	71	26	5	
Aluminum, wrought, 2014-T4	2.80	280	280	160	430	<i>b</i>	120	73	28	20	22.5
Aluminum, wrought, 2024-T4	2.77	330	330	190	470	<i>b</i>	120	73	28	19	22.5
Aluminum, wrought, 6061-T6	2.71	270	270	180	310	<i>b</i>	93	70	26	17	22.5



Magnesium, extrusion, AZ80X	1.83	240	180	340	<sup>b</sup>	140	130	45	16	12	25.9
Magnesium, sand cast, AZ63-HT	1.83	100	96	270	<sup>b</sup>	130	100	45	16	12	25.9
Monel, wrought, hot-rolled	8.84	340	<sup>b</sup>	620	<sup>b</sup>		270	180	65	35	14.0
Red brass, cold-rolled	8.75	410		520				100	39	4	17.6
Red brass, annealed	8.75	100	<sup>b</sup>	270	<sup>b</sup>			100	39	50	17.6
Bronze, cold-rolled	8.86	520		690				100	45	3	16.9
Bronze, annealed	8.86	140	<sup>b</sup>	340	<sup>b</sup>			100	45	50	16.9
Titanium alloy, annealed	4.63	930	<sup>b</sup>	1070	<sup>b</sup>			96	36	13	
Invar, annealed	8.09	290	<sup>b</sup>	480	<sup>b</sup>			140	56	41	1.1
Nonmetallic materials											
Douglas fir, green <sup>c</sup>	0.61	33	23		27	6.2		11			
Douglas fir, air dry <sup>c</sup>	0.55	56	44		51	7.6		13			
Red oak, green <sup>c</sup>	1.02	30	18		24	8.3		10			3.4
Red oak, air dry <sup>c</sup>	0.69	58	32		48	12.4		12			
Concrete, medium strength	2.41		8		21			21			10.8
Concrete, fairly high strength	2.41		14		34			31			10.8

<sup>a</sup> Elastic strength may be represented by proportional limit, yield point, or yield strength at a specified offset (usually 0.2 percent for ductile metals).

<sup>b</sup> For ductile metals (those with an appreciable ultimate elongation), it is customary to assume the properties in compression have the same values as those in tension.

<sup>c</sup> Rotating beam.

<sup>d</sup> Elongation in 200 mm.

<sup>e</sup> All timber properties are parallel to the grain.

# *Mechanics of Materials*



# Preface to the Instructor

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## INTRODUCTION

The primary objectives of a course in mechanics of materials are: (1) to develop a working knowledge of the relations between the loads applied to a nonrigid body made of a given material and the resulting deformations of the body; (2) to develop a thorough understanding of the relations between the loads applied to a nonrigid body and the stresses produced in the body; (3) to develop a clear insight into the relations between stress and strain for a wide variety of conditions and materials; and (4) to develop adequate procedures for finding the required dimensions of a member of a specified material to carry a given load subject to stated specifications of stress and deflection. These objectives involve the concepts and skills that form the foundation of all structural and machine design.

The principles and methods used to meet the general objectives are drawn largely from prerequisite courses in mechanics, physics, and mathematics together with the basic concepts of the theory of elasticity and the properties of engineering materials. This book is designed to emphasize the required fundamental principles, with numerous applications to demonstrate and develop logical orderly methods of procedure. Instead of deriving numerous formulas for all types of problems, we have stressed the use of free-body diagrams and the equations of equilibrium, together with the geometry of the deformed body, and the observed relations between stress and strain, for the analysis of the force system acting on a body.

This book is designed for a first course in mechanics of deformable bodies. Because of the extensive subdivision into different topics, the book will provide flexibility in the choice of assignments to cover courses of different length and content. The developments of structural applications include the inelastic as well as the elastic range of stress; however, the material is organized so that the book will be found satisfactory for elastic coverage only.

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## ORGANIZATION OF THE TEXT

Since most mechanics of materials problems begin with a statics problem (finding the forces in structural members or the forces in pins connecting structural members), we have included a review of statics in Chapter 1 of this book. The coverage is perhaps more complete and comprehen-

sive than would be necessary for review so that the book could be used for both statics and mechanics of materials if desired.

After the review of statics in Chapter 1, Chapters 2 and 3 consist of a thorough discussion of material stress and strain including principal stresses and principal strains. We choose to present principal stresses and principal strains at this early position to make it easier to talk about maximum stresses in the axial, torsional, and flexural applications that follow. It also allows us to talk about the maximum stresses in combined loading situations in Chapters 5, 6, and 7, rather than waiting until the end of the book.

Material properties and the relationship between stress and strain are presented in Chapter 4. In Chapters 5, 6, and 7, we consider the stresses and strains in axial, torsional, and flexural loading applications. In addition to calculating the stresses in members subjected to axial loading and the stresses in pressure vessels subjected to internal pressure, in Chapter 5 we also calculate the stresses in pressure vessels subjected to an axial load. In addition to calculating the stresses in circular shafts subjected to torsional loading, in Chapter 6 we also calculate the stresses in circular shafts subjected to axial loads and in pressure vessels subjected to torsional loads. In Chapter 7 we first calculate the normal and shear stresses in beams subjected to flexural loading. We conclude Chapter 7 with the calculation of stresses in circular shafts subjected to a combination of axial, torsional, and flexural loads.

In Chapter 8 we calculate the deflection of beams due to various loading situations and also cover the calculation of support reactions for and stresses in statically indeterminate beams. In Chapter 9 we consider the tendency of columns to buckle. Finally, in Chapter 10 we discuss theories of failure and the use of energy methods.

Every chapter opens with a brief Introduction and ends with a Summary of important concepts covered in the chapter followed by a set of Review Problems. All principles are illustrated by one or more Example Problems and several Homework Problems. The Homework Problems are graded in difficulty and are separated into groups of Introductory, Intermediate, and Challenging problems. Several sections of Homework Problems also have a set of Computer Problems. While the computation could be accomplished by the student writing a FORTRAN program, the computation could just as easily be carried out using MathCAD, Mathematica, or a spreadsheet program. The important concept of the Com-

puter problems is that they require students to analyze how the solution depends on some parameter of the problem.

Most chapters conclude with a section on Design which includes Example Problems and a set of Homework Problems. The emphasis in these problems is that there are often more than just one criteria to be satisfied in a design specification. An acceptable design must satisfy all specified criteria. In addition, standard lumber, pipes, beams, etc. come in specific sizes. The student must choose an appropriate structural member from these standard materials. Since each different choice of a beam or a piece of lumber has a different specific weight and affects the overall problem differently, students are also introduced to the idea that design is an iterative process.

---

## FREE-BODY DIAGRAMS

We strongly feel that a proper free-body diagram is just as important in mechanics of materials as it is in statics. It is our approach that, whenever an equation of equilibrium is written, it must be accompanied by a complete, proper free-body diagram. Furthermore, since the primary purpose of a free-body diagram is to show the forces acting on a body, the free-body diagram should not be used for any other purpose. We encourage students to draw separate diagrams to show deformation and compatibility relationships.

---

## PROBLEMS-SOLVING PROCEDURES

Students are urged to develop the ability to reduce problems to a series of simpler component problems that can be easily analyzed and combined to give the solution of the initial problem. Along with an effective methodology for problem decomposition and solution, the ability to present results in a clear, logical, and neat manner is emphasized throughout the text.

---

## HOMEWORK PROBLEMS

The illustrative examples and problems have been selected with special attention devoted to problems that require an understanding of the principles of mechanics of materials without demanding excessive time for computational work. A large number of homework problems are included so that problem assignments may be varied from term to term. The problems in each set represent a considerable range of diffi-

culty and are grouped according to this range of difficulty. Mastery, in general, is not achieved by solving a large number of simple but similar problems. While the solution of simple problems is necessary to build a student's problem solving skills and confidence, we believe that a student gains mastery of a subject through application of basic theory to the solution of problems that appear somewhat difficult.

---

## SI VERSUS USCS UNITS

U.S. customary units and SI units are used in approximately equal proportions in the text for both Example Problems and Homework Problems. To help the instructor who wants to assign problems of one type or the other, odd-numbered Homework Problems are in U.S. customary units and even-numbered Homework Problems are in SI units.

---

## ANSWERS PROVIDED

Answers to about half of the Homework Problems are included at the end of this book. Since the convenient designation of problems for which answers are provided is of great value to those who make up assignment sheets, the problems for which answers are provided are indicated by means of an asterisk (\*) after the problem number.

---

## ACKNOWLEDGMENTS

We are grateful for comments and suggestions received from colleagues and from users of the earlier editions of this book. Special thanks go to the following people who provided input and comments: Subhash Anand, Clemson U.; Walid Thabel, Union College; Douglas Winslow, Purdue U.; Daniel R. White, U. Missouri—Rolla; Mahera S. Philobos, GA Tech.; and John B. Ligon, Michigan Tech. U. Final judgments concerning organization of material and emphasis of topics, however, were made by the authors. We will be pleased to receive comments from readers and will attempt to acknowledge all such communications. Comments can be sent by email to sturges @ iastate.edu or to dhmorris @ vt.edu.

**William F. Riley**  
**Leroy D. Sturges**  
**Don H. Morris**

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# **Preface to the Student**

---

## **INTRODUCTION**

The primary objectives of a course in mechanics of materials are: (1) to develop a working knowledge of the relations between the loads applied to a nonrigid body made of a given material and the resulting deformations of the body; (2) to develop a thorough understanding of the relations between the loads applied to a nonrigid body and the stresses produced in the body; (3) to develop a clear insight into the relations between stress and strain for a wide variety of conditions and materials; and (4) to develop adequate procedures for finding the required dimensions of a member of a specified material to carry a given load subject to stated specifications of stress and deflection. These objectives involve the concepts and skills that form the foundation of all structural and machine design.

The principles and methods used to meet the general objectives are drawn largely from prerequisite courses in mechanics, physics, and mathematics together with the basic concepts of the theory of elasticity and the properties of engineering materials. This book is designed to emphasize the required fundamental principles, with numerous applications to demonstrate and develop logical orderly methods of procedure. Instead of deriving numerous formulas for all types of problems, we have stressed the use of free-body diagrams and the equations of equilibrium, together with the geometry of the deformed body, and the observed relations between stress and strain, for the analysis of the force system acting on a body.

Extensive use has been made of prerequisite course material in statics and calculus. A working knowledge of these subjects is considered an essential prerequisite to a successful study of mechanics of materials as presented in this book.

---

## **FREE-BODY DIAGRAMS**

Most engineers consider the free-body diagram to be the single most important tool for the solution of mechanics problems. The free-body diagram is just as important in mechanics of materials as it is in statics. It is our approach that, whenever an equation of equilibrium is written, it must be accompanied by a complete, proper free-body diagram.

In many problems, it will also be necessary to draw deformation diagrams and/or compatibility diagrams. Since the

primary purpose of a free-body diagram is to show the forces acting on a body, the free-body diagram should not be used for any other purpose. Separate diagrams should be drawn to show deformation and compatibility relationships.

---

## **PROBLEM-SOLVING PROCEDURES**

Success in engineering mechanics courses depends, to a surprisingly large degree, on a well-disciplined method of problem solving and on the solution of a large number of problems. Students are urged to develop the ability to reduce problems to a series of simpler component problems that can be easily analyzed and combined to give the solution of the initial problem. Along with an effective methodology for problem decomposition and solution, the ability to present results in a clear, logical, and neat manner is emphasized throughout the text. A first course in mechanics is an excellent place to begin development of this disciplined approach which is so necessary in most engineering work.

---

## **HOMEWORK PROBLEMS**

The illustrative examples and problems have been selected with special attention devoted to problems that require an understanding of the principles of mechanics of materials without demanding excessive time for computational work. The problems in each set represent a considerable range of difficulty and are grouped according to this range of difficulty. Mastery, in general, is not achieved by solving a large number of simple but similar problems. While the solution of simple problems is necessary to build a student's problem-solving skills and confidence, we believe that a student gains mastery of a subject through application of basic theory to the solution of problems that appear somewhat difficult.

---

## **SIGNIFICANT FIGURES**

Results should always be reported as accurately as possible. However, results should not be reported to 10 significant figures merely because the calculator displays that many digits. One of the tasks in all engineering work is to

determine the accuracy of the given data and the expected accuracy of the final answer. Results should always reflect the accuracy of the given data.

In a textbook, however, it is not possible for students to examine or question the accuracy of the given data. It is also impractical, in an introductory course, to give error bounds on every number. Therefore, since an accuracy greater than about 0.2% is seldom possible for practical engineering problems, all given data in Example Problems and Homework Problems, regardless of the number of figures shown, will be assumed sufficiently accurate to justify rounding off the final answer to approximately this degree of accuracy (three to four significant figures).

---

## **SI VERSUS USCS UNITS**

Most large engineering companies deal in an international marketplace. In addition, the use of the International System of Units (SI) is gaining acceptance in the United States. As a result, most engineers must be proficient in both the SI system and the U.S. customary system (USCS) of units. In response to this need, both U.S. customary units and SI units are used in approximately equal proportions

in the text for both Example Problems and Homework Problems.

---

## **ANSWERS PROVIDED**

Answers to about half of the Homework Problems are included at the end of this book. The problems for which answers are provided are indicated by means of an asterisk (\*) after the problem number.

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## **ACKNOWLEDGMENTS**

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**William F. Riley**  
**Leroy D. Sturges**  
**Don H. Morris**

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# *Chapter 1*

## *Introduction and Review of Statics*

### **1-1 INTRODUCTION**

The primary objective of a course in mechanics of materials is the development of relationships between the loads applied to a nonrigid body and the internal forces and deformations induced in the body. Ever since the time of Galileo Galilei (1564–1642), scientists and engineers have studied the problem of the load-carrying capacity of structural members and machine components, and have developed mathematical and experimental methods of analysis for determining the internal forces and the deformations induced by the applied loads. The experiences and observations of these scientists and engineers of the last three centuries are the heritage of the engineer of today. The fundamental knowledge gained over the last three centuries, together with the theories and analysis techniques developed, permit the modern engineer to design, with complete competence and assurance, structures and machines of unprecedented size and complexity.

The subject matter of this book forms the basis for the solution of three general types of problems:

1. Given a certain function to perform (the transporting of traffic over a river by means of a bridge, conveying scientific instruments to Mars in a space vehicle, the conversion of water power into electric power), of what materials should the machine or structure be constructed, and what should be the sizes and proportions of the various elements? This is the designer's task, and obviously there is no single solution to any given problem.
2. Given the completed design, is it adequate? That is, does it perform the function economically and without excessive deformation? This is the checker's problem.
3. Given a completed structure or machine, what is its actual load-carrying capacity? The structure may have been designed for some purpose other than the one for which it is now to be used. Is it adequate for the proposed use? For example, a building may have been designed as an office building, but is later found to be desirable for use as a warehouse. In such a case, what maximum loading may the floor safely support? This is the rating problem.

Because the complete scope of these problems is obviously too comprehensive for mastery in a single course, this book is restricted to a study of individual members and very simple structures or machines. The design courses that follow will consider the entire structure or machine, and will provide essential background for the complete analysis of the three problems.



The principles and methods used to meet the objective stated at the beginning of this chapter depend to a great extent on prerequisite courses in mathematics and mechanics, supplemented by additional concepts from the theory of elasticity and the properties of engineering materials. The equations of equilibrium from statics are used extensively, with one major change in the free-body diagrams; namely, most free bodies are isolated by cutting through a member instead of simply removing a pin or some other connection. The internal force on the cut section is related to the stresses (force per unit area) generated by the cohesive forces holding the member together. The size and shape of the member must be adjusted to keep the stress below the limiting value for the type of material from which the member is constructed.

In some instances, the specified maximum deformation and not the specified maximum stress will govern the maximum load that a member may carry. In other instances, it may be found that the equations of equilibrium (or motion) are not sufficient to determine all of the unknown loads or reactions acting on a body. In such cases it is necessary to consider the geometry (the change in size or shape) of the body after the loads are applied. The deformation per unit length in any direction or dimension is called *strain*.

Some knowledge of the physical and mechanical properties of materials is required in order to create a design, to properly evaluate a given design, or even to write the correct relation between an applied load and the resulting deformation of a loaded member. Essential information will be introduced as required, and more complete information can be obtained from textbooks and handbooks on properties of materials.

## 1-2 CLASSIFICATION OF FORCES

Force is one of the most important of the basic concepts in the study of mechanics of materials (or the mechanics of deformable bodies). Force is the action of one body on another; forces always exist in equal magnitude, opposite direction pairs. Forces may result from direct physical contact between two bodies, or from two bodies that are not in direct contact. For example, consider a person standing on a sidewalk. The person exerts a force on the sidewalk through direct physical contact between the soles of his or her shoes and the sidewalk; the sidewalk in turn exerts an equal magnitude, opposite direction force on the soles of the person's shoes. If the person were to jump, the contact force would vanish but there would still be a gravitational attraction (force between two bodies not in direct contact) between the person and the earth. The gravitational attraction force exerted on the person by the earth is called the *weight* of the person; an equal magnitude, opposite direction, attraction force is exerted on the earth by the person. Another type of force that exists without direct physical contact is an electromagnetic force.

Contact forces are called *surface forces*, since they exist at surfaces of contact between two bodies. If the area of contact is small compared to the size of the body, the force is called a *concentrated force*; this type of force is assumed to act at a point. For example, the force applied by a car wheel to the pavement on a bridge (see Fig. 1-1) is often modeled as a concentrated force. Also, a contact force may be distributed over a narrow region in a uniform or nonuniform manner. This situation would exist where floor decking contacts a floor joist, as shown in Fig. 1-2a. Here, the floor decking exerts a uniformly distributed load

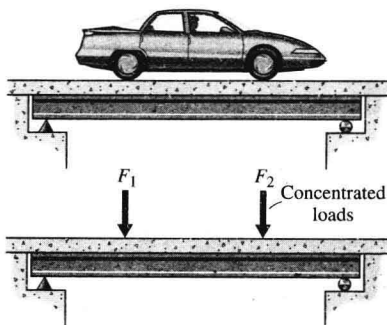


Figure 1-1

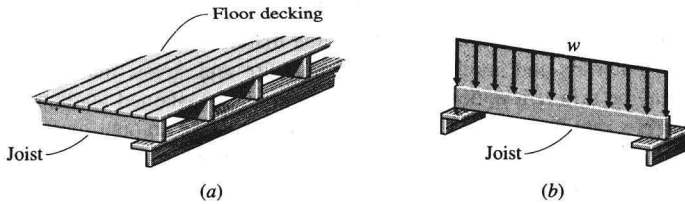


Figure 1-2

(force) on the joist, as shown in Fig. 1-2b. The intensity of the distributed load is  $w$  and has dimensions of force per unit length.

Other common types of forces are external, internal, applied, and reaction. To illustrate, consider the beam loaded and supported, as shown in Fig. 1-3a. A free-body diagram of the beam is shown in Fig. 1-3b. All forces acting on the free-body diagram are *external* forces; that is, they represent the interaction between the beam (the object shown in the free-body diagram) and the external world (everything else that has been discarded). Force  $F$  is a concentrated force, whereas  $w$  is a uniformly distributed load with dimensions of force/length. The forces  $F$  and  $w$  are called *applied* forces or loads. They are the forces that the beam is designed to carry. Forces  $A_x$ ,  $A_y$ , and  $B$  are necessary to prevent movement of the beam. Such supporting forces are called *reactions*. Force distributions at supports are complicated, and reactions are usually modeled as concentrated forces.

Once again, all the forces shown in Figure 1-3 are external forces. At every section along the beam, there also exists a system of equal magnitude, opposite direction, pairs of internal forces between the atoms on either side of the section. The study of mechanics of materials or mechanics of deformable bodies, depends on the calculation of these internal forces at various sections of a structure or machine element and how these forces are distributed over the sections. The determination of internal forces is discussed in Section 1-5.

In our previous discussion of loads (forces), we saw that the loads might be concentrated forces, or distributed forces. Furthermore, we assumed that the forces did not vary with time, that is, they were *static* loads. In many situations, loads may be a function of time. For example, a sustained load is a load that is

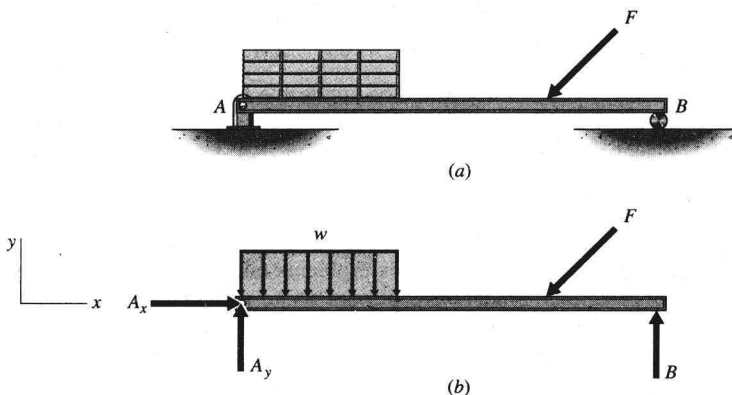


Figure 1-3