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METHODS FOR SOLVING INVERSE PROBLEMS IN MATHEMATICAL PHYSICS

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Preface

The theory of inverse problems for differential equations is being extensively developed within the framework of mathematical physics. In the study of the so-called direct problems the solution of a given differential equation or system of equations is realised by means of supplementary conditions, while in inverse problems the equation itself is also unknown. The determination of both the governing equation and its solution necessitates imposing more additional conditions than in related direct problems.

The sources of the theory of inverse problems may be found late in the 19th century or early 20th century. They include the problem of equilibrium figures for the rotating fluid, the kinematic problems in seismology, the inverse Sturm–Liuville problem and more. Newton's problem of discovering forces making planets move in accordance with Kepler's laws was one of the first inverse problems in dynamics of mechanical systems solved in the past. Inverse problems in potential theory in which it is required to determine the body's position, shape and density from available values of its potential have a geophysical origin. Inverse problems of electromagnetic exploration were caused by the necessity to elaborate the theory and methodology of electromagnetic fields in investigations of the internal structure of Earth's crust.

The influence of inverse problems of recovering mathematical physics equations, in which supplementary conditions help assign either the values of solutions for fixed values of some or other arguments or the values of certain functionals of a solution, began to spread to more and more branches as they gradually took on an important place in applied problems arising in "real-life" situations. From a classical point of view, the problems under consideration are, in general, ill-posed. A unified treatment and advanced theory of ill-posed and conditionally well-posed problems are connected with applications of various regularization methods to such problems in mathematical physics. In many cases they include the subsidiary information on the structure of the governing differential equation, the type of its coefficients and other parameters. Quite often the unique solvability of an inverse problem is ensured by the surplus information of this sort. A definite structure of the differential equation coefficients leads to an inverse problem being well-posed from a common point of view. This book treats the subject of such problems containing a sufficiently complete and systematic theory of inverse problems and reflecting a rapid growth and development over recent years. It is based on the original works of the authors and involves an experience of solving inverse problems in many branches of mathematical physics: heat and mass transfer, elasticity theory, potential theory, nuclear physics, hydrodynamics, etc. Despite a great generality of the presented research, it is of a constructive nature and gives the reader an understanding of relevant special cases as well as providing one with insight into what is going on in general.

In mastering the challenges involved, the monograph incorporates the well-known classical results for direct problems of mathematical physics and the theory of differential equations in Banach spaces serving as a basis for advanced classical theory of well-posed solvability of inverse problems for the equations concerned. It is worth noting here that plenty of inverse problems are intimately connected or equivalent to nonlocal direct problems for differential equations of some combined type, the new problems arising in momentum theory and the theory of approximation, the new types of linear and nonlinear integral and integro-differential equations of the first and second kinds. In such cases the well-posed solvability of inverse problem entails the new theorems on unique solvability for nonclassical direct problems we have mentioned above. Also, the inverse problems under consideration can be treated as problems from the theory of control of systems with distributed or lumped parameters.

It may happen that the well-developed methods for solving inverse problems permit one to establish, under certain constraints on the input data, the property of having fixed sign for source functions, coefficients and solutions themselves. If so, the inverse problems from control theory are in principal difference with classical problems of this theory. These special inverse problems from control theory could be more appropriately referred to as problems of the "forecast-monitoring" type. The property of having fixed sign for a solution of "forecast-monitoring" problems will be of crucial importance in applications to practical problems of heat and mass transfer, the theory of stochastic diffusion equations, mathematical economics, various problems of ecology, automata control and computerized tomography. In many cases the well-posed solvability of inverse problems is established with the aid of the contraction mapping principle, the Birkhoff–Tarsky principle, the Newton–Kantorovich method and other effective operator methods, making it possible to solve both linear and nonlinear problems following constructive iterative procedures.

The monograph covers the basic types of equations: elliptic, parabolic and hyperbolic. Special emphasis is given to the Navier–Stokes equations as well as to the well-known kinetic equations: Bolzman equation and neutron transport equation.

Being concerned with equations of parabolic type, one of the wide-spread inverse problems for such equations amounts to the problem of determining an unknown function connected structurally with coefficients of the governing equation. The traditional way of covering this is to absorb some additional information on the behavior of a solution at a fixed point $u(x_0, t) = \varphi(t)$. In this regard, a reasonable interpretation of problems with the overdetermination at a fixed point is approved. The main idea behind this approach is connected with the control over physical processes for a proper choice of parameters, making it possible to provide at this point a required temperature regime. On the other hand, the integral overdetermination

$$\int_{\Omega} u(x,t)\, w(x)\; dx = \varphi(t)\,,$$

where w and φ are the known functions and u is a solution of a given parabolic equation, may also be of help in achieving the final aim and comes first in the body of the book. We have established the new results on uniqueness and solvability. The overwhelming majority of the Russian and foreign researchers dealt with such problems merely for linear and semilinear equations. In this book the solvability of the preceding problem is revealed for a more general class of quasilinear equations. The approximate methods for constructing solutions of inverse problems find a wide range of applications and are gaining increasing popularity.

One more important inverse problem for parabolic equations is the problem with the final overdetermination in which the subsidiary information is the value of a solution at a fixed moment of time: $u(x, T) = \varphi(x)$. Recent years have seen the publication of many works devoted to this canonical problem. Plenty of interesting and profound results from the explicit formulae for solutions in the simplest cases to various sufficient conditions of the unique solvability have been derived for this inverse problem and gradually enriched the theory parallel with these achievements. We offer and develop a new approach in this area based on properties of Fredholm's solvability of inverse problems, whose use permits us to establish the well-known conditions for unique solvability as well.

It is worth noting here that for the first time problems with the integral overdetermination for both parabolic and hyperbolic equations have been completely posed and analysed within the Russian scientific school headed by Prof. Aleksey Prilepko from the Moscow State University. Later the relevant problems were extensively investigated by other researchers including foreign ones. Additional information in such problems is provided in the integral form and admits a physical interpretation as a result of measuring a physical parameter by a perfect sensor. The essense of the matter is that any sensor, due to its finite size, always performs some averaging of a measured parameter over the domain of action.

Similar problems for equations of hyperbolic type emerged in theory and practice. They include symmetric hyperbolic systems of the first order, the wave equation with variable coefficients and the system of equations in elasticity theory. Some conditions for the existence and uniqueness of a solution of problems with the overdetermination at a fixed point and the integral overdetermination have been established.

Let us stress that under the conditions imposed above, problems with the final overdetermination are of rather complicated forms than those in the parabolic case. Simple examples help motivate in the general case the absence of even Fredholm's solvability of inverse problems of hyperbolic type. Nevertheless, the authors have proved Fredholm's solvability and established various sufficient conditions for the existence and uniqueness of a solution for a sufficiently broad class of equations.

Among inverse problems for elliptic equations we are much interested in inverse problems of potential theory relating to the shape and density of an attracting body either from available values of the body's external or internal potentials or from available values of certain functionals of these potentials. In this direction we have proved the theorems on global uniqueness and stability for solutions of the aforementioned problems. Moreover, inverse problems of the simple layer potential and the total potential which do arise in geophysics, cardiology and other areas are discussed. Inverse problems for the Helmholz equation in acoustics and dispersion theory are completely posed and investigated. For more general elliptic equations, problems of finding their sources and coefficients are analysed in the situation when, in addition, some or other accompanying functionals of solutions are specified as compared with related direct problems.

In spite of the fact that the time-dependent system of the Navier–Stokes equations of the dynamics of viscous fluid falls within the category of equations similar to parabolic ones, separate investigations are caused by some specificity of its character. The well-founded choice of the inverse problem statement owes a debt to the surplus information about a solution as supplementary to the initial and boundary conditions. Additional information of this sort is capable of describing, as a rule, the indirect manifestation of the liquid motion characteristics in question and admits plenty of representations. The first careful analysis of an inverse problem for the Navier–Stokes equations was carried out by the authors and provides proper guidelines for deeper study of inverse problems with the overdetermination at a fixed point and the same of the final observation conditions. This book covers fully the problem with a perfect sensor involved, in which the subsidiary information is prescribed in the integral form. Common settings of inverse problems for the Navier–Stokes system are similar to parabolic and hyperbolic equations we have considered so far and may also be treated as control problems relating to viscous liquid motion.

The linearized Bolzman equation and neutron transport equation are viewed in the book as particular cases of kinetic equations. The linearized Bolzman equation describes the evolution of a deviation of the distribution function of a one-particle-rarefied gas from an equilibrium. The statements of inverse problems remain unchanged including the Cauchy problem and the boundary value problem in a bounded domain. The solution existence and solvability are proved. The constraints imposed at the very beginning are satisfied for solid sphere models and power potentials of the particle interaction with angular cut off.

For a boundary value problem the conditions for the boundary data reflect the following situations: the first is connected with the boundary absorption, the second with the thermodynamic equilibrium of the boundary with dissipative particles dispersion on the border. It is worth noting that the characteristics of the boundary being an equilibria in thermodynamics lead to supplementary problems for investigating inverse problems with the final overdetermination, since in this case the linearized collision operator has a nontrivial kernel. Because of this, we restrict ourselves to the stiff interactions only.

Observe that in studying inverse problems for the Bolzman equation we employ the method of differential equations in a Banach space. The same method is adopted for similar problems relating to the neutron transport. Inverse problems for the transport equation are described by inverse problems for a first order abstract differential equation in a Banach space. For this equation the theorems on existence and uniqueness of the inverse problem solution are proved. Conditions for applications of these theorems are easily formulated in terms of the input data of the initial transport equation. The book provides a common setting of inverse problems which will be effectively used in the nuclear reactor theory.

Differential equations in a Banach space with unbounded operator coefficients are given as one possible way of treating partial differential equations. Inverse problems for equations in a Banach space correspond to abstract forms of inverse problems for partial differential equations. The method of differential equations in a Banach space for investigating various inverse problems is quite applicable. Abstract inverse problems are considered for equations of first and second orders, capable of describing inverse problems for partial differential equations.

It should be noted that we restrict ourselves here to abstract inverse problems of two classes: inverse problems in which, in order to solve the differential equation for $u(t)$, it is necessary to know the value of some operator or functional $B\,u(t) = \varphi(t)$ as a function of the argument t, and problems with pointwise overdetermination: $u(T) = u_T$.

For the inverse problems from the first class (problems with evolution overdetermination) we raise the questions of existence and uniqueness of a solution and receive definite answers. Special attention is being paid to the problems in which the operator B possesses some smoothness properties. In context of partial differential equations, abstract inverse problems are suitable to problems with the integral overdetermination, that is, for the problems in which the physical value measurement is carried out by a perfect sensor of finite size. For these problems the questions of existence and uniqueness of strong and weak solutions are examined, and the conditions of differentiability of solutions are established. Under such an approach the emerging equations with constant and variable coefficients are studied.

It is worth emphasizing here that the type of equation plays a key role in the case of equations with variable coefficients and, therefore, its description is carried out separately for parabolic and hyperbolic cases. Linear and semilinear equations arise in the hyperbolic case, while parabolic equations include quasilinear ones as well. Semigroup theory is the basic tool adopted in this book for the first order equations. Since the second order equations may be reduced to the first order equations, we need the relevant elements of the theory of cosine functions.

A systematic study of these problems is a new original trend initiated and well-developed by the authors.

The inverse problems from the second class, from the point of possible applications, lead to problems with the final overdetermination. So far they have been studied mainly for the simplest cases. The authors began their research in a young and growing field and continue with their pupils and colleagues. The equations of first and second orders will be of great interest, but we restrict ourselves here to the linear case only. For second order equations the elliptic and hyperbolic cases are extensively investigated. Among the results obtained we point out sufficient conditions of existence and uniqueness of a solution, necessary and sufficient conditions for the existence of a solution and its uniqueness for equations with a self-adjoint main part and Fredholm's-type solvability conditions. For differential equations in a Hilbert structure inverse problems are studied and conditions of their solvability are established. All the results apply equally well to inverse problems for mathematical physics equations, in particular, for parabolic equations, second order elliptic and hyperbolic equations, the systems of Navier–Stokes and Maxwell equations, symmetric hyperbolic systems, the system of equations from elasticity theory, the Bolzman equation and the neutron transport equation.

The overview of the results obtained and their relative comparison are given in concluding remarks. The book reviews the latest discoveries of the new theory and opens the way to the wealth of applications that it is likely to embrace.

In order to make the book accessible not only to specialists, but also to students and engineers, we give a complete account of definitions and notions and present a number of relevant topics from other branches of mathematics.

It is to be hoped that the publication of this monograph will stimulate further research in many countries as we face the challenge of the next decade.

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