A PITMAN INTERNATIONAL TEXT

Electrical Network Theory

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Preface

This book is intended to be an intermediate level text on linear electrical network theory. It assumes that the student has already had a basic course in d.c. and steady-state a.c. analysis and is familiar with the basic network theorems and laws. The bulk of the material has been given for a number of years as a one-term subject at the second-year level of a three-year degree course. It is hoped that the student will gain from this book the necessary foundation in network analysis to follow advanced courses on analysis and synthesis.

The first chapter defines the scope of the book and reviews some important fundamental network laws and theorems. I also consider an elementary treatment of network topology useful in so far as it enables the student to determine the essential structure of a network and hence to choose an approach to network analysis that results in the minimum number of equations to solve. The next two chapters discuss the methods of loop and nodal analysis and the application of signal flow graphs in network analysis. Chapters 4 and 5 are concerned with developing various methods for characterizing signals. In chapter 4, time series description and frequency domain description of signals via the Fourier series and Fourier transform are discussed. The more versatile Laplace transform is introduced in chapter 5 and its application in the study of the transient and steady-state response of networks is treated in chapter 6. In this chapter we also look at the important aspect of network stability and develop simple criteria for stability of a linear network. This is discussed with reference to feedback systems and oscillators. The concept of a transfer function to describe a two-terminal pair or two-port network is also introduced here as a ratio of the Laplace transforms of the response and the input. Its limitations lead us to study the more useful parametric description of two-port networks in chapter 7.

Worked examples are used to help the student understand new concepts, and these are supplemented by a range of problems at the end of each chapter. In this connection, I am indebted to the University of Warwick for permission to use questions from the second-year examination papers

of their Bachelor of Science course.

I wish to acknowledge the invaluable help of Dr. R.K.L. Gay of the University of Singapore who reviewed the manuscript and offered helpful criticisms and useful suggestions for improvement of the presentation. Particular thanks is due to Mrs Pady Goh who typed the manuscript. Finally I wish to express appreciation to my wife who showed much patience and gave much encouragement during the many hours of manuscript writing.

K. C. NG
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1 Electrical Networks

Introduction

An electrical system is an interconnection of electrical elements which may vary in complexity from the very simple resistor-capacitor filter to the electrical power distribution network of a state or country. Usually a system is activated by some sort of external stimuli or signals which we shall call inputs or excitations. In response to these excitations, the system will perform certain functions and meet specific objectives in the form of responses or outputs. This general situation is depicted in Fig. 1.1 where only one input and one output are shown. In general the system may have more than one input and one output.

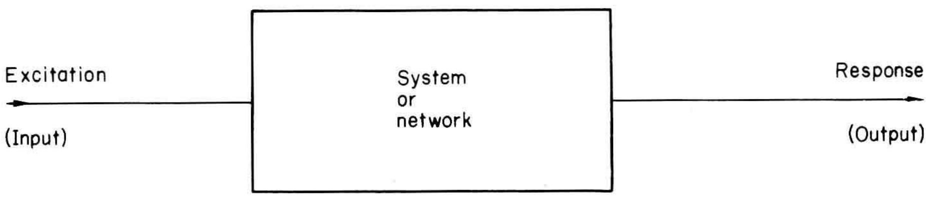


Fig.1.1 Excitation-response relationship

An electrical system will be a network of physical elements such as resistors, inductors, capacitors, sources of electrical energy and other devices such as valves, transistors, diodes. Henceforth, in this book the terms system and network will be used interchangeably. To be able to begin to understand the function of a complex system, we have to understand first the input-output relationships of the component parts and the interaction between these parts when connected together. Now all physical elements are nonlinear to a greater or lesser extent. The response of nonlinear elements to excitations is dependent on the magnitude of the excitations, so that no useful general conclusions can be made of their behaviour in response to excitations. Fortunately in the majority of cases we find that either the nonlinear nature of the element is not too pronounced or the signal levels are low or appreciably constant. In these circumstances a linear approximation generally yields analytical results which are in close agreement with the observed behaviour. It is for this reason that linear systems theory is so useful a discipline to master. We assume in this book that the reader is already familiar with the characteristic behaviour of linear resistors, capacitors, inductors and sources and with elementary a.c. and d.c. network theory, although we shall review some of the basic theorems of linear networks later in this chapter.

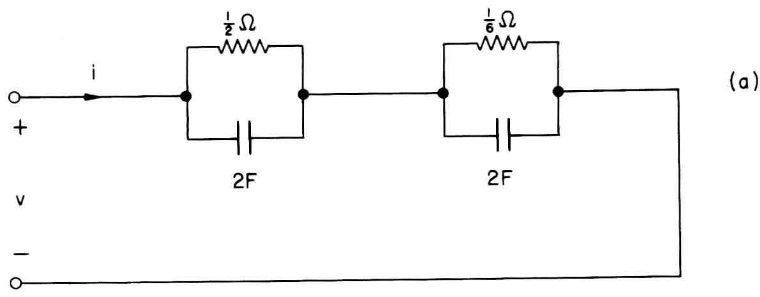
In the study of electrical networks we can distinguish between two broad classes of problems. In the first category we are concerned with determining the response, given the excitation and the network. This is *network analysis*. One part of the analysis problem is the characterization of the excitation and response signals. For electrical networks these signals are voltages and currents which are functions of, and can be describe in terms of, time t. However, the signals can be described equally well in terms of spectral or frequency information. The other part of the analysis problem is the characterization of the network itself in terms of time and frequency and the determination of the behaviour of the network as a signal processor.

The second class of network problems is the converse of the analysis problem. In *network synthesis*, we are concerned with the design of a network which will give the desired response to a specific excitation signal. This is generally more difficult than network analysis. A given linear network has an unique input-output relationship. However, a given input-output relationship generally can be satisfied by more than one network. For example, the two networks in Fig. 1.2 have the same input (current)-response (voltage) relationship

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 3v = \frac{di}{dt} + 2i$$

From a theoretical point of view, either of these two networks would be acceptable. Physical constraints and considerations of availability of components may be deciding factors in the choice of the design.

It is not intended in a book of this size to consider network synthesis and the reader is referred to the many excellent introductory books on network synthesis (see, for example, Guillemin, Kuo and Van Valkenburg in the Bibliography). We shall deal only with the analysis of linear time-invariant causal networks assumed to be made up of ideal elements. We now go on to describe the general properties of this type of network.



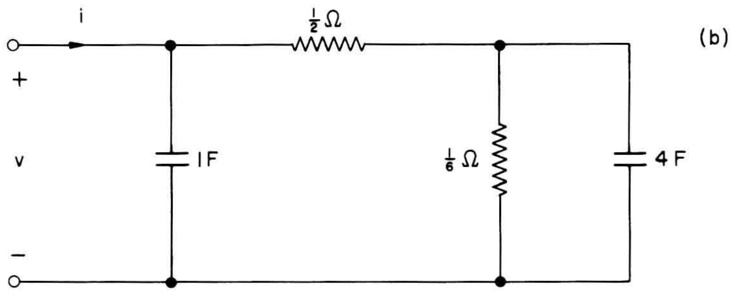


Fig.1.2 Two networks with the same input-response relationship

1.2 Basic Definitions

Linearity A system is linear if it obeys the following laws.

(i) *Additivity*. The response to a sum of independent excitations is equal to the sum of the responses due to each of these excitations acting alone, the other excitations being suppressed.

In other words, if the response to the ith excitation E_i is $a_i E_i$, then the total response to the sum of n excitations $\sum_{i=1}^{n} E_i$ may be written as

$$R = \sum_{i=1}^{n} a_i E_i \tag{1.1}$$

where the a_i's are constants determined by the network. This property is commonly known as the *superposition theorem*.

(ii) *Homogeneity*. The principle of homogeneity or proportionality states that if the excitation is multiplied by a constant, the response is also multiplied by the same constant. This result follows naturally from the superposition theorem.

(iii) *Associativity*. Consider the three networks A, B, and C in Fig.1.3(a). Let the response of these networks to the excitations E_a, E_b and E_c be aE_a, bE_b, cE_c respectively. If the responses of networks A and B are additively applied to network C as shown in Fig.1.3(b), then by the associative law the response of network C is

$$R = c(aE_a + bE_b)$$
$$= acE_a + bcE_b$$

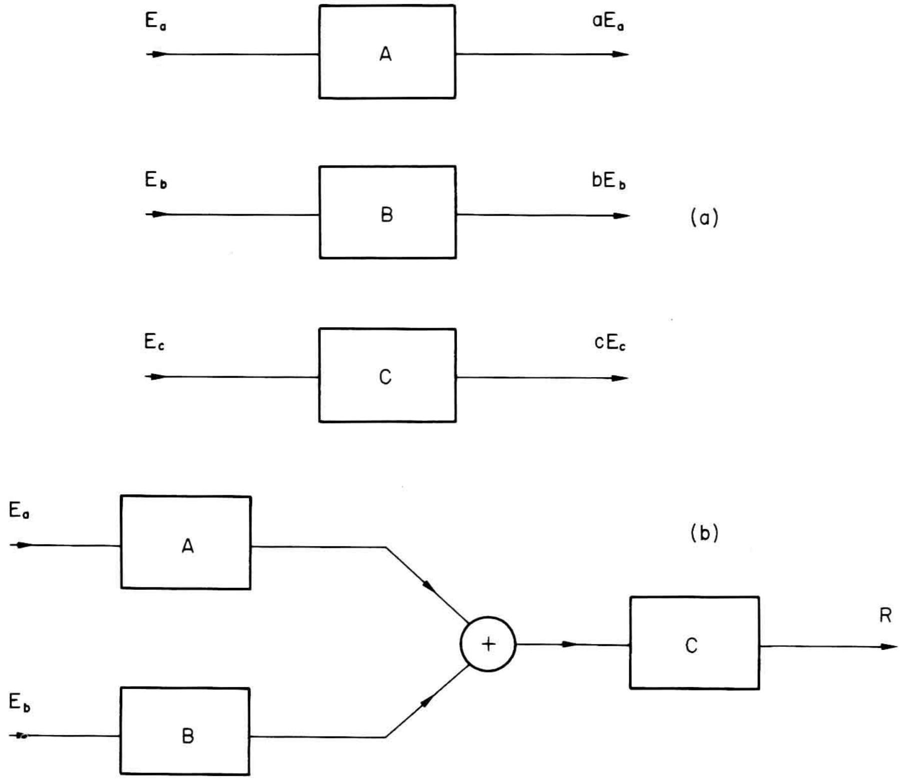


Fig.1.3 The Associativity law

Time Invariance. A time-invariant network is one whose characteristics do not change with time. Suppose an excitation e(t) applied to a network at t = 0 produces a response r(t). If the network is time-invariant, then when the same excitation is applied at any other time t_1, the response will still be of the same waveshape but delayed in time by t_1. This is illustrated clearly in Fig.1.4. We should note here that a linear system need not be time-invariant.

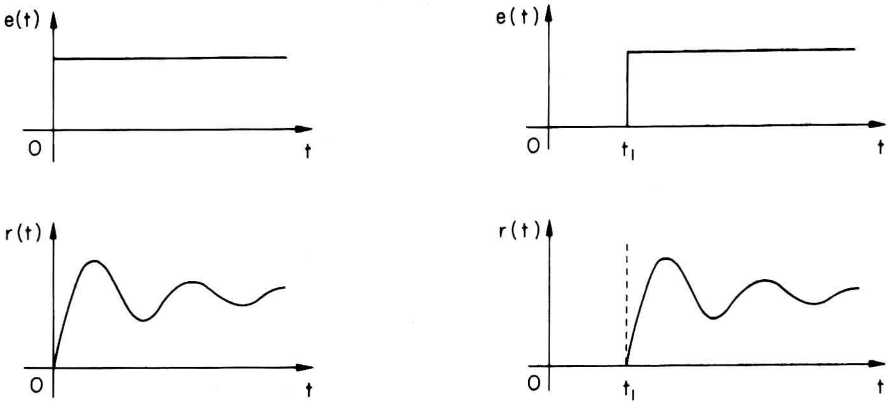


Fig.1.4 Responses of a time-invariant system

Causality. A network is described as causal if its response to an excitation is zero until after the excitation is applied. That is, for a causal network, if

$$e(t) = 0 \qquad t < T$$

then

$$r(t) = 0 \qquad t < T$$

as shown in Fig. 1.5. A causal network is therefore nonpredictive. All

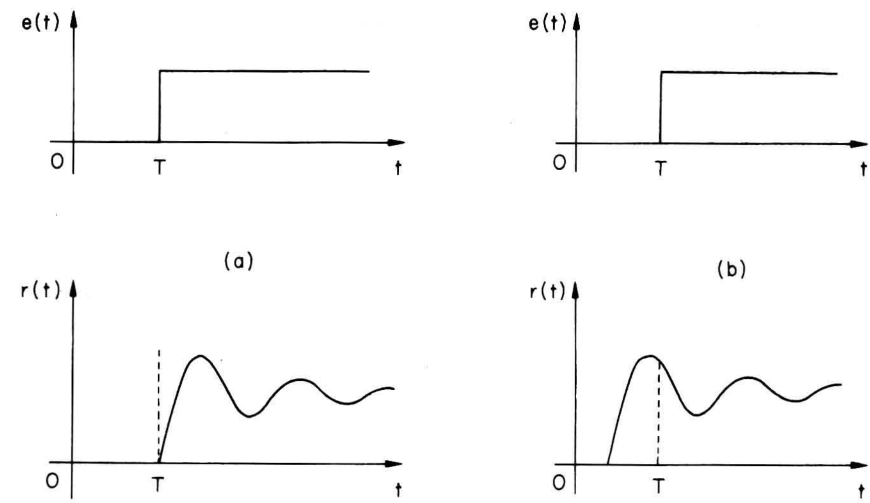


Fig.1.5 (a) Response of a causal system (b) Response of a noncausal system

networks made up of physical elements (that is, physically realizable networks) are causal. The concept of causality is therefore of fundamental importance in the study of network synthesis. We cannot synthesize networks which are specified by noncausal input-response relationships.

Passive Networks. A network that does not contain a source of electrical energy is passive. An active network will have at least one energy source in it. We shall be considering both types of networks in this book. The two sources of electrical energy are the voltage source and the current source. It is useful to distinguish between an *ideal* and a *practical* source. We can further classify the sources as *dependent* or *independent.*

An ideal current source is a source of energy capable of delivering any amount of electrical energy at constant current. This implies that the internal impedance of an ideal current source is infinite, so that the current it delivers to an external load impedance is independent of the magnitude of the load impedance. A practical current source, however, will have a large but finite internal impedance z_g, so that the current delivered to the external load is now dependent on the load. The circuit symbols for an ideal current source and a practical one are shown in Fig. 1.6.

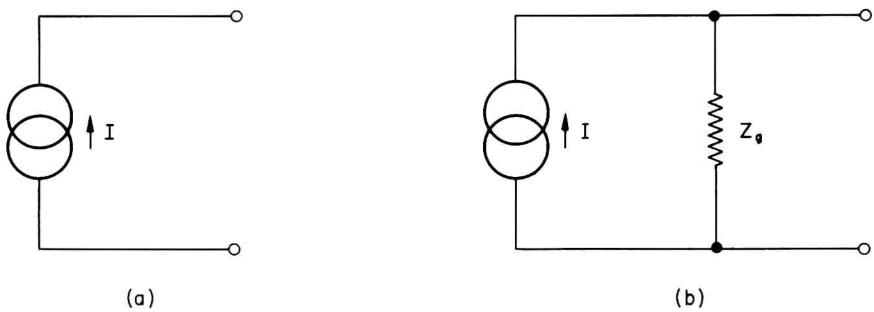


Fig.1.6 (a) Ideal current source (b) Practical current source

An ideal voltage source is defined as a source which can delivery any amount of electrical energy at constant voltage. This implies that it has zero internal impedance. A physical voltage source, on the other hand, has a small but finite internal impedance z_g, so that as current is drawn from the source the terminal voltage of the source falls. The circuit symbols for these sources are shown in Fig. 1.7.

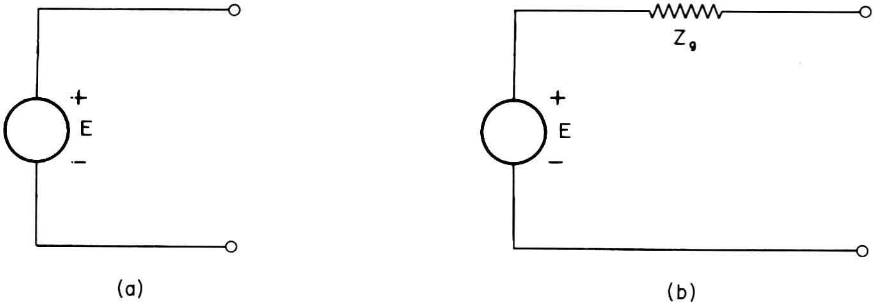


Fig.1.7 (a) Ideal voltage source (b) Practical voltage source

The sources described above are independent sources, in that the source voltage E and the source current I are independent of the voltages and currents in the network in which they exist. In the case of dependent sources, the source voltage or current is a function of voltages or currents that appear at another part of the network. Typical examples are illustrated in Fig. 1.8. Such sources are generally used to represent

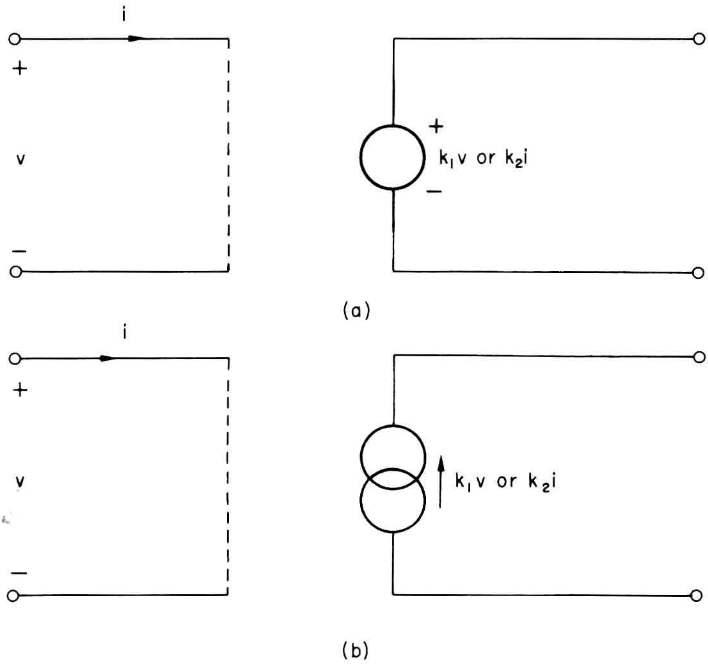


Fig.1.8 Illustration of dependent sources

active devices: transistors, valves and amplifiers, for example.

1.3 Basic Network Theorems

In this section we consider four network theorms which are of fundamental importance in network theory. We assume that the reader has been exposed to Kirchhoff's Laws, but discuss these briefly for the prime purpose of laying down the convention of signs which will be followed throughout this book. Kirchhoff's voltage and current laws are the basic network theorems which govern the interconnection of electrical elements to form networks and on which depend basic analytical methods, such as loop and nodal analyses. The equations necessary to describe a network can always be found by the appropriate application of these two laws. Thévenin's theorem and Norton's theorem express the conditions of equivalence between networks and will be found to be useful in deriving equivalent networks of complex networks and for transforming voltage sources into their equivalent current sources and vice versa.

1.3.2 Kirchhoff's Current and Voltage Laws

Kirchhoff's Current Law. By the continuity principle, all the currents entering a junction point in a network must leave instantaneously. In other words, the algebraic sum of the currents at the junction will be zero; that is

$$\Sigma i = 0 \tag{1.2}$$

In applying this equation, we shall use the following sign convention:

Assign a positive sign to each current leaving the junction point and a negative sign to each current entering the point.

Therefore, in Fig. 1.9 we have

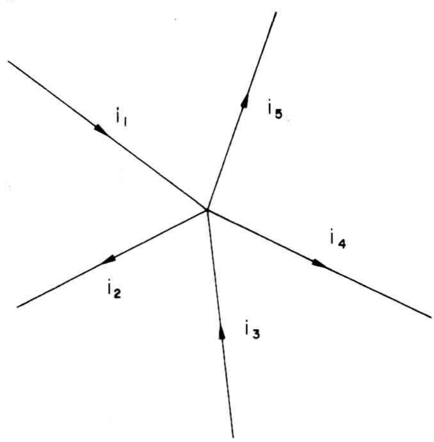


Fig.1.9 Kirchhoff's current law