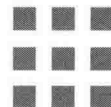


# PRINCIPLES OF FILTRATION

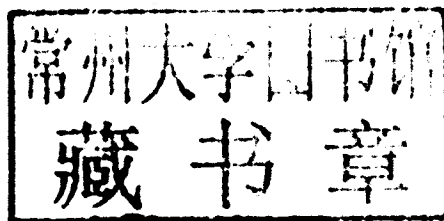


CHI TIEN



# Principles of Filtration

Chi Tien  
Syracuse University  
Syracuse, NY



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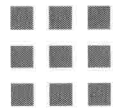
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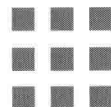
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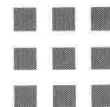


# Principles of Filtration



# Dedication

To Julia C. Tien



# Preface

Filtration as a fluid–particle separation process is an engineering practice of long standing. Separation in filtration is effected by passing a fluid–particle mixture through a medium with particles retained by the medium and passage of clear fluid through it. To be more specific and descriptive of the process conducted, depending upon factors such as the operative particle retention mechanism, the type of medium used, the flow configuration and/or the system treated, different terminologies such as cake filtration, surface filtration, depth filtration, deep bed filtration, cross-flow filtration, aerosol filtration, water (liquid) filtration, granular filtration, fibrous filtration, fabric filtration, cartridge filtration, membrane filtration, etc. have been introduced over the years. As an unintended consequence, that these specific filtration processes are not totally different is often ignored, even among filtration experts.

The compartmentalization of filtration into different types of operation in terminology is reflected in published texts and monographs as well. In most unit operations textbooks for undergraduate teaching, discussion of filtration is invariably restricted to cake filtration of liquid suspensions. Similarly, textbooks on air pollution control engineering and water treatment limit their presentations mainly to fibrous or fabric aerosol filtration (in air pollution control) or deep bed filtration of aqueous suspensions (in water treatment). By considering these specific types of filtration as independent and unrelated subjects in teaching, the students are denied the opportunity of gaining a broader horizon and greater understanding of the subject matter and the advantage of applying results of one type of filtration to the solutions of others.

Technically speaking, one may well argue that separating and classifying filtration based on factors mentioned above is arbitrary and may not be realistic. For cake filtration, in which filter cakes formed play a major role in determining filtration performance, penetration of particles into filter medium cannot be ruled out in many cases. In fact, better understanding of the temporal evolution of filter medium resistance is a key factor to exact cake filtration analysis and accurate predictions of filtration performance, requires the knowledge of deep bed filtration (with the medium considered as a deep bed filter). Similarly, for proper design of deep bed filters of water treatment, in order to insure prolonged operation and to reduce the frequency of backwashing, knowledge of the variables affecting cake formation at deep bed inlet is necessary. Equally, if not more important, for membrane filtration carried out in the cross-flow mode, particle separation is often caused by a combination of particle deposition over membrane surfaces and particle retention within membrane pores; therefore, blurring the distinctiveness of one type of filtration from another. With these considerations, teaching cake filtration and

deep bed filtration and treating them separately and independently in engineering texts and monographs have become an obsolete and ineffective practice.

The practice of analyzing filtration according to the medium used or the type of the system treated is equally questionable. In spite of the medium and/or system differences, problems arising from the flow of fluid suspensions (liquid or gas) through porous media (granular, fibrous, fabric) and the attendant particle deposition can be analyzed using the same set of equations and procedures. In other words, there exists a common core of knowledge and information, which form the basis for teaching filtration as a single subject.

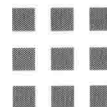
I have designed this volume in order to provide a unified treatment of filtration operations in different guise. Major emphasis is placed on presenting basic and well-established principles for the description of the various types of filtration operations, some common procedures for data treatment and correlation, and a collection of filtration rate parameter correlations. A substantial number of illustrative examples are given throughout the text in order to demonstrate the principle/procedures discussed. The problems given at the end of each chapter provide further opportunities for readers to better understand these principles and procedures for their applications.

This book is written as a text/reference book for both engineering students and practitioners. Its purpose is to equip readers with certain basic knowledge and information of filtration at a level higher than those found in elementary texts and to enable them to undertake meaningful engineering work in filtration and/or begin their research in this area. As a textbook, it may be used for either undergraduate (senior) or graduate (first year) teaching for chemical, environmental, and mechanical engineering students. Different parts of the book can also be easily incorporated into courses such as fluid–solid separation, air pollution control, environmental engineering analysis, water treatment, membrane process and technology found in many engineering curricula. The level of presentation is consistent with what is taught at ABET accredited B.Sc. degree programs in chemical, environmental, and mechanical engineering. In spite of a fairly large number of mathematical equations present throughout the text, the book can be easily comprehended by students with elementary competence in calculus plus some knowledge of differential equations except perhaps those with extreme number-phobia.

In preparing the text, I have benefitted greatly from the discussions I have had with a large number of colleagues especially those with Professor B.V. Ramarao (SUNY-ESF). The preliminary draft of the work was read by a number of people. In particular, I would like to express my gratitude to Professor George G. Chase (University of Akron), Professor Rolf Gimbel (University of Duisburg-Essen), Professor W.P. Johnson (University of Utah), Professor Y.-W. Jung (Inha University), Professor Wallace W. Leung (Polytechnic University, Hong Kong), Professor Dominique Thomas (University of Nancy), Professor K.-L. Tung (Chun Yuan Christian University), and Professor Eugene Vorobiev (Universite de Technologie de Compiegne) for their insightful suggestions, critical comments, and moreover careful proofreading!!! Without their help and efforts, the book would not appear in its present form.

Finally, I would like to express my thanks to my editors, Dr. Kostas Marinakis and Dr. Anita Koch of Elsevier for their efforts and help which make prompt publication of this volume possible, Kathy Datthyn-Madigan for her efforts in assembling the manuscript, and last but not the least, my wife, Julia, for all the help and support she has given me during the past half century.

Chi Tien



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## Notation

$K_0$	coefficient of Equation (1.1)
$k$	exponent of Equation (1.1)
$t$	time (s)
$V$	cumulative filtrate volume per unit media surface area ( $\text{m}^3 \text{m}^{-2}$ )

Filtration may be described as an operation in which solids (particles) present in a solid–fluid mixture are separated from the liquid by forcing the flow of the mixture through a supported mesh or cloth (Walker et al., 1937). The mixture is caused to flow by various forces: gravity, pressure, vacuum, or centrifugal force. The products of the separation consist of a fluid stream (filtrate) free or nearly free of particles, a solid phase with some entrained liquid and possibly a solid–fluid mixture with enhanced solid concentration (as in the case of crossflow cake filtration).

Giddings (1991) advanced the premise that separation of a mixture of several components is effected by the relative displacements of the various components present in the mixture. Earlier, King (1980) stated that the working of a separation process is accomplished by the application of a separating agent as shown in Fig. 1.1. The agent may be either energy or matter or both. Through the action of the agent, a feed is split into several streams of different compositions. Using King's description or Gidding's premise, filtration is a process employing energy (for the flow of the suspension to be treated) and matter (filter media) as separating agent, leading to a relative solid/fluid displacement from the flow of the suspension through the medium with particle retention at the surface of the medium, or particle deposition throughout the medium.

Filtration may be applied to both gas/solid and liquid/solid suspensions. In the following sections, a brief discussion on its use in certain industrial applications is presented as background information.

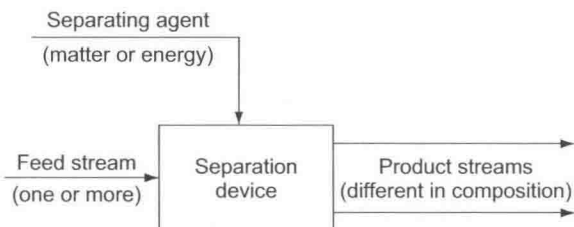


FIGURE 1.1 Representation of separation process.

## 1.1 Filtration as a Liquid–Solid Separation Technology

Liquid–solid separation technology, as the name implies, refers to a collection of processes for removing, separating, and recovering particles from liquid–solid mixtures. While the processes known as liquid–solid separation are too numerous to be cited individually, it is generally accepted that liquid–solid separation encompasses filtration, sedimentation, cycloning, thickening, flocculation, and expression. Tiller (1974) proposed a classification scheme based on liquid–solid separation functions (see Fig. 1.2), namely, liquid–solid separation may be viewed as a system consisting of one or more stages: pretreatment, solid concentration, solid separation, and post-treatment. According to this scheme, filtration is used in both the separation stage and post-treatment.

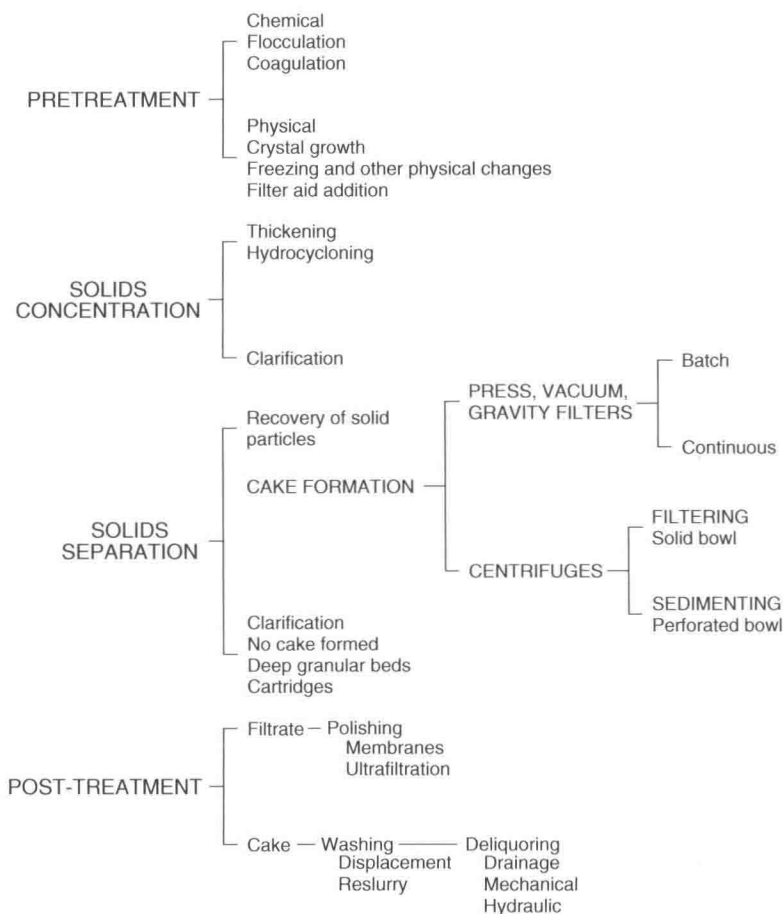


FIGURE 1.2 Stages of solid/liquid separation according to Tiller.