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Dynamic Analysis of Open Economies

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Preface

Most economic systems are dynamically coupled to one another. Although systems may sometimes be examined in "isolation" on the assumption that interactions among them and with the environments in which they function can be ignored, such analysis can be only approximate. The interactions exist, of course. Because our knowledge of an economy is only approximate, and because the models we employ only dimly reflect very complex economic reality, such approximate analysis is sometimes justified. We cannot, however, neglect interdependence between the object of our examination and the rest of the world entirely. The study of open economies is a case in point.

While some economies may be analyzed as "small open economies" in which the rest of the world is treated as exogenous, not all economies may be analyzed this way; and not all questions can be analyzed this way either. Influences that some economies exert internationally and the feedback effects or repercussions must be explicitly considered in the analysis.

Because models of open economies provide very important and interesting examples of interconnected macroeconomic systems, I have for some time now been interested in the dynamic behavior of open economies in general, and dynamic interactions among several interconnected economies in particular. This has led me to examine related questions of how economies with different structures are affected by common exogenous disturbances and how these disturbances spread throughout the world as well as questions of policy coordination among nations and of how decisions made in one country affect other economies, when each country's policy maker acts more or less independently of policy makers elsewhere.

The emphasis here is on dynamic responses of models of open economies. I find that most results on open economies available in economic literature refer either to impact or to comparative static analysis. I have therefore found it necessary to conduct most of the dynamic analysis myself.

I record here both the results of my examination of the dynamic behavior of models of open economies and an exposition of the techniques of dynamic analysis that I used, since these techniques are not standard and merit wider use.

Both simple and relatively more fully specified models are used to illustrate my procedures. The latter models reflect recent developments in models of deterministic macroeconomic open economies. Although various behavioral relations in the models of this book are not explicitly derived by intertemporal optimizations by economic agents as in some more recent and simple models, this fact does not preclude a possibility that some of the behavioral relations may be derivable or be suggested by these optimization procedures.

I have confined this book to deterministic models partly because stochastic models require additional tools that are best discussed separately, and partly because what I discuss here serves as a basis to develop my treatment of stochastic models. Deterministic models can also serve as a certainty equivalent version of stochastic models. Most of the important concepts and techniques, such as the variational analysis and structural perturbation examination, apply to stochastic systems as well. Since some of the economic applications of the techniques I describe here are new, I feel that it is best to avoid in this book the additional complications associated with stochastic models.

Before we begin, let me add a word about the use of algebra and elementary differential or difference equations as analytical tools. The effects of open market operations are customarily evaluated by using graphs that draw (straight) lines to represent various asset equilibrium conditions in a plane, typically with an interest rate and the exchange rate as coordinates. (See Girton and Henderson (1973) as a representative of such analysis.) The graphical approach works well when there is a small number of variables to keep track of, as in the case of impact analysis of open market operations or of intervention into the exchange market of the asset sector composed of a few financial assets. However, when the goods sector is included in the analysis in order to consider the influences of changes in the output or its price level, when wage rates are variable, or when effects of fiscal expansion not matched by real tax increase (unbalanced budget fiscal expansion) are to be examined, the graphical approach becomes a messy way to keep track of variables.

In writing this book, I have benefited from comments by and discussions with my colleagues and students. My special thanks go to Professor W. H. Branson, who has been very helpful in many ways. Professor S. Kamiyama, Messrs. K. Mashiyama and B. A. Jensen read an early version of the manuscript. I have benefited from comments and discussion with Professors R. C. Marston, Y. Shinkai, Y. Murata, A. Takayama, D. Freedman, J. Frankel, J. B. deMacedo, and Dr. M. Canzoneri.

A grant from the National Science Foundation enabled me to pursue a research program, some results of which are included in this book, for which I am grateful.

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Scope of the Investigation

We study how macroeconomic policy instruments affect open economies under flexible exchange rate regimes and examine the extent to which interdependence of national economies affects assessment of national policy effectiveness in a dynamic context. Traditionally, questions related to policy effectiveness have been examined only at impacts and at steady-state (long-run) equilibria. In this study we evaluate behavior of open economies not only at the instant of exogenous shocks or changes in instruments, but also after some time has elapsed since the last impacts. We answer the question, How much do dynamics matter? by examining behavior of a wide range of models and drawing general conclusions. To carry out this study, we develop a set of techniques associated with variational analysis and theory of perturbation. These methods, ideally suited for conducting comparative dynamic analysis of economic models, are systematically applied to the models of the open economies we consider.

The study reported here is theoretical, dealing with analytical rather than econometric models. We do not study nor do we draw policy implications for any particular country, although assumptions embedded in the models may reflect some of the structural, institutional constraints or special circumstances in some countries.

Much literature exists on exchange rate dynamics, internal and external balances, transmissions of inflation, effects of wage indexation, and the like. These topics also interest us. We attempt to examine these problems more systematically by constructing our basic analytical general equilibrium models more explicitly and by paying closer attention to various sources of dynamics. The novel methods we employ to compare consequences of alternative policies are also more powerful, having been designed particularly for comparative dynamic studies.

The emphasis on dynamic analysis distinguishes this book from several others with similar objectives. We approach dynamics via equilibrium dynamics, i.e., dynamics that describe how momentary equilibrium states shift over time because of changing stocks of assets and changing expectations. More specifically, by systematic applications of variational analysis, we examine how endogenous variables deviate from a trend or from other reference time paths because of changes in instruments or exogenous disturbances. Various studies suggest rudiments of this procedure, where "detrended" log-linear specifications of models are frequently used to compare consequences of alternative policy regimes, or of exogenous shocks. Econometricians employ a similar procedure, comparing a "control" simulation run to other simulation runs on digital computers. We make systematic and explicit comparisons for the analytical models of the open economies constructed in this book.

Certain simple models are examined as benchmark cases. Dynamic behaviors of models that are elaborations on the simple models are then analyzed by treating the elaborations as perturbations on the original models. These two methods, variational dynamic analysis and analysis by theory of perturbation, enable us to get some insight into the nature of the dynamic interdependence of macroeconomic policies. Loosely speaking, we use variational analysis to examine dynamic behavior of a model near a reference time path and to apply perturbation theory to compare the dynamic behaviors of models that are similar in structure.

Besides systematically applying variational and perturbation techniques to obtain comparative dynamic results, we offer an analytical innovation for dealing with models of the world that are composed of several countries and show the usefulness of path controllability. We find that several other concepts from system theory are particularly well qualified to assess the interdependence of national economies and, more specifically, to investigate how the influences of instruments of national policies spread internationally. These concepts, together with analytical tools, are developed in the first part of the book and then applied to models of a small open economy, to two-country models of the world, and to a three-country world model.

The dynamic analysis proper of open economies begins in Part Two, which is devoted to models of small open economies. Two- and multiple-country models of the world are considered in Part Three.

The models analyzed incorporate the more recent portfolio balance approach to the asset sectors. In each case, the dynamics of the basic model are analyzed first and then various elaborations are discussed. Our models consistently observe stock-flow relationships. The influence of government budget imbalances and current account on stocks of domestic and foreign financial assets are modeled explicitly. The effects of wealth and interest earnings on domestic government bonds and foreign financial assets are included whenever practical.

This classification of models into the so-called small-country model and multiple-country model of the world notwithstanding, we recognize that there are several other possible groupings. Helliwell (1979), for example, makes these divisions: (i) models that impose the purchasing-power parity condition, (ii) those with the interest-rate parity condition, and (iii) those in which neither condition is imposed. We can superimpose onto our basic classification of open economies into small open and multiple-country models a dichotomy with and without wage indexation, with and without imported intermediate goods, or the distinction of the level of disaggregation of outputs by traded or nontraded goods.

The most complete model of a small open economy that we discuss in Part Two has domestic money, domestic bonds, and foreign bonds as the financial assets available to domestic residents. In the real sector, the goods produced in the economy are assumed to be an imperfect substitute for the world goods. We impose neither the purchasing-power parity condition, nor the (covered) interest-rate parity condition. Although we do not wish to be exhaustively taxonomic in our dealings, we proceed from a simple model and deal with progressively complex models. Basic models for a small open economy, two- and three-country models of the world, are later elaborated by adding further refinements in the course of our discussions of them.

How do symmetric specifications of individual countries modify dynamic behavior of the world models? Do national disturbances become synchronized? How? What are the distributional effects of exogenous shocks and policy instrument changes in the world models? We address these and other related questions in Part Three.

Long-run comparative static properties of models are well documented and have not therefore been emphasized here. Long-run equilibrium is examined only as necessary to assure that the models have not been improperly specified as far as their long-run behavior goes, that the models do not exhibit pathological properties, and to determine long-run equilibrium states or paths that can serve as reference states or reference paths in our variational analysis.

Although the basic notions of variational dynamics and the structural perturbation method are also applicable to stochastic models, we do not cover stochastic models of open economies. This topic has been reserved for a separate work in order to keep our treatment of the dynamics of deterministic open economies reasonably self-contained without being encumbered by technicalities of stochastic dynamic analysis. This is not to deny the importance of stochastic considerations in open economies. Stochastic considerations should be paramount in stabilization discussions and in discussions of choices of regimes since any proposed schemes must be evaluated for possible performance in random environments. We point out that the procedures developed in this book can be interpreted as "certainty equivalent" approaches to stochastic systems. Furthermore, the ideas of comparative dynamics and sensitivity analysis are valid for stochastic systems and will be important in assessing stochastic system dynamic behavior and policy effects, especially in conjunction with Ito's lemma.

Summaries of individual chapters can be found at the beginning of each part.

Notation

To distinguish impact analysis, which analyzes changes induced on endogenous variables at the same instant that instrument changes take place, from short-run analysis, which investigates changes of endogenous variables some time after such instrument changes occur, we use the notation Δ and δ to indicate variation of variables in impact analysis and short-run analysis, respectively. For example, $\delta i(t)$ is defined by

$$i(t) = i^0(t) + \delta i(t), \qquad t \geq t_0,$$

where i and i^0 are the respective perturbed and reference time paths of the interest rate; i.e., $\delta i(t)$ is the difference between the perturbed path and a reference time path of i. We generally use superscript 0 to refer to a value on the reference path. See Chapter 1 for the notion of reference paths. If an open market operation is conducted at time t_0, then $\delta i(t_0) = \Delta i(t_0)$ is the discontinuous (instantaneous) change in i at the time of the open market operation conducted at t_0, but $\delta i(t_0 + \delta t) \neq \Delta i(t_0)$, generally, where $\delta t > 0$ denotes a small time interval. Here we note that variables governed by differential equations do not change discontinuously.

Except for the interest rate, we use lowercase letters to denote relative deviations. For example,

$$M(t) = M^0(t)(1 + m(t)) + o(t),$$

where M denotes money stock, $o(t)$ higher order of smallness, and where we shall denote proportional changes by lowercase letters, e.g., $m(t) = \delta M(t)/M^0(t)$, where $\delta M(t) = M(t) - M^0(t)$.

The initial condition for m is given by

$$m(t_0) = \delta M(t_0)/M^0(t_0) = \Delta M(t_0)/M^0(t_0),$$

where ΔM is subject to stock constraints and is nonzero only if an open market operation is conducted at time t_0. See Section 2.3 for a more complete explanation.

In our notation, a dot over a variable denotes differentiation with respect to time, i.e., $\dot{x} = dx/dt$, $\dot{B} = dB/dt$, etc. [not to be confused with rates of change $(dx/dt)/x$ or $(dB/dt)/B$].

An overbar denotes either an exogenously fixed constant or a long-run equilibrium value or a constant value along the reference path of a variable. The caret symbol ^ over a variable indicates expected values, not rates of changes. The caret is also used to denote Laplace transforms.

In addition, a numerical subscript on a function usually refers to the partial derivative with respect to one of the arguments. For example, the derivative with respect to the first argument is represented by the subscript one. The total derivative of a function with respect to its argument is denoted by a prime. For example, the second derivative of $f(x)$ is written as $f''(x)$. Also ( ) is used to denote arguments of functions. Finally, it is assumed that the derivatives of the functions postulated exist.

To avoid repetitious writings of integral signs in expressing solutions of differential equations, we use the following shorthand notation: We write the solution of a linear differential equation

$$\dot{\mathbf{z}}(t) = A(t)\mathbf{z}(t) + B(t)\mathbf{x}(t)$$

as

$$\mathbf{z}(t) = Z(t, 0)\mathbf{z}(0) + (Z, B\mathbf{x})(t, 0),$$

where the matrix $Z(t, s)$ is the transition matrix (fundamental solution matrix) of the differential equation. When the matrix A is constant, then the transition matrix $Z(t, s)$ equals $\exp(A(t - s))$. The second term of the solution expresses the effect of the instrument $\mathbf{x}$ on the solution and is given in our shorthand notation. It stands for the integral

$$(Z, B\mathbf{x})(t, 0) = \int_0^t Z(t, s)B(s)\mathbf{x}(s)\, ds.$$

We also write this as $(ZB, \mathbf{x})$.

Because of linearity of integration, the following relations are valid for our notation:

(P1) $$(Z, aB\mathbf{x}) = a(Z, B\mathbf{x}) \qquad \text{for any scalar } a,$$

(P2) $$(Z, B\mathbf{x} + D\mathbf{v}) = (Z, B\mathbf{x}) + (Z, D\mathbf{v}),$$

(P3) $$F(Z, B\mathbf{x}) = (FZ, B\mathbf{x})$$

for a constant matrix F which is compatible with Z, i.e., for which FZ is defined.

To illustrate the use of this notation, we write the solutions of the next set of two coupled differential equations in our notation. Given

$$\dot{\mathbf{z}} = A\mathbf{z} + B\mathbf{w} + C\mathbf{x}, \qquad \dot{\mathbf{w}} = D\mathbf{w} + E\mathbf{v},$$

where $\mathbf{x}$ and $\mathbf{v}$ are the exogenous inputs (instruments), the solution of the second differential equation is, in our notation,

$$\mathbf{w}(t) = W(t, 0)\mathbf{w}(0) + (W, E\mathbf{v}),$$

where $W(t, s)$ is the transition matrix of the second equation. The first differential equation has the solution of the form

$$\mathbf{z}(t) = Z(t, 0)\mathbf{z}(0) + (Z, B\mathbf{w} + C\mathbf{x}) = Z(t, 0)\mathbf{z}(0) + (Z, B\mathbf{w}) + (Z, C\mathbf{x}),$$

where (P2) is used and $Z(t, s)$ is the transition matrix of the first equation. The term $(Z, B\mathbf{w})$ can further be written by substituting the solution for $\mathbf{w}$ as

$$(Z, B\mathbf{w}) = (Z, BW)\mathbf{w}(0) + (Z, B(W, E\mathbf{v})),$$

where the first term stands for $[\int_0^t Z(t, s)B(s)W(s, 0)\, ds]\mathbf{w}(0)$.

We introduce another notational convention to express the last term of the above equation so that the effect of the instrument $\mathbf{v}$ is more directly exhibited. This can be done by changing the order of integration for which this shorthand notation stands:

$$\int_0^t Z(t, \tau)B\left[\int_0^t W(\tau, s)E\mathbf{v}(s)\, ds\right] d\tau = \int_0^t\left[\int_s^t Z(t, \tau)BW(\tau, s)\, d\tau\right]E\mathbf{v}(s)\, ds.$$

Thus we define

$$(BW)^*Z(t, s) = \int_s^t Z(t, \tau)BW(\tau, s)\, d\tau.$$

With a constant B, $(Z, B\mathbf{w})$ equals $(ZB, \mathbf{w})$. Hence we have the equality $(BW)^*Z = W^*(ZB)$. We can write for a constant E[1]

(P4) $$(Z, B(W, E\mathbf{v})) = (ZB, (W, E\mathbf{v})) = (W^*ZB, E\mathbf{v})$$
$$= (W^*ZBE, \mathbf{v}) = \int_0^t (W^*ZB)(t, s)E\mathbf{v}(s)\, ds,$$

where

$$(W^*ZB)(t, s) = \int_s^t Z(t, \tau)B(\tau)W(\tau, s)\, d\tau.$$

[1] If we strictly follow our notational convention, $(Z, B\mathbf{w}) = (B^*Z, \mathbf{w}) = (B^*Z, (W, E\mathbf{v})) = (W^*(B^*Z), E\mathbf{v}) = (E^*(W^*(B^*Z)), \mathbf{v})$. However, only for a transition matrix is the operation * crucial. We can always resolve any doubt by returning to the original integral expressions, e.g.,

$$(E^*(W^*(B^*Z)), \mathbf{v}) = \int_0^t \left\{\int_s^t Z(t, \tau)B(\tau)W(\tau, s)E(s)\mathbf{v}(s)\, d\tau\right\} ds.$$

List of Symbols

Asset sector variables

A	Nominal wealth of the private sector
B	Stock of domestic bonds held by domestic private sector[1]
B_g	Stock of government bonds held by the domestic private sector[2]
D	Domestic component of domestic money stock (domestic credit)
E	Exchange rate (domestic currency price of a unit of foreign currency)
F	Stock of international bonds domestically held, i.e., either by the domestic government and the private sector or by the private sector alone[3]
R	Foreign exchange (international reserve) held by the domestic government (central bank)
V	$B + B_g$ when private and government bonds are assumed to be perfect substitutes
i	Domestic interest rate
π	Expected domestic output price inflation
ε	Expected rate of depreciation of the exchange rate

Real sector variables

B_T	Balance of trade in foreign exchange, i.e., $PX/E - P^*I_m$

[1] Bonds are sometimes treated as consols and at other times as fixed in nominal value with variable rate of return. In the former, the symbol B stands for the number of consols so that B/i is the market value in domestic currency.

[2] This distinction between B and B_g is not always made for simple models.

[3] B^* is sometimes used to denote stock of international bonds held by the domestic private sector.

Real sector variables

C	Real consumption demand for domestic goods by domestic residents
D_f	Domestic government budget deficit
$F(\ )$	Production function
G	Government expenditure on current goods and services
I	Real investment
I_m	Real imports demand
K	Capital stock
qK	Real value of the stock of equity capital
P	Output good price
P_I	Consumer price index
T	Real tax receipts
X	Real export demand
W	Money wage rate (except in Chapter 3, where W is used to denote real private wealth)
l	Coefficients in variational real demand equations
Ω	Coefficients in variational expressions for balance of trade

Various Greek letters are used to indicate exogenous parameters such as adjustment speeds.

Dynamics

Φ, Ψ, A	Usually denote dynamic matrices in state-space representation of variational dynamics
z	State vector (except in Chapter 14)
μ_i or m_i	Coefficients in the variational government budget constraint equation

$\tau = (\partial T/\partial Y)(Y/T)$

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