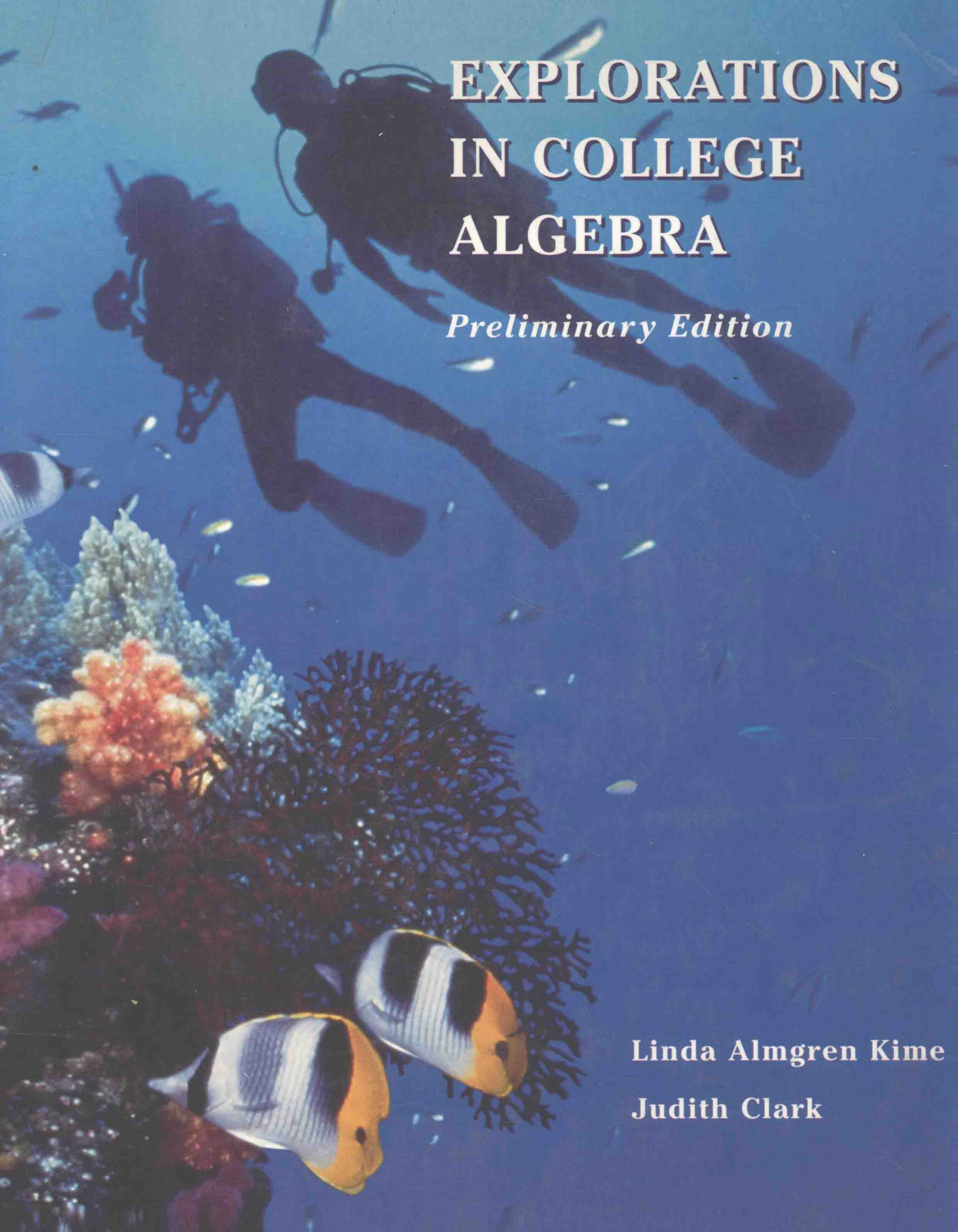

EXPLORATIONS IN COLLEGE ALGEBRA

Preliminary Edition

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Explorations in College Algebra

Discovering Algebra from Data Based Applications

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Funded by a National Science Foundation Grant

This material is based upon work supported by the National Science foundation under Grant No. USE- 9254117

This project was supported, in part, by the
National Science Foundation
Opinions expressed are those of the authors and not necessarily those of the Foundation

ISBN: 0471-10699-2

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

Dedicated to:

Theresa Mortimer, Associate Provost at UMass/Boston, who initially brought us together and whose generous and unwavering support has been critical throughout this project

and

to all of our students who have been thoughtful critics and enthusiastic participants in the development of these materials.. Their sincere and candid comments on all phases of the course have indelibly shaped its content.

Preface

This book has been developed by a consortium of schools from an experimental course, Explorations in College Algebra, funded in part by a grant from the National Science Foundation, and initially taught at the University of Massachusetts, Boston. The course materials have now been tested in fifteen different schools. This text and the accompanying software are the result of this ongoing collaboration.

The course was designed in the spirit of the calculus reform movement and we intend the course to be effective preparation for other reformed college mathematics courses. Our goal is to give students sufficient mathematical confidence and skills so they can continue on in more advanced mathematics or other quantitatively based courses. Equally important, we hope students come to understand and appreciate the basic uses of mathematics in our society.

The content and approach was shaped by the reform guidelines that have been laid out by the various mathematical professional societies (including the MAA and AMATYC). The materials are designed to shift the focus from learning a set of discrete mechanical rules to exploring how algebra is used in the social, physical, and life sciences.

Our approach

The materials are designed around the following general principles:

- the development of algebra from data based applications
  We start with quantitative questions from the social, physical and life sciences and learn how mathematics can provide powerful strategies to help answer them. Real problems and real data motivate students to learn how to use algebraic and technological tools.

- the active involvement of students
  Our course advocates actively engaging students in class discussions and team work. The explorations included at the end of each Chapter are designed for use by small groups of students working together in or out of class. The explorations often pose open-ended problems with no single "correct" solution. Students work collaboratively to synthesize information from class lectures, the text and readings, and most importantly from their own discoveries.

- the use of multiple representations
  Graphs, tables and symbolic expressions are used to explore patterns in data, test conjectures and make generalizations about the properties of functions. Students construct their own meaning of algebraic rules and procedures by making connections among these representations.

- the communication of ideas
  Communication of ideas is stressed through use of class discussions, small group work, student presentations and written reports. Suggestions for writing strategies are included in Chapter 1, and many of the assignments require students to

describe their conclusions in writing. Students are encouraged to read the writings of others as represented in the variety of essays, articles, and reports in the Anthology of Readings.

- the integration of technology

  While we encourage the use of technology and have built many explorations and exercises around graphing calculators and computers, there are no specific technology requirements. Some schools use graphing calculators, others use computers and some a combination of both. The course was originally designed to be taught in a computer lab (Mac or IBM) equipped with a generic spreadsheet and function graphing program. The materials have now been expanded for use with a graphing calculator.

  In addition, the book comes with easy to use interactive software (for both Mac and IBM) that helps students visualize particular mathematical concepts. The software can be used by students in a lab or at home or by the instructor for in-class demonstrations.

Our process of development

The text has been developed and refined over the last three years by experts from many different disciplines. We were able to write materials and then test them out the next day in class. Our students graciously allowed us to experiment, and sometimes fail, as we tried out new materials and new approaches. Their input, more than any other, shaped our ideas.

John Wiley & Sons conducted an extensive survey of faculty members teaching college algebra courses. The results helped us decide which topics to include in the text, but also convinced us that there is no such thing as a generic college algebra course.

The materials are still under development and we hope that those of you who are using this preliminary edition will help us to make the materials stronger. Revisions will be based on your comments and suggestions.

Content

The course is motivated by the need to answer practical questions arising in science and society: Is there a relationship between income and years of education? How fast do bacteria grow? How can we describe to ourselves and our children the size and age of our universe ?

The text includes more material than college algebra courses usually include in one semester. Instructors will need to make choices and may wish to cover the chapters in a different order. It is recommended that students become familiar with the material covered in Chapters 1, 2, 3 & 4, before attempting to do the other chapters. The rest of the chapters may be covered in any order, with the exception that the material on exponents in Chapter 7 should be covered prior to material in the other chapters in Part II. We assume that students have had elementary high school algebra and are familiar with the use of symbols to represent unknown numerical quantities.

Part I focuses on algebraic applications in the social sciences. Chapter 1 introduces the basic issues of working with and writing about data. This chapter is the least like a traditional algebra chapter, but perhaps the most relevant to students' everyday lives. We have found that plunging students the first day of class into collecting data about themselves, is a successful strategy for immediately setting the tone of active student participation throughout the course. Students learn the basics of using a graphing calculator or computer. The amount of time spent on this chapter (from 2 days to 3 weeks) will vary a great deal from instructor to instructor. The time spent is probably dictated by what mathematics courses most of the students would take next and how long it takes for students to become familiar with a particular technology.

The fundamental concept of functions and their representations in tables, graphs and equations are introduced in Chapter 2 . In Chapter 3, students encounter the notion of the *average rate of change*, which becomes a central thread throughout the course.

Chapter 4 is the first of three chapters dedicated to *linear functions* and their applications. It is motivated by looking at phenomena in which the rate of change is constant and focuses on constructing, graphing and interpreting linear functions. Chapter 5 introduces a major tool of the social sciences - fitting lines to data. Students use real data about 1000 families in the US. to describe and analyze the relationship between education and income. Chapter 6 deals with systems of linear equations and their uses in a number of contexts, many economic; for example, break even costs for gas, electric or solar heating and the proposed graduated vs. flat income tax plans for the state of Massachusetts.

Part II starts with Chapter 7, "Deep Time and Deep Space." Studying the age and size of objects in the known universe provides a context for learning *scientific notation* and strategies for comparing objects of widely differing sizes. Students learn to estimate answers, a critical skill in this age of calculators and computers. The basic laws of manipulating *exponents* are extended to different bases and powers. *Logarithms* are used to scale graphs and to construct measurement systems for numbers with different orders of magnitude.

The growth of *E. coli* bacteria motivates the study of exponential functions in Chapter 8 ("Growth and Decay.") The model of exponential growth and decay is applied to human populations, radioactive decay and Medicare costs. *Exponential functions* are plotted on a semi-log graph. The text currently deals with exponential functions in the form $y = C a^{x}$, but future editions will include functions of the type $y = C e^{rx}$. Chapter 9 introduces the family of *polynomial functions* by first studying the *power functions*. Power functions are introduced through studying the behavior of gases and are applied to a scuba diving problem, which was the inspiration for the cover of our book.

Chapters 10 and 11 introduce the family of *quadratic functions*. A classic free fall experiment in Chapter 10 gives students some insights into how Galileo uncovered the basic laws of motion. Instructors are encouraged to arrange for students to visit a physics lab or use a motion sensor attached to a graphing calculator to collect the data for themselves. If this is not possible, data collected by other students is provided on the course disk. In Chapter 11 ("Parabolic Reflections") the properties of quadratics (symmetry, vertices, x and y-intercepts) are examined.

Part III provides a collection of diverse experiments from the social and physical sciences for use by small groups of students. Different groups can work on different explorations, or the entire class could work on the same exploration. You may want to design your own exploration and share it with us. The instructor does not have to have covered all the chapters in the book, as long as the basic notion of curve fitting has been discussed. At this point students hopefully have the basic knowledge to apply algebraic techniques developed throughout the course. These explorations do not occur at the end of a chapter focused on a particular function. Students must make their own decisions about the kinds of patterns that occur. Is there a relationship between infant mortality rate and GNP? If so, of what kind? How can we describe the rate at which sucrose is converted to fructose and glucose? What affects this rate? Groups are asked to present their findings and discuss them with the class.

Special features

Our curriculum mandates an unusual book format. As other books do, our book contains explanatory text and related exercises; in addition there are:

- explorations
- an anthology of pertinent readings
- graphing calculator instructions
- course software
- data files that can be used with graphing calculators or with spreadsheets

Text supplements include: Instructor's Manual, an Instructor's Solution Manual and a Student's Solution Manual. The supplements along with the course software and data files are available upon request from your Wiley representative or by contacting John Wiley & Sons (see address below.)

Readers' comments

The authors would be delighted to receive reactions to this preliminary edition and welcome any suggestions for homework problems or explorations that could be included in the first edition. Please contact us by mail at the University of Massachusetts at Boston, 100 Morrissey Blvd, Boston, MA 02125 or by e-mail at *algebra@umbsky.cc.umb.edu*

Readers who are interested in further information about the materials or in site testing the course, should contact: John Wiley & Sons, Inc. by mail at 605 Third Avenue New York, NY 10158 or by e-mail at *math@wiley.com.*

Acknowledgments

We wish to express our appreciation to all those who have helped and supported us in this collaborative endeavor. We are grateful for the support of the National Science Foundation, whose funding made this project possible, and for the generous help from our program officers, Elizabeth Teles and Marjorie Enneking, We wish to thank the members of our Advisory Board: Deborah Hughes Hallett, Philip and Phylis Morrison, Nicholas Rubino and Ethan Bolker for providing encouragement and invaluable advice in each phase of our work.

The text could not have been produced without the generous and on-going support we received from students, faculty, staff and administrators at UMass/Boston. Our thanks especially to the administrators who provided resources for many aspects of the project: Chancellor Sherry Penney, Provost Louis Esposito, former Provost Fuad Safwat, Associate Provost Theresa Mortimer, and Deans Patricia Davidson, Christine Armett-Kibel, Ellie Kutz. and Martin Quitt. Special thanks to Paul O'Keefe, Mike Larsen, and Carol DeSouza for helping us with the endless paperwork and procedures that are an inevitable part of obtaining and running a grant.

We are especially grateful to the following colleagues at UMass and elsewhere, for their contributions to the text. Bob Seeley, George Lukas, Rachel Skvirsky, Ron Etter, Lowell Schwartz, Max West, Tony Roman, Joe Check, Suzy Groden, Joan Lukas, Bernice Auslander, Karen Callaghan, Lou Ferleger, Art MacEwan, Randy Albelda, Rosanne Donahue, Gourish Hosangady, Bob Lee, Ken Kustin, John Looney, Brenda Cherry, Peg Cronin, Bob Morris, Margaret Zaleskas, Debra Borkovitz, Paul Foster, Elizabeth Cavicchi, Eric Entemann, Bob Martin, David Hruby, and Steve Rodi. Particular thanks are due to Barry Bluestone from whom we borrowed the concept of the FAM 1000 data set used in Chapter Five "Looking for Links."

A project cannot function without an efficient and forgiving Project Administrator. In Theresa Fougere we found a woman with a divine combination of attributes: common sense, dedication, attention to detail and a sense of humor. Our special thanks to her and others who have shared administrative duties: Clare Crawford, Marie Coleman, Aurora Alamariu, David Wilson, Val Goktuk, Jonathan Rose, Wendy Sanders, Matt Gunderson, Karen Sullivan, Eric Dunlap, Jan McLeod, Matt Smith, Lynne Bowen, Patricia Dognazzi, and John Harper.

A text designed around the application of real world data would have been impossible if not for the long and selfless hours put in by Myrna Kustin, researching and acquiring copyright permissions from around the globe.

We also wish to thank our terrific student teaching assistants: John Koveos, Tony Beckwith, Irene Blach, Philip Wan, Kristen Demopoulos, Tony Horne, Cathy Briggs and Arlene Russo. They excelled as students and as teachers.

We are deeply indebted to Peter Renz, Madalyn Stone and Dick Cluster for their gracious and supportive editorial help.

Kudos to our publisher, John Wiley & Sons. We are grateful that William Pesce gave us a chance to present our ideas to the staff at Wiley. He asked good questions and we are still searching for some of the answers. Eileen Navagh, Wayne Anderson, and many others at Wiley have been most helpful. Particular thanks goes to our wonderful editor, Ruth Baruth. After our first two hour lunch in Harvard Square, we knew that we would have a pleasant long term relationship. Ruth has provided the gentle guidance necessary for first time authors to publish a text with multiple contributors. She has generous given us many hours of practical advice and is always cheerful about helping with the endless details of publishing a book.

We are especially grateful to the 14 beta sites scattered across the U.S.. One of the joys of this project has been working with so many dedicated faculty who are searching for new ways to reach out to students. These faculty all offered incredible support and encouragement, and a wealth of helpful suggestions. Our heartful thanks to: Jean Prendergast and Keith Desrosiers, Bridgewater State College; Sandi Athanssiou, Erika Kwiatkowski, and Mark Yannotta, University of Missouri-Columbia; Judy Stubblefield, Garden City Community College; Josie Hamer, Robert Hoburg, and Bruce King, Western Connecticut State University; Peggy Tibbs and John Watson, Arkansas Tech University; Peg McPartland, Golden Gate University; Russell Reich, Sierra Nevada College; Christopher Olsen, George Washington High School; Lida McDowell, Jan Davis, and Jeff Stuart, University of Southern Mississippi; Judy Jones and Beverly Taylor, Valencia Community College.

Our families couldn't help but become caught up in this time consuming endeavor. Linda's husband, Milford, cheerfully helped subsidize a sabbatical year and her son, Kristian, compiled the text's index. Judy's husband, Gerry, became our Consortium lawyer, and her daughters, Rachel, Caroline and Kristin provided support, understanding, laughter and "whatever." All our family members ran errands, made dinners, listened to our concerns and gave us the time and space to work on the text. Our love and thanks.

Finally, we wish to thank all of our students. It is for them that this book was really written.

A Note to the Students

Mathematics is a filter, a way of looking at the world. Just as musical training increases your awareness of sounds, or knowledge of history gives a deeper perspective on the present, mathematics heightens your perception of underlying order and systematic patterns in numerical information.

Numbers are adjectives, one of many ways that help us describe things. For instance, we might say an object is yellow, red and brown, is in the shape of a 12 inch circle and smells like pepperoni with extra cheese. All of these descriptions are potentially useful facts, but the 12 inch circle is the only piece of data which gives us quantitative information. From this fact we can calculate the area of the pizza, the size of a box that can contain it, and a fair price for it relative to a 9 inch pizza. Unfortunately there are many people who are afraid to make such use of quantitative facts. If you are like many of the intelligent students we've encountered, who have little confidence in their ability to do math, this course is intended to help you overcome your fears and become comfortable describing things quantitatively.

Many skills are involved in being able to generate a useful quantitative description. In this course, the focus is much broader than in a traditional skills-based college algebra course. Much of the work may seem at first unrelated to the kind of mathematics you've done in previous courses. Our hope is that it will enable you to use quantitative reasoning throughout the rest of your life. You will be asked to *read* and analyze mathematical arguments from a variety of different sources and viewpoints, from contemporary newspaper articles and essays, to publications from the federal government. You will be asked to *reason* through your own quantitative arguments, using both traditional algebraic tools and contemporary technological ones. And you will be asked to *write* about your conclusions.

The classes may not look much like the ones you're used to. The professor might lecture a lot less, and you may find yourself working with your classmates in small groups. Our exploratory approach results in a lot more questioning and discussion than is common in math classes. Sometimes you might wonder why the professor doesn't tell you straight out what you're supposed to learn, instead of allowing you to explore the issue. The answer is that sometimes there is no "straight out" solution to the problem, and that the process of exploring can be as important as memorizing the "facts."

A former student: wrote to us that

> Doing problem solving and learning about the computer and the rules of algebra all at once was a struggle - but now that we are almost done, I can say that I thoroughly enjoyed this class. I learned how to relax and to struggle with finding answers to hard questions.

We hope that you too will enjoy doing math and that mathematics will become one of the tools you use to understand and participate in the world around you. We also hope to hear from you about your ideas for exploring algebra.

With warm regards,
Linda Kime and Judy Clark
Cambridge, Massachusetts
June, 1996

HOW TO USE DIFFERENT PARTS OF THE TEXT

This book has many parts: chapters on specific topics, explorations, exercises, readings and graphing calculator instructions. We encourage you to be flexible in how you use the materials and spend some time just browsing through the text.

CHAPTERS

The text is designed in a way that allows you to be thinking and doing as you read it. Each chapter contains sections called *Algebra Aerobics* and *Something to think about.* The Algebra Aerobics provide a chance for you to practice algebraic rules and procedures. These are skill building exercises and we encourage you to do the problems as you are reading the text. We have provided answers at the bottom of the page so you can check your work. The Something *to think about* sections pose provocative questions for you to ponder alone or discuss with others in or outside of class.

EXPLORATIONS

The Explorations allow you to experiment with the ideas presented in each chapter. They are more open-ended than the exercises and are designed to be used in parallel with reading the text. They can be done in small groups inside or outside of class. Some of the explorations and exercises assume you have access to graphing calculators or computers. We have noted in the text when this is the case. You may be asked to present your results to the class. We have found that after students' initial fears are dispelled, the presentations often became the most interesting part of the course.

EXERCISES

The exercises contain many different types of problems: skill building exercises where you practice using algebraic rules and procedures and problems that require you to reason critically and exercises that ask you to express your ideas in writing.

GRAPHING CALCULATOR APPENDIX

The Graphing Calculator Appendix provides instructions for using the TI-82 and TI-83 graphing calculators along with exercises to practice specific techniques. If you are using a graphing calculator for the first time, you should do these exercises before doing the explorations or exercises in each chapter.

ANTHOLOGY OF READINGS

The Anthology of Readings are meant to supplement the ideas developed in the text and to give you a glimpse of the complexity of the issues raised in each chapter. You might want to look at the Table of Contents and read those articles of interest whether or not they are assigned by your instructor.

We hope that you explore the ideas presented in each chapter by talking, thinking and working together.

Part I: Algebra in the Social Sciences

Exploring Data

Exploring Functions

Measuring Change

Linear Functions

Power Functions and Polynomials

Quadratic Functions

Part III: Exploring On Your Own

Appendices

Part I

Algebra in the Social Sciences

Introduction to Part I

Secretary of Labor Robert Reich pronounces "learning as the key to earning" in a speech on the eve of Labor Day 1994. He asserts that "The fundamental fault line running through today's workforce is based on education and skills. ... As recently as 1979 a male college graduate earned 49 percent more than a similar man with only a high school diploma... By 1992, however, the average male college graduate was earning 83 percent more than his high-school graduate counterpart, and the notion of common prospects had faded considerably."

Presidential hopefuls battle over the virtues of a flat income tax. Republican analyst Kevin Phillips claims that the public fascination is based on the misconception that "the flat tax, by closing loopholes, will make the rich pay more." Yet according to Steve Forbes "Everyone gets a tax break with the flat tax."

Life expectancy at birth for both sexes has increased dramatically in the last century for Americans. In 1900 the average life expectancy was slightly over 49 years. By 1990 the average life expectancy had risen to 75.8 years. This is due in large part to the fact that the United States is the world's richest nation and it spends far more of its income on health care than does any other. Yet according to the United Nations' 1994 Human Development Report, people in other countries live longer and get more care.

Political and personal decisions depend on understanding the benefits of education, analyzing the merits of a flat tax, or knowing the impact of gains in life expectancy. These issues are addressed by the social sciences: political science, economics, history, demography and sociology, the disciplines that deal with the functioning of our society. Unlike physical scientists who try to discover fundamental laws of the universe, social scientists do not expect to discover exact principles that govern society. But general societal trends can be uncovered by collecting numerical data and using algebraic strategies to search for patterns. The conclusions from such research often form the basis for decisions about how our society should operate.

Part I approaches the standard topics in college algebra from the perspective of the social sciences. The tools required to deal with real data are developed as needed: verbal and visual methods of data presentation; algebraic methods of single and two-variable data analysis including numerical data summaries, rate of change calculations, linear functions and linear regression models; and systems of equations. Throughout this process we raise questions about how good the data and accompanying conclusions are.