RICE · STRANGE

PLANE TRIGONOMETRY

FOURTH EDITION

Plane Trigonometry Fourth Edition

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Metric and English Measurements

Metric Equivalents		English Equivalents
1 meter = 1000 millimeters = 100 centimeters = 0.001 kilometer	Length	1 foot = 12 inches 1 yard = 3 feet
1 hectare = 10,000 square meters	Area	1 acre = 43,560 square feet
1 liter = 1000 milliliters = 0.001 kiloliter	Volume	1 quart = 2 pints
1 gram = 1000 milligrams = 0.001 kilogram	Weight	1 pound = 16 ounces

Approximate Equivalents

Metric-English

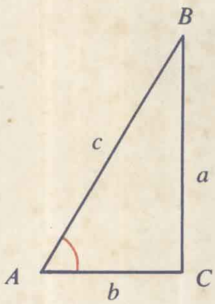
1 centimeter =·0.3937 inch
1 meter = 39.37 inches
1 meter = 3.281 feet
1 kilometer = 0.62 mile
1 liter = 1.057 U.S. quarts
1 kilogram = 2.2 pounds
1 hectare = 2.47 acres

English-Metric

1 inch = 2.54 centimeters
1 yard = 0.914 meter
1 foot = 0.3048 meter
1 mile = 1.6 kilometers
1 quart = 0.946 liter
1 pound = 0.45 kilogram
1 acre = .404 hectare

Triangles

Right Triangles

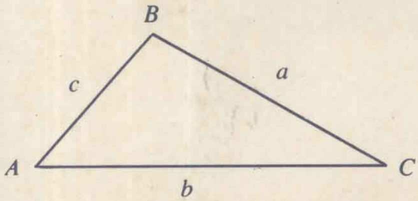


$$\sin\theta = \frac{a}{c} \qquad \cot\theta = \frac{b}{a}$$

$$\cos\theta = \frac{b}{c} \qquad \sec\theta = \frac{c}{b}$$

$$\tan\theta = \frac{a}{b} \qquad \csc\theta = \frac{c}{a}$$

Oblique Triangles



Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Trigonometric Identities

(1) $\sin A = \dfrac{1}{\csc A}$

(2) $\cos A = \dfrac{1}{\sec A}$

(3) $\tan A = \dfrac{1}{\cot A}$

(4) $\tan A = \dfrac{\sin A}{\cos A}$

(5) $\cot A = \dfrac{\cos A}{\sin A}$

(6) $\sin^2 A + \cos^2 A = 1$

(7) $1 + \tan^2 A = \sec^2 A$

(8) $1 + \cot^2 A = \csc^2 A$

(9) $\sin (A + B) = \sin A \cos B + \cos A \sin B$

(10) $\sin (A - B) = \sin A \cos B - \cos A \sin B$

(11) $\cos (A + B) = \cos A \cos B - \sin A \sin B$

(12) $\cos (A - B) = \cos A \cos B + \sin A \sin B$

(13) $\tan (A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(14) $\tan (A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Trigonometric Identities

(15) $\sin 2A = 2 \sin A \cos A$

(16) $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

(17) $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

(18) $\sin \tfrac{1}{2}A = \pm\sqrt{\dfrac{1 - \cos A}{2}}$

(19) $\cos \tfrac{1}{2}A = \pm\sqrt{\dfrac{1 + \cos A}{2}}$

(20) $\tan \tfrac{1}{2}A = \dfrac{\sin A}{1 + \cos A}$

(21) $\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$

(22) $\sin A - \sin B = 2 \cos \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$

(23) $\cos A + \cos B = 2 \cos \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$

(24) $\cos A - \cos B = -2 \sin \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$

(25) $\sin A \cos B = \tfrac{1}{2} \sin [(A + B) + \sin (A - B)]$

(26) $\cos A \sin B = \tfrac{1}{2} \sin [(A + B) - \sin (A - B)]$

(27) $\cos A \cos B = \tfrac{1}{2} \cos [(A + B) + \cos (A - B)]$

(28) $\sin A \sin B = \tfrac{1}{2} \cos [(A - B) - \cos (A + B)]$

Preface

This fourth edition of *Plane Trigonometry* retains the essential features that made the first three editions successful. They are (1) an early emphasis on the trigonometric definitions for acute angles imposed on the coordinate plane with the applications to the right triangle as a special focus, (2) an abundance of examples and exercises, and (3) a large variety of applications.

To simplify the early discussion of angles and the evaluation of the trigonometric ratios, we have delayed the introduction of radian measure until needed for arc length measurements, angular velocity and transition to the trigonometry of real numbers. Other changes to this edition include:

- an expanded discussion of vectors in Chapter 3 and a new section on sinusoidal modeling in Chapter 5.
- the use of the calculator as the primary tool for evaluating the trigonometric functions. (Tables and related topics such as interpolation are included as optional material.) We believe the change from the use of tables to the use of the calculator is both efficient and realistic since students are now using calculators at all levels of mathematics education.
- the addition of new applications to almost every exercise set. These range in emphasis from the natural sciences, to aerospace, to psychology.
- the addition of a geometry appendix. This appendix provides a handy reference for the elementary definitions and formulas of plane geometry.
- the inclusion of *WARNING* and *COMMENT* labels to highlight common errors or some pertinent explanation.

As with most books, there are more topics than even the most ambitious instructor could comfortably cover in a one-semester course. Not all of the applications in Chapter 3 need be covered and certain formulas (such as the formula for the Area) may be omitted without loss of continuity. Chapter 9 on complex numbers and polar coordinates is dependent on earlier chapters but is not used elsewhere in the book. Chapter 10 on logarithms may be covered at any time.

A special thanks in this edition goes to Professor Lois Miller of El Camino College for her suggestions for improving the text. The section in Chapter 5 on sinusoidal modeling was significantly influenced by her suggestions on including more and different applications.

This edition of *Plane Trigonometry* has benefited from critical reviews and comments from the following: Professor Arthur P. Dull, Diablo Valley College; Professor Ralph Esparza, Richland College; Professor Allen E. Hansen, Riverside City College; Professor Ferdinand Haring, North Dakota State University; Professor Don. L. Merrill, Utah Technical College; Professor John Spellman, Southwest Texas State University; Professor Donna M. Szott, Community College of Allegheny County–South Campus. We wish to thank all of these people for their valuable comments.

It is also a pleasure to acknowledge the fine cooperation of the staff of Prindle, Weber & Schmidt—particularly our editor Dave Pallai and production editor Susan London. Finally, we want to express our thanks to the production staff of Lifland et al., Bookmakers who handled much of the work in actually producing the text.

Bernard J. Rice
Jerry D. Strange

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1

Some Fundamental Concepts

Historical Background

Trigonometry is one of the oldest branches of mathematics. An ancient scroll called the Ahmes Papyrus, written about 1550 B.C., contains problems that are solved by using similar triangles, the heart of the trigonometric idea. There is historical verification that, in about 1100 B.C., the Chinese made measurements of distance and height using what is essentially right-triangle trigonometry. The subject eventually became intertwined with the study of astronomy. In fact, the Greek astronomer Hipparchus (180–125 B.C.) is credited with compiling the first trigonometric tables and thus has earned the right to be known as "the father of trigonometry." The trigonometry of Hipparchus and the other astronomers was strictly a tool of measurement, and it is, therefore, difficult to classify the early uses of the subject as either mathematics or astronomy.

In the fifteenth century, trigonometry was developed as a discipline within mathematics by Johann Muller (1436–1476). This development created an interest in trigonometry throughout Europe and thus placed Europe in a position of prominence with respect to astronomy and trigonometry.

In the eighteenth century, trigonometry was systematically developed in a completely different direction, highlighted by the publication in 1748 of the now-famous "Introduction to Infinite Analysis" by Leonhard Euler (1707–1783). From this new viewpoint, trigonometry did not necessarily have to be considered in relation to a right triangle. Rather, the analytic or functional properties became paramount. As this wider outlook of the subject evolved, many new applications arose, especially for describing physical phenomena that are "periodic."

To read this book profitably, you should have some ability with elementary algebra, particularly manipulative skills. Some of the specific background knowledge you will need is presented in this chapter.

1.1 The Rectangular Coordinate System

We often wish to make an association between points on a line (or in a plane) and numbers, a process called coordinatization. The number (or numbers) assigned to a point is called the **coordinate** (or coordinates) of the point.

To associate points with real numbers, choose any straight line, and then choose any point on the line to be the starting point, or origin. Take any unit distance and measure that distance to the right of the origin. Then the number 0 is associated with the origin, the number 1 with the point a unit distance to the right of the origin, the number 2 with the point two units to the right of the origin, etc. In this way, the so-called **integral points** are determined. Points in between are coordinated by noninteger real numbers. The line, illustrated in Figure 1.1, is called a **real number line**.

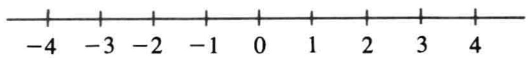

Figure 1.1 Real Number Line

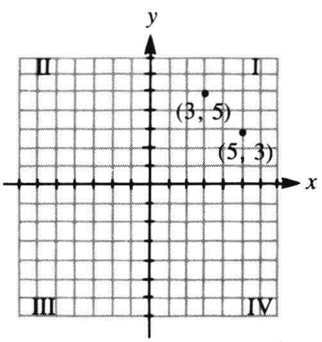

Figure 1.2 Cartesian Coordinate System

For purposes of elementary trigonometry, the most important type of coordinatization is the association of each point in the plane with a pair of numbers. In this case, we choose two mutually perpendicular intersecting lines, as shown in Figure 1.2. Normally, the horizontal line is called the ***x*-axis**, the vertical line is called the ***y*-axis**, and their intersection is called the **origin**. When considered together, the two axes form a rectangular coordinate system.* As you can see, the coordinate axes divide the plane into four zones, or **quadrants**. The upper right quadrant is called *the first quadrant*, and the others are numbered consecutively in a counterclockwise direction from this one, as in Figure 1.2. The coordinate axes are not considered to be in any quadrant.

To locate points in the plane, use the origin as a reference point and lay off a suitable scale on each of the coordinate axes. The displacement of a point in the plane to the right or left of the y-axis is called the ***x*-coordinate**, or **abscissa**, of the point, and is denoted by x. Values of x measured to the right of the y-axis are *positive* and to the left are *negative*. The displacement of a point in the plane above or below the x-axis is called the ***y*-coordinate**, or **ordinate**, of the point, and is denoted by y. Values of y above the x-axis are *positive* and below the x-axis are *negative*. Together, the abscissa and the ordinate of a point are called the **coordinates** of the point. The coordinates of a point are conventionally written in parentheses, with the abscissa written first and separated from the ordinate by a comma—that is, (x, y).

* This system is also called the Cartesian coordinate system in honor of René Descartes, who invented it.

We see that a point (x, y) lies

- in quadrant I if both coordinates are positive,
- in quadrant II if the x-coordinate is negative and the y-coordinate is positive,
- in quadrant III if both coordinates are negative,
- in quadrant IV if the x-coordinate is positive and the y-coordinate is negative.

Since the first number represents the horizontal displacement and the second the vertical displacement, order is significant. For example, the ordered pair (3, 5) represents a point that is displaced 3 units to the right of the origin and 5 units up, whereas the ordered pair (5, 3) represents a point that is 5 units to the right and 3 units up. The association of points in the plane with ordered pairs of real numbers in an obvious extension of the concept of the real line.

To be precise, we should always distinguish between the point and the ordered pair; however, it is common practice to blur the distinction and say "the point (x, y)" instead of "the point whose coordinates are (x, y)."

Each point in the plane can be described by a unique ordered pair of numbers (x, y), and each ordered pair of numbers (x, y) can be represented by a unique point in the plane called the **graph** of the ordered pair.

Example 1. Locate the points (a) $P(-1, 2)$, (b) $Q(2, 3)$, (c) $R(-3, -4)$, (d) $S(3, -5)$, and (e) $T(\pi, 0)$ in the plane.

Solution

(a) $P(-1, 2)$ is in quadrant II because the x-coordinate is negative and the y-coordinate is positive.
(b) $Q(2, 3)$ is in quadrant I because both coordinates are positive.
(c) $R(-3, -4)$ is in quadrant III because both coordinates are negative.
(d) $S(3, -5)$ is in quadrant IV because the x-coordinate is positive and the y-coordinate is negative.
(e) $T(\pi, 0)$ is not in any quadrant, but lies on the positive x-axis.

The points are plotted in Figure 1.3.

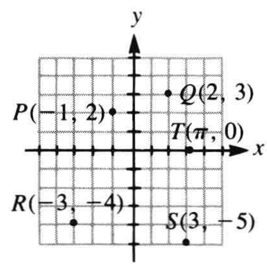

Figure 1.3 Locating Points in the Plane

When an entire set of ordered pairs is plotted, the corresponding set of points in the plane is called the **graph** of the set.

Example 2. Graph the set of points whose abscissas are greater than -1 and whose ordinates are less than or equal to 4.

Solution. This set is described by the two inequalities

$$x > -1$$

$$y \leq 4$$

The shaded region in Figure 1.4 is the graph of the set. The solid line is part of the region, whereas the broken line is not.

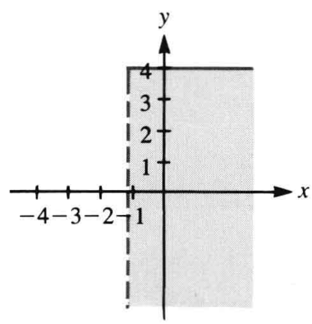

Figure 1.4