

# CONTEMPORARY MATHEMATICS

364

**Israel Mathematical Conference Proceedings**

## Complex Analysis and Dynamical Systems

Proceedings of an International Conference on  
Complex Analysis and Dynamical Systems

June 19–22, 2001

ORT Braude College

Karmiel, Israel

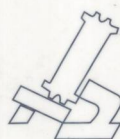
Mark Agranovsky

Lavi Karp

David Shoikhet

Lawrence Zalcman

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# CONTEMPORARY MATHEMATICS

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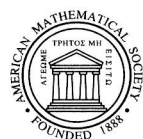
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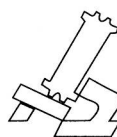
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# Complex Analysis and Dynamical Systems

## Preface

Israel Mathematical Conference Proceedings (IMCP) is a publication, part of the Contemporary Mathematics Series, devoted to the proceedings of conferences, symposia and seminars. Collections of papers focusing on a certain subject will also be published. Prospective volumes may be submitted to any member of the editorial board. Each volume has an editor (or editors) responsible for its preparation. In order to ensure inexpensive and timely distribution, authors are requested to submit to the Editor of the volume an electronic TEX file of their manuscript in AMS-LATEX, using the Contemporary Mathematics style file which can be downloaded at <http://www.ams.org/tex/author-info.html> For further information, contact the Managing Editor, IMCP, Department of Mathematics, Bar-Ilan University, Ramat-Gan 52900, Israel; e-mail: rowen@macs.biu.ac.il

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In recent years, ORT Braude College in Karmiel, Israel has hosted a series of conferences and workshops devoted to aspects of mathematical analysis. The third of these, an International Conference on Complex Analysis and Dynamical Systems, was held during June 19–22, 2001. Altogether, 46 mathematicians from 11 countries participated in a program which included 41 lectures. The nineteen papers collected in this volume form a partial record of that Conference. Many of them are based on the actual lectures presented, while others provide extensive elaborations of those lectures. In a few instances, contributors have chosen to submit papers different from those delivered at the Conference. Taken together, they provide a snapshot of activity in analysis at the outset of the twenty-first century.

Lawrence Zalcman

## Program

### TUESDAY

19th of June

14:30 - 14:45	OPENING
<u>Afternoon Session</u>	VIP Room
14:45 - 15:25	<i>B. Korenblum</i> F.Wiener and H.Bohr-type theorems for $L^p$ and $H^p$
15:30 - 16:10	<i>M. Reissig</i> Klein-Gordon decay rates for wave equations with time-dependent mass term
16:10 - 16:30	Coffee Break
16:30 - 17:10	<i>L. Zalcman</i> Beyond Bloch's Principle
17:15 - 17:55	<i>B.-W. Schulze</i> Long-time asymptotics on manifolds with geometric singularities

### WEDNESDAY

20th of June

<u>Morning Session</u>	VIP Room
09:00 - 09:40	<i>A. Hinkkanen</i> On Painlevé's third equation
09:45 - 10:25	<i>S. Dineen</i> Quasi-invertible manifolds
10:30 - 11:10	<i>M. Klein</i> Spectral theory and metastability for Markov chains
11:10 - 11:30	Coffee Break
<u>Session A</u>	Room 505
11:30 - 12:10	<i>M. Shapiro</i> On the Koppelman formula
12:15 - 12:55	<i>A. Solynin</i> Local variations and minimal area problems
<u>Session B</u>	Room 506
11:30 - 12:10	<i>D. Lenz</i> Uniform spectral properties of quasicrystals
12:15 - 12:55	<i>V. Rabinovich</i> Pseudodifferential operators with analytical symbols and estimates of eigenfunctions of the Schrödinger operators

Afternoon SessionSession A

Room 505

14:30 - 15:00

*R. Brooks*

Random construction of Riemann surfaces

15:05 - 15:35

*S. Krushkal*

Grunsky inequalities of higher rank with applications to complex geometry and function theory

15:40 - 16:10

*M. Sodin*

Dimension free estimates of polynomials and analytic functions

Session B

Room 506

14:30 - 15:00

*L. Rodman*Symmetric interpolation in matrix  $H_2$  spaces

15:05 - 15:35

*D. Alpay*

Some finite dimensional spaces of analytic functions in the ball and a related interpolation problem

15:40 - 16:10

*W. Kaczor*

Fixed points of asymptotically regular nonexpansive mappings on nonconvex sets

THURSDAY21st of JuneMorning Session

VIP Room

09:30 - 10:10

*L. Harris*

Invertibility preserving linear maps of operator algebras

10:15 - 10:55

*S. Simons*

Hahn-Banach and minimax theorems

11:00 - 11:40

*Yu. Lyubich*

Dynamical systems and functional equations

11:40 - 12:00

Coffee Break

Session A

Room 505

12:00 - 12:40

*M. Lanza de Cristoforis*

Asymptotic behaviour of the conformal representation of a Jordan domain with a small hole, and relative capacity

12:45 - 13:25

*Y. Lyubarskii*

Interpolation in weighted spaces of entire functions

Session B

VIP Room

12:00 - 12:40

*H. Render*

Multiresolutional analysis via cardinal polysplines

12:45 - 13:25

*Th. Krainer*

On the inverse of parabolic boundary value problems in an infinite space-time cylinder



Afternoon SessionSession A

Room 505

15:00 - 15:30

*L. Aizenberg*Generalization of the Carathéodory inequality  
and Bohr's phenomenon for power series

15:35 - 16:05

*G. Levin*

On a class of first-order differential equations

Session B

VIP Room

15:00 - 15:30

*T. Kuczumow*Holomorphic retracts in bounded, convex domains and  
locally uniformly convex in the Kobayashi distance domains

15:35 - 16:05

*M. Budzynska*

Nonexpansive retracts in product spaces

16:10 - 16:40

Coffee Break

Session A

Room 505

16:30 - 17:00

*D. Aharonov*

A minimal area problem in conformal mapping

17:05 - 17:35

*U. Srebro*

Q-homeomorphisms

Session B

VIP Room

16:30 - 17:00

*K. Włodarczyk*Random iterations of holomorphic maps in locally convex  
and Banach spaces and weaker contractions in uniform spaces

17:05 - 17:35

*J. Zemanek*

On quasi-regular points of holomorphic mappings

FRIDAY22nd of JuneMorning Session

VIP Room

09:00 - 09:40

*B. Paneah*Sharp estimates for analytic functions of exponential type  
and uncertainty principle

09:45 - 10:15

*E. Malinnikova*

Measures on the sphere orthogonal to harmonic vector fields

10:20 - 10:50

*M. Elin*

Spirallike functions with respect to a boundary point

10:55 - 11:10

CLOSING

## List of Participants

Agranovsky Mark (Israel, Bar-Ilan University)  
Aharonov Dov (Israel, Technion)  
Aizenberg Lev (Israel, Bar-Ilan University)  
Alpay Daniel (Israel, Ben-Gurion University)  
Ben-Artzi Matania (Israel, Hebrew University)  
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Elin Mark (Israel, ORT Braude College)  
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Korenblum Boris (USA, SUNY at Albany)  
Krainer Thomas (Germany, University of Potsdam)  
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Lenz Daniel (Germany, Johann Wolfgang Goethe-University)  
Levin Genadi (Israel, Hebrew University)  
Lin Vladimir (Israel, Technion)  
Lyubarskii Yurii (Norway, University of Trondheim)  
Lyubich Yuri (Israel, Technion)  
Malinnikova Eugenia (Norway, University of Trondheim)  
Mayer Dieter (Germany, Technical University of Clausthal)  
Paneah Boris (Israel, Technion)  
Rabinovich Vladimir (Mexico, ESIME del IPN)  
Reich Simeon (Israel, Technion)  
Reissig Michael (Germany, Technical University Bergakademie)  
Render Hermann (Germany, University of Duisburg)  
Rodman Leiba (USA, College of William and Mary)  
Schulze B.-Wolfgang (Germany, University of Potsdam)  
Shapiro Michael (Mexico, National Polytechnic Institute of Mexico)

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Sodin Mikhail (Israel, Tel-Aviv University)  
Solynin Alexander (Russia, Stekhlov Mathematics Institute)  
Spitkovsky Ilya (USA, College of William and Mary)  
Srebro Uri (Israel, Technion)  
Włodarczyk Kazimierz (Poland, University of Lodz)  
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Zemánek Jaroslav (Poland, Polish Academy of Sciences)

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## REMARKS ON THE “ASYMPTOTIC MAXIMUM PRINCIPLE”

by LEV AIZENBERG

Department of Mathematics, Bar-Ilan University, Ramat-Gan 52900 Israel  
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**Abstract.** New results on the maximum principles in multidimensional complex analysis are obtained by means of an extended version of the one-dimensional Maximum Principle.

1.

For  $z = (z_1, \dots, z_n) \in \mathbb{C}^n$ , let  $\|z\|_q = (|z_1|^q + \dots + |z_n|^q)^{1/2}$  ( $0 < q < \infty$ ) and  $\|z\|_\infty = \max_{1 \leq j \leq n} |z_j|$ . Consider domains  $\mathcal{D}_q = \{z : \|z\|_q < 1\}$ . The simplest cases of such domains are the ball  $\mathcal{D}_2 = \{z : |z| < 1\}$ , the hypercone  $\mathcal{D}_1 = \{z : |z_1| + \dots + |z_n| < 1\}$ , and the polydisk  $\mathcal{D}_\infty = \{z : |z_j| < 1, j = 1, \dots, n\}$ . Let  $S(\mathcal{D})$  be the Shilov boundary of  $\mathcal{D}$ , and let  $E$  be a circular set, i.e.,  $z \in E$  implies  $ze^{i\varphi} \in E$ ,  $0 \leq \varphi < 2\pi$ ; we also assume that  $\overline{E} \supset S(\mathcal{D})$ .

PROPOSITION 1. *If  $f(z)$  is holomorphic in  $\mathcal{D}_q$  and*

$$(1) \quad \limsup_{\|z\|_q \rightarrow 1} \{(1 - \|z\|_q)^2 \log |f(z)|\} = 0,$$

*then the Asymptotic Maximum Principle (AMP) is valid, i.e.,*

$$(2) \quad \sup_{\zeta \in E} \limsup_{r \rightarrow 1} |f(r\zeta)| \leq c$$

*implies  $|f(z)| \leq c$  in  $\mathcal{D}_q$ . Condition (1) cannot be replaced by*

$$(3) \quad \limsup_{\|z\|_q \rightarrow 1} \{(1 - \|z\|_q)^2 \log |f(z)|\} \leq t,$$

*with  $t > 0$ .*

PROOF. Consider the intersections of  $\mathcal{D}_q$  with complex lines  $d_a = z : \{z_j = a_j t, 1 \leq j \leq n, t \in \mathbb{C}\}$  where  $\|a\|_q = 1$ . We then apply an extended version of the one-dimensional Maximum Principle from [1],<sup>1</sup> where it is called the “Asymptotic

2000 *Mathematics Subject Classification.* Primary 32A40; Secondary 32A05, 32A07, 32A10.

<sup>1</sup>After submitting this note for publication, the author was informed that Korenblum’s result readily follows from the earlier published studies by B.E.J. Dahlberg [2, 3] (see also [4, 5]).

Maximum Principle" to those intersections such that  $\partial\mathcal{D}_q \cap d_a \subset E$ . Since (1) yields the one-dimensional condition of the same type, we get  $|f(z)| \leq c$  on each such intersection  $\mathcal{D}_q \cap d_a$ . Our function is holomorphic on  $r\overline{\mathcal{D}_q}$  where  $r\mathcal{D}_q$ ,  $0 < r < 1$ , is the homothetic contraction of  $\mathcal{D}_q$ . Since  $r\overline{E} \supset rS(\mathcal{D}) = S(\mathcal{D}_r)$ , we obtain  $|f(z)| \leq c$  in  $r\mathcal{D}_q$  for every  $0 < r < 1$ , and  $|f(z)| \leq c$  then follows. Korenblum's example for the unit disk

$$f(z) = \exp \left\{ ci \left( \frac{1+z}{1-z} \right)^2 \right\},$$

which shows that in the disk the analogue of (1) cannot be replaced by the analogue of (3), remains valid for our multidimensional generalization: take, e.g.,  $f = f(z_1)$ .

Proposition 1 can be generalized in two directions:

- 1) We can consider more general domains than  $\mathcal{D}_q$ ;
- 2) we can require growth restriction (2) to hold not everywhere but only on a certain set of intersections of the domain with complex lines.

## 2.

Let  $\mathcal{D}$  be a complete circular domain (Cartan domain), which means that  $z \in \mathcal{D}$  implies  $\lambda z \in \mathcal{D}$  for all  $\lambda \in \mathbb{C}$ ,  $|\lambda| \leq 1$ . In a Cartan domain, holomorphic polynomials can be developed in a series of homogeneous polynomials. Let, in addition,  $\mathcal{D}$  be strongly starlike, i.e.,  $\lambda\overline{\mathcal{D}} \subset \mathcal{D}$  where  $0 \leq \lambda < 1$ ; also, let the set  $E$  satisfy the same requirements as in section 1.

**PROPOSITION 2.** *Let  $f(z)$  be holomorphic in a Cartan domain  $\mathcal{D}$  and let the one-dimensional condition*

$$(4) \quad \limsup_{|t| \rightarrow R_{d_a}} \{(R_{d_a} - |t|)^2 \log |f(at)|\} = 0$$

*hold in each intersection of  $\mathcal{D}$  with the complex line  $d_a = \{z : z_j = a_j t, 1 \leq j \leq n, t \in \mathbb{C}\}$ , where  $\partial\mathcal{D} \cap d_a \subset E$  and  $R_{d_a}$  is the radius of the disk  $\mathcal{D} \cap d_a$ . Then the AMP holds, i.e., (2) implies  $|f(z)| \leq c$  in  $\mathcal{D}$ .*

## 3.

Let  $\mathcal{D}$  be a complete semicircular strongly starlike domain (Hartogs' domain):  $\mathcal{D} = \{z : |z_1| < R(z'), z' \in \mathbb{Q}\}$  where  $R(z')$  is a positive function in a domain  $Q \subset \mathbb{C}^{n-1}$ ,  $z = (z_1, z')$ ,  $z' = (z_2, \dots, z_n)$ . In such a domain, holomorphic functions are representable by Taylor series in  $z_1$  with coefficients that are holomorphic functions in  $z'$ . Further, let  $E_1 \subset \partial\mathcal{D}$  be a set such that  $z \in E_1$  implies  $(e^{i\varphi} z_1, z') \in E_1$ ,  $0 \leq \varphi < 2\pi$ , and  $\overline{E}_1 \supset S(\mathcal{D})$ .

**PROPOSITION 3.** *If  $f(z)$  is holomorphic in a Hartogs' domain  $\mathcal{D}$ , and for each fixed  $z' \in \mathbb{Q}$  such that  $z \in E_1$ , the condition*

$$(5) \quad \limsup_{|z_1| \rightarrow R(z')} \{(R(z') - |z_1|^2) \log |f(z_1, z')|\} = 0$$

holds, then the AMP is valid, i.e.,

$$\sup_{\zeta \in E_1} \limsup_{r \rightarrow 1} |f(r\zeta_1, \zeta')| \leq c$$

implies  $|f(z)| \leq c$  in  $\mathcal{D}$ .

Note that in Propositions 2 and 3 the growth restrictions (4) and (5) cannot be replaced by weaker restrictions analogous to (3).

The proofs of Propositions 2 and 3 are analogous to that of Proposition 1; the details are left to the reader.

I thank B. Korenblum for helpful discussions.

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