CONTEMPORARY MATHEMATICS

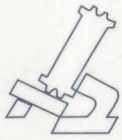
364

Israel Mathematical Conference Proceedings

Complex Analysis and Dynamical Systems

Proceedings of an International Conference on Complex Analysis and Dynamical Systems
June 19–22, 2001
ORT Braude College
Karmiel, Israel

Mark Agranovsky
Lavi Karp
David Shoikhet
Lawrence Zalcman
Editors



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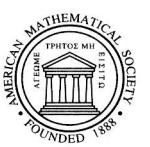

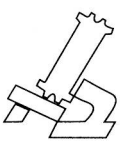

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This volume contains contributions from participants at the Third International Conference on Complex Analysis and Dynamical Systems. The conference was held from June 19–22, 2001, at ORT Braude College in Karmiel, Israel. This is the first volume of the Israel Mathematics Conference Proceedings (IMCP) published as a subseries of the Contemporary Mathematics series.

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Complex Analysis and Dynamical Systems

Preface

Israel Mathematical Conference Proceedings (IMCP) is a publication, part of the Contemporary Mathematics Series, devoted to the proceedings of conferences, symposia and seminars. Collections of papers focusing on a certain subject will also be published. Prospective volumes may be submitted to any member of the editorial board. Each volume has an editor (or editors) responsible for its preparation. In order to ensure inexpensive and timely distribution, authors are requested to submit to the Editor of the volume an electronic TEX file of their manuscript in AMS-LATEX, using the Contemporary Mathematics style file which can be downloaded at http://www.ams.org/tex/author-info.html For further information, contact the Managing Editor, IMCP, Department of Mathematics, Bar-Ilan University, Ramat-Gan 52900, Israel; e-mail: rowen@macs.biu.ac.il

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In recent years, ORT Braude College in Karmiel, Israel has hosted a series of conferences and workshops devoted to aspects of mathematical analysis. The third of these, an International Conference on Complex Analysis and Dynamical Systems, was held during June 19–22, 2001. Altogether, 46 mathematicians from 11 countries participated in a program which included 41 lectures. The nineteen papers collected in this volume form a partial record of that Conference. Many of them are based on the actual lectures presented, while others provide extensive elaborations of those lectures. In a few instances, contributors have chosen to submit papers different from those delivered at the Conference. Taken together, they provide a snapshot of activity in analysis at the outset of the twenty-first century.

Lawrence Zalcman

Program

TUESDAY
19th of June

14:30 - 14:45	OPENING
Afternoon Session	VIP Room
14:45 - 15:25	*B. Korenblum*
	F.Wiener and H.Bohr-type theorems for L^p and H^p
15:30 - 16:10	*M. Reissig*
	Klein-Gordon decay rates for wave equations with time-dependent mass term
16:10 - 16:30	Coffee Break
16:30 - 17:10	*L. Zalcman*
	Beyond Bloch's Principle
17:15 - 17:55	*B.-W. Schulze*
	Long-time asymptotics on manifolds with geometric singularities

WEDNESDAY
20th of June

Morning Session	VIP Room
09:00 - 09:40	*A. Hinkkanen*
	On Painlevé's third equation
09:45 - 10:25	*S. Dineen*
	Quasi-invertible manifolds
10:30 - 11:10	*M. Klein*
	Spectral theory and metastability for Markov chains
11:10 - 11:30	Coffee Break
Session A	Room 505
11:30 - 12:10	*M. Shapiro*
	On the Koppelman formula
12:15 - 12:55	*A. Solynin*
	Local variations and minimal area problems
Session B	Room 506
11:30 - 12:10	*D. Lenz*
	Uniform spectral properties of quasicrystals
12:15 - 12:55	*V. Rabinovich*
	Pseudodifferential operators with analytical symbols and estimates of eigenfunctions of the Schrödinger operators

Afternoon Session

Session A	Room 505
14:30 - 15:00	*R. Brooks* Random construction of Riemann surfaces
15:05 - 15:35	*S. Krushkal* Grunsky inequalities of higher rank with applications to complex geometry and function theory
15:40 - 16:10	*M. Sodin* Dimension free estimates of polynomials and analytic functions
Session B	Room 506
14:30 - 15:00	*L. Rodman* Symmetric interpolation in matrix H_2 spaces
15:05 - 15:35	*D. Alpay* Some finite dimensional spaces of analytic functions in the ball and a related interpolation problem
15:40 - 16:10	*W. Kaczor* Fixed points of asymptotically regular nonexpansive mappings on nonconvex sets

THURSDAY
21st of June

Morning Session	VIP Room
09:30 - 10:10	*L. Harris* Invertibility preserving linear maps of operator algebras
10:15 - 10:55	*S. Simons* Hahn-Banach and minimax theorems
11:00 - 11:40	*Yu. Lyubich* Dynamical systems and functional equations
11:40 - 12:00	Coffee Break
Session A	Room 505
12:00 - 12:40	*M. Lanza de Cristoforis* Asymptotic behaviour of the conformal representation of a Jordan domain with a small hole, and relative capacity
12:45 - 13:25	*Y. Lyubarskii* Interpolation in weighted spaces of entire functions
Session B	VIP Room
12:00 - 12:40	*H. Render* Multiresolutional analysis via cardinal polysplines
12:45 - 13:25	*Th. Krainer* On the inverse of parabolic boundary value problems in an infinite space-time cylinder

Afternoon Session	
Session A	Room 505
15:00 - 15:30	*L. Aizenberg* Generalization of the Carathéodory inequality and Bohr's phenomenon for power series
15:35 - 16:05	*G. Levin* On a class of first-order differential equations
Session B	VIP Room
15:00 - 15:30	*T. Kuczumow* Holomorphic retracts in bounded, convex domains and locally uniformly convex in the Kobayashi distance domains
15:35 - 16:05	*M. Budzynska* Nonexpansive retracts in product spaces
16:10 - 16:40	Coffee Break
Session A	Room 505
16:30 - 17:00	*D. Aharonov* A minimal area problem in conformal mapping
17:05 - 17:35	*U. Srebro* Q-homeomorphisms
Session B	VIP Room
16:30 - 17:00	*K. Wlodarczyk* Random iterations of holomorphic maps in locally convex and Banach spaces and weaker contractions in uniform spaces
17:05 - 17:35	*J. Zemanek* On quasi-regular points of holomorphic mappings
FRIDAY	
22nd of June	
Morning Session	VIP Room
09:00 - 09:40	*B. Paneah* Sharp estimates for analytic functions of exponential type and uncertainty principle
09:45 - 10:15	*E. Malinnikova* Measures on the sphere orthogonal to harmonic vector fields
10:20 - 10:50	*M. Elin* Spirallike functions with respect to a boundary point
10:55 - 11:10	CLOSING

List of Participants

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Contents

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REMARKS ON THE "ASYMPTOTIC MAXIMUM PRINCIPLE"

by LEV AIZENBERG

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Abstract. New results on the maximum principles in multidimensional complex analysis are obtained by means of an extended version of the one-dimensional Maximum Principle.

1.

For $z = (z_1, \dots, z_n) \in \mathbb{C}^n$, let $\|z\|_q = (|z_1|^q + \cdots + |z_n|^q)^{1/2}$ $(0 < q < \infty)$ and $\|z\|_\infty = \max\limits_{1 \le j \le n} |z_j|$. Consider domains $\mathcal{D}_q = \{z : \|z\|_q < 1\}$. The simplest cases of such domains are the ball $\mathcal{D}_2 = \{z : |z| < 1\}$, the hypercone $\mathcal{D}_1 = \{z : |z_1| + \cdots + |z_n| < 1\}$, and the polydisk $\mathcal{D}_\infty = \{z : |z_j| < 1,\ j = 1, \dots, n\}$. Let $S(\mathcal{D})$ be the Shilov boundary of $\mathcal{D}$, and let E be a circular set, i.e., $z \in E$ implies $ze^{i\varphi} \in E$, $0 \le \varphi < 2\pi$; we also assume that $\overline{E} \supset S(\mathcal{D})$.

PROPOSITION 1. *If $f(z)$ is holomorphic in $\mathcal{D}_q$ and*

$$\limsup_{\|z\|_q \to 1} \{(1 - \|z\|_q)^2 \log |f(z)|\} = 0, \tag{1}$$

then the Asymptotic Maximum Principle (AMP) is valid, i.e.,

$$\sup_{\zeta \in E} \limsup_{r \to 1} |f(r\zeta)| \le c \tag{2}$$

implies $|f(z)| \le c$ in $\mathcal{D}_q$. Condition (1) *cannot be replaced by*

$$\limsup_{\|z\|_q \to 1} \{(1 - \|z\|_q)^2 \log |f(z)|\} \le t, \tag{3}$$

with $t > 0$.

PROOF. Consider the intersections of $\mathcal{D}_q$ with complex lines $d_a = z : \{z_j = a_j t,\ 1 \le j \le n,\ t \in \mathbb{C}\}$ where $\|a\|_q = 1$. We then apply an extended version of the one-dimensional Maximum Principle from [**1**],[1] where it is called the "Asymptotic

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[1]After submitting this note for publication, the author was informed that Korenblum's result readily follows from the earlier published studies by B.E.J. Dahlberg [**2, 3**] (see also [**4, 5**]).

Maximum Principle" to those intersections such that $\partial\mathcal{D}_q \cap d_a \subset E$. Since (1) yields the one-dimensional condition of the same type, we get $|f(z)| \leq c$ on each such intersection $\mathcal{D}_q \cap d_a$. Our function is holomorphic on $\overline{r\mathcal{D}_q}$ where $r\mathcal{D}_q$, $0 < r < 1$, is the homothetic contraction of $\mathcal{D}_q$. Since $\overline{rE} \supset rS(\mathcal{D}) = S(\mathcal{D}_r)$, we obtain $|f(z)| \leq c$ in $r\mathcal{D}_q$ for every $0 < r < 1$, and $|f(z)| \leq c$ then follows. Korenblum's example for the unit disk

$$f(z) = \exp\left\{ci\left(\frac{1+z}{1-z}\right)^2\right\},$$

which shows that in the disk the analogue of (1) cannot be replaced by the analogue of (3), remains valid for our multidimensional generalization: take, e.g., $f = f(z_1)$.

Proposition 1 can be generalized in two directions:

1) We can consider more general domains than $\mathcal{D}_q$;
2) we can require growth restriction (2) to hold not everywhere but only on a certain set of intersections of the domain with complex lines.

2.

Let $\mathcal{D}$ be a complete circular domain (Cartan domain), which means that $z \in \mathcal{D}$ implies $\lambda z \in \mathcal{D}$ for all $\lambda \in \mathbb{C}$, $|\lambda| \leq 1$. In a Cartain domain, holomorphic polynomials can be developed in a series of homogeneous polynomials. Let, in addition, $\mathcal{D}$ be strongly starlike, i.e., $\lambda\overline{\mathcal{D}} \subset \mathcal{D}$ where $0 \leq \lambda < 1$; also, let the set E satisfy the same requirements as in section 1.

PROPOSITION 2. *Let $f(z)$ be holomorphic in a Cartan domain $\mathcal{D}$ and let the one-dimensional condition*

$$\limsup_{|t| \to R_{d_a}} \{(R_{d_a} - |t|)^2 \log |f(at)|\} = 0 \tag{4}$$

hold in each intersection of $\mathcal{D}$ with the complex line $d_a = \{z : z_j = a_j t,\ 1 \leq j \leq n,\ t \in \mathbb{C}\}$, where $\partial\mathcal{D} \cap d_a \subset E$ and R_{d_a} is the radius of the disk $\mathcal{D} \cap d_a$. Then the AMP holds, i.e., (2) implies $|f(z)| \leq c$ in $\mathcal{D}$.

3.

Let $\mathcal{D}$ be a complete semicircular strongly starlike domain (Hartogs' domain): $\mathcal{D} = \{z : |z_1| < R(z'),\ z' \in \mathbb{Q}\}$ where $R(z')$ is a positive function in a domain $Q \subset \mathbb{C}^{n-1}$, $z = (z_1, z')$, $z' = (z_2, \ldots, z_n)$. In such a domain, holomorphic functions are representable by Taylor series in z_1 with coefficients that are holomorphic functions in z'. Further, let $E_1 \subset \partial\mathcal{D}$ be a set such that $z \in E_1$ implies $(e^{i\varphi} z_1, z') \in E_1$, $0 \leq \varphi < 2\pi$, and $\overline{E}_1 \supset S(\mathcal{D})$.

PROPOSITION 3. *If $f(z)$ is holomorphic in a Hartogs' domain $\mathcal{D}$, and for each fixed $z' \in \mathbb{Q}$ such that $z \in E_1$, the condition*

$$\limsup_{|z_1| \to R(z')} \{(R(z') - |z_1|^2) \log |f(z_1, z')|\} = 0 \tag{5}$$

holds, then the AMP is valid, i.e.,

$$\sup_{\zeta\in E_1}\limsup_{r\to 1}|f(r\zeta_1,\zeta')|\le c$$

implies $|f(z)|\le c$ *in* $\mathcal{D}$.

Note that in Propositions 2 and 3 the growth restrictions (4) and (5) cannot be replaced by weaker restrictions analogous to (3).

The proofs of Propositions 2 and 3 are analogous to that of Proposition 1; the details are left to the reader.

I thank B. Korenblum for helpful discussions.

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