

The background of the book cover is an abstract composition of layered, textured paper. The top half features a blue and green marbled pattern, while the bottom half is dominated by warm, earthy tones of yellow, orange, and brown, suggesting a cross-section of the Earth's interior.

**Introduction to
the Physics of the**

Earth's Interior

Second Edition

Jean-Paul Poirier

INTRODUCTION TO THE PHYSICS OF THE EARTH'S INTERIOR

SECOND EDITION

JEAN-PAUL POIRIER

Institut de Physique du Globe de Paris



CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS

The Edinburgh Building, Cambridge CB2 2RU, UK <http://www.cup.cam.ac.uk>
40 West 20th Street, New York, NY 10011-4211, USA <http://www.cup.org>
10 Stamford Road, Oakleigh, Melbourne 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain

© Jean-Paul Poirier 2000

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 2000

Printed in the United Kingdom at the University Press, Cambridge

Typeset in Times 10/13pt [vN]

A catalogue record for this book is available from the British Library

Library of Congress Cataloging in Publication data

Poirier, Jean Paul.

Introduction to the physics of the Earth's interior / Jean-Paul Poirier. – 2nd ed.
p. cm.

Includes bibliographical references and index.

ISBN 0 521 66313 X (hardbound). – ISBN 0 521 66392 X (pbk.)

1. Earth – Core. 2. Earth – Mantle. 3. Geophysics. I. Title.

QE509.2.P65 2000

551.1'1 – dc21 99-30070 CIP

ISBN 0 521 66313 X hardback

ISBN 0 521 66392 X paperback

Introduction to the Physics of the Earth's Interior describes the structure, composition and temperature of the deep Earth in one comprehensive volume.

The book begins with a succinct review of the fundamentals of continuum mechanics and thermodynamics of solids, and presents the theory of lattice vibration in solids. The author then introduces the various equations of state, moving on to a discussion of melting laws and transport properties. The book closes with a discussion of current seismological, thermal and compositional models of the Earth. No special knowledge of geophysics or mineral physics is required, but a background in elementary physics is helpful. The new edition of this successful textbook has been enlarged and fully updated, taking into account the considerable experimental and theoretical progress recently made in understanding the physics of deep-Earth materials and the inner structure of the Earth.

Like the first edition, this will be a useful textbook for graduate and advanced undergraduate students in geophysics and mineralogy. It will also be of great value to researchers in Earth sciences, physics and materials sciences.

Jean-Paul Poirier is Professor of Geophysics at the Institut de Physique du Globe de Paris, and a corresponding member of the Académie des Sciences. He is the author of over one-hundred-and-thirty articles and six books on geophysics and mineral physics, including *Creep of Crystals* (Cambridge University Press, 1985) and *Crystalline Plasticity and Solid-state flow of Metamorphic Rocks* with A. Nicolas (Wiley, 1976).

Preface to the first edition

Not so long ago, Geophysics was a part of Meteorology and there was no such thing as Physics of the Earth's interior. Then came Seismology and, with it, the realization that the elastic waves excited by earthquakes, refracted and reflected within the Earth, could be used to probe its depths and gather information on the elastic structure and eventually the physics and chemistry of inaccessible regions down to the center of the Earth.

The basic ingredients are the travel times of various phases, on seismograms recorded at stations all over the globe. Inversion of a considerable amount of data yields a seismological earth model, that is, essentially a set of values of the longitudinal and transverse elastic-wave velocities for all depths. It is well known that the velocities depend on the elastic moduli and the density of the medium in which the waves propagate; the elastic moduli and the density, in turn, depend on the crystal structure and chemical composition of the constitutive minerals, and on pressure and temperature. To extract from velocity profiles self-consistent information on the Earth's interior such as pressure, temperature, and composition as a function of depth, one needs to know, or at least estimate, the values of the physical parameters of the high-pressure and high-temperature phases of the candidate minerals, and relate them, in the framework of thermodynamics, to the Earth's parameters.

Physics of the Earth's interior has expanded from there to become a recognized discipline within solid earth geophysics, and an important part of the current geophysical literature can be found under such key words as "equation of state", "Grüneisen parameter", "adiabaticity", "melting curve", "electrical conductivity", and so on.

The problem, however, is that, although most geophysics textbooks devote a few paragraphs, or even a few chapters, to the basic concepts of the physics of solids and its applications, there still is no self-contained book

that offers the background information needed by the graduate student or the non-specialist geophysicist to understand an increasing portion of the literature as well as to assess the weight of physical arguments from various parties in current controversies about the structure, composition, or temperature of the deep Earth.

The present book has the, admittedly unreasonable, ambition to fulfill this role. Starting as a primer, and giving at length all the important demonstrations, it should lead the reader, step by step, to the most recent developments in the literature. The book is primarily intended for graduate or senior undergraduate students in physical earth sciences but it is hoped that it can also be useful to geophysicists interested in getting acquainted with the mineral physics foundations of the phenomena they study.

In the first part, the necessary background in thermodynamics of solids is succinctly given in the framework of linear relations between intensive and extensive quantities. Elementary solid-state theory of vibrations in solids serves as a basis to introduce Debye's theory of specific heat and anharmonicity. Many definitions of Grüneisen's parameter are given and compared.

The background is used to explain the origin of the various equations of state (Murnaghan, Birch–Murnaghan, etc.). Velocity–density systematics and Birch's law lead to seismic equations of state. Shock-wave equations of state are also briefly considered. Tables of recent values of thermodynamic and elastic parameters of the most important mantle minerals are given. The effect of pressure on melting is introduced in the framework of anharmonicity, and various melting laws (Lindemann, Kraut–Kennedy, etc.) are given and discussed. Transport properties of materials – diffusion and viscosity of solids and of liquid metals, electrical and thermal conductivity of solids – are important in understanding the workings of the Earth; a chapter is devoted to them.

The last chapter deals with the application of the previous ones to the determination of seismological, thermal, and compositional Earth models.

An abundant bibliography, including the original papers and the most recent contributions, experimental or theoretical, should help the reader to go further than the limited scope of the book.

It is a pleasure to thank all those who helped make this book come into being: First of all, Bob Liebermann, who persuaded me to write it and suggested improvements in the manuscript; Joël Dyon, who did a splendid job on the artwork; Claude Allègre, Vincent Courtillot, François Guyot,

Jean-Louis Le Mouël, and Jean-Paul Montagner, who read all or parts of the manuscript and provided invaluable comments and suggestions; and last but not least, Carol, for everything.

1991

Jean-Paul Poirier

Preface to the second edition

Almost ten years ago, I wrote in the introduction to the first edition of this book: ‘It will also probably become clear that the simplicity of the inner Earth is only apparent; with the progress of laboratory experimental techniques as well as observational seismology, geochemistry and geomagnetism, we may perhaps expect that someday “Physics of the Inner Earth” will make as little sense as “Physics of the Crust”’. We are not there yet, but we have made significant steps in this direction in the last ten years. No geophysicist now would entertain the idea that the Earth is composed of homogeneous onion shells. The analysis of data provided by more and better seismographic nets has, not surprisingly, revealed the heterogeneous structure of the depths of the Earth and made clear that the apparent simplicity of the lower mantle was essentially due to its remoteness. We also know more about the core.

Mineral physics has become an essential part of geophysics and the progress of experimental high-pressure and high-temperature techniques has provided new results, solved old problems and created new ones. Samples of high-pressure phases prepared in laser-heated diamond-anvil cells or large-volume presses are now currently studied by X-ray diffraction, using synchrotron beams, and by transmission electron microscopy. In ten years, we have thus considerably increased our knowledge of the deep minerals, including iron at core pressures. We know more about their thermoelastic properties, their phase transitions and their melting curves. Concurrently, quantum mechanical *ab-initio* computer methods have made such progress as to be able to reproduce the values of physical quantities in the temperature- and pressure-ranges that can be experimentally reached, and therefore predict with confidence their values at deep-Earth conditions.

In this new edition, I have therefore expanded the chapters on equations

of state, on melting, and the last chapter on Earth models. Close to two-hundred-and-fifty new references have been added.

I thank Dr Brian Watts of CUP, my copy editor, for a most thorough review of the manuscript.

1999

Jean-Paul Poirier

Contents

<i>Preface to the first edition</i>	<i>page ix</i>
<i>Preface to the second edition</i>	<i>xii</i>
Introduction to the first edition	1
1 Background of thermodynamics of solids	4
1.1 Extensive and intensive conjugate quantities	4
1.2 Thermodynamic potentials	6
1.3 Maxwell's relations. Stiffnesses and compliances	8
2 Elastic moduli	11
2.1 Background of linear elasticity	11
2.2 Elastic constants and moduli	13
2.3 Thermoelastic coupling	20
2.3.1 Generalities	20
2.3.2 Isothermal and adiabatic moduli	20
2.3.3 Thermal pressure	25
3 Lattice vibrations	27
3.1 Generalities	27
3.2 Vibrations of a monatomic lattice	27
3.2.1 Dispersion curve of an infinite lattice	27
3.2.2 Density of states of a finite lattice	33
3.3 Debye's approximation	36
3.3.1 Debye's frequency	36
3.3.2 Vibrational energy and Debye temperature	38
3.3.3 Specific heat	39

3.3.4	Validity of Debye's approximation	41
3.4	Mie–Grüneisen equation of state	44
3.5	The Grüneisen parameters	46
3.6	Harmonicity, anharmonicity and quasi-harmonicity	57
3.6.1	Generalities	57
3.6.2	Thermal expansion	58
4	Equations of state	63
4.1	Generalities	63
4.2	Murnaghan's integrated linear equation of state	64
4.3	Birch–Murnaghan equation of state	66
4.3.1	Finite strain	66
4.3.2	Second-order Birch–Murnaghan equation of state	70
4.3.3	Third-order Birch–Murnaghan equation of state	72
4.4	A logarithmic equation of state	74
4.4.1	The Hencky finite strain	74
4.4.2	The logarithmic EOS	76
4.5	Equations of state derived from interatomic potentials	77
4.5.1	EOS derived from the Mie potential	77
4.5.2	The Vinet equation of state	78
4.6	Birch's law and velocity–density systematics	79
4.6.1	Generalities	79
4.6.2	Bulk-velocity–density systematics	82
4.7	Thermal equations of state	90
4.8	Shock-wave equations of state	94
4.8.1	Generalities	94
4.8.2	The Rankine–Hugoniot equations	96
4.8.3	Reduction of the Hugoniot data to isothermal equation of state	100
4.9	First principles equations of state	102
4.9.1	Thomas–Fermi equation of state	102
4.9.2	<i>Ab-initio</i> quantum mechanical equations of state	107
5	Melting	110
5.1	Generalities	110
5.2	Thermodynamics of melting	115
5.2.1	Clausius–Clapeyron relation	115
5.2.2	Volume and entropy of melting	115
5.2.3	Metastable melting	118

5.3	Semi-empirical melting laws	120
5.3.1	Simon equation	120
5.3.2	Kraut–Kennedy equation	121
5.4	Theoretical melting models	123
5.4.1	Shear instability models	123
5.4.2	Vibrational instability: Lindemann law	125
5.4.3	Lennard-Jones and Devonshire model	132
5.4.4	Dislocation-mediated melting	139
5.4.5	Summary	143
5.5	Melting of lower-mantle minerals	144
5.5.1	Melting of MgSiO_3 perovskite	145
5.5.2	Melting of MgO and magnesiowüstite	145
5.6	Phase diagram and melting of iron	146
6	Transport properties	156
6.1	Generalities	156
6.2	Mechanisms of diffusion in solids	162
6.3	Viscosity of solids	174
6.4	Diffusion and viscosity in liquid metals	184
6.5	Electrical conduction	189
6.5.1	Generalities on the electronic structure of solids	189
6.5.2	Mechanisms of electrical conduction	194
6.5.3	Electrical conductivity of mantle minerals	203
6.5.4	Electrical conductivity of the fluid core	212
6.6	Thermal conduction	213
7	Earth models	221
7.1	Generalities	221
7.2	Seismological models	223
7.2.1	Density distribution in the Earth	223
7.2.2	The PREM model	227
7.3	Thermal models	230
7.3.1	Sources of heat	230
7.3.2	Heat transfer by convection	231
7.3.3	Convection patterns in the mantle	236
7.3.4	Geotherms	241
7.4	Mineralogical models	244
7.4.1	Phase transitions of the mantle minerals	244
7.4.2	Mantle and core models	259

Appendix	PREM model (1s) for the mantle and core	272
<i>Bibliography</i>		275
<i>Index</i>		309

Introduction to the first edition

The interior of the Earth is a problem at once fascinating and baffling, as one may easily judge from the vast literature and the few established facts concerning it.

F. Birch, J. Geophys. Res., 57, 227 (1952)

This book is about the inaccessible interior of the Earth. Indeed, it is because it is inaccessible, hence known only indirectly and with a low resolving power, that we can talk of the physics of the interior of the Earth. The Earth's crust has been investigated for many years by geologists and geophysicists of various persuasions; as a result, it is known with such a wealth of detail that it is almost meaningless to speak of the crust as if it were a homogeneous medium endowed with averaged physical properties, in a state defined by simple temperature and pressure distributions. We have the physics of earthquake sources, of sedimentation, of metamorphism, of magnetic minerals, and so forth, but no physics of the crust.

Below the crust, however, begins the realm of inner earth, less well known and apparently simpler: a world of successive homogeneous spherical shells, with a radially symmetrical distribution of density and under a predominantly hydrostatic pressure. To these vast regions, we can apply macroscopic phenomenologies such as thermodynamics or continuum mechanics, deal with energy transfers using the tools of physics, and obtain Earth models – seismological, thermal, or compositional. These models, such as they were until, say, about 1950, accounted for the gross features of the interior of the Earth: a silicate mantle whose density increased with depth as it was compressed, with a couple of seismological discontinuities inside, a liquid iron core where convection currents generated the Earth's magnetic field, and a small solid inner core.

The physics of the interior of the Earth arguably came of age in the 1950s,

when, following Bridgman's tracks, Birch at Harvard University and Ringwood at the Australian National University started investigating the high-pressure properties and transformations of the silicate minerals. Large-volume multi-anvil presses were developed in Japan (see Akimoto 1987) and diamond-anvil cells were developed in the United States (see Bassett 1977), allowing the synthesis of minerals at the static pressures of the lower mantle, while shock-wave techniques (see Ahrens 1980) produced high dynamic pressures. It turn out, fortunately, that the wealth of mineral architecture that we see in the crust and uppermost mantle reduces to a few close-packed structures at very high pressures.

It is now possible to use the arsenal of modern methods (e.g. spectroscopies from the infrared to the hard X-rays generated in synchrotrons) to investigate the physical properties of the materials of the Earth at very high pressures, thus giving a firm basis to the averaged physical properties of the inner regions of the Earth deduced from seismological or geomagnetic observations and allowing the setting of constraints on the energetics of the Earth.

It is the purpose of this book to introduce the groundwork of condensed matter physics, which has allowed, and still allows, the improvement of Earth models. Starting with the indispensable, if somewhat arid, phenomenological background of thermodynamics of solids and continuum mechanics, we will relate the macroscopic observables to crystalline physics; we will then deal with melting, phase transitions, and transport properties before trying to synthetically present the Earth models of today.

The role of laboratory experimentation cannot be overestimated. It is, however, beyond the scope of this book to present the experimental techniques, but references to review articles will be given.

In a book such as this one, which topic to include or reject is largely a matter of personal, hence debatable, choice. I give only a brief account of the phase transitions of minerals in a paragraph that some readers may well find somewhat skimpy; I chose to do so because this active field is in rapid expansion and I prefer outlining the important results and giving recent references to running the risk of confusing the reader. Also, little is known yet about the mineral reactions in the transition zone and the lower mantle, so I deal only with the polymorphic, isochemical transitions of the main mantle minerals, thus keeping well clear of the huge field of experimental petrology.

It is hoped that this book may help with the understanding of how condensed matter physics may be of use in improving Earth models. It will

also probably become clear that the simplicity of the inner Earth is only apparent; with the progress of laboratory experimental techniques as well as observational seismology, geochemistry, and geomagnetism, we may perhaps expect that someday “physics of the interior of the Earth” will make as little sense as “physics of the crust.”

1

Background of thermodynamics of solids

1.1 Extensive and intensive conjugate quantities

The physical quantities used to define the state of a system can be scalar (e.g. volume, hydrostatic pressure, number of moles of constituent), vectorial (e.g. electric or magnetic field) or tensorial (e.g. stress or strain). In all cases, one may distinguish extensive and intensive quantities. The distinction is most obvious for scalar quantities: extensive quantities are size-dependent (e.g. volume, entropy) and intensive quantities are not (e.g. pressure, temperature).

Conjugate quantities are such that their product (scalar or contracted product for vectorial and tensorial quantities) has the dimension of energy (or energy per unit volume, depending on the definition of the extensive quantities), (Table 1.1). By analogy with the expression of mechanical work as the product of a force by a displacement, the intensive quantities are also called *generalized forces* and the extensive quantities, *generalized displacements*.

If the state of a single-phase system is defined by N extensive quantities e_k and N intensive quantities i_k , the differential increase in energy per unit volume of the system for a variation of e_k is:

$$dU = \sum_k i_k de_k \quad (1.1)$$

The intensive quantities can therefore be defined as partial derivatives of the energy with respect to their conjugate quantities:

$$i_k = \frac{\partial U}{\partial e_k} \quad (1.2)$$

For the extensive quantities, we have to introduce the Gibbs potential