PRELIMINARY EDITION

Calculus

VOLUME 1

Thomas P. Dick

Charles M. Patton

THE OREGON STATE UNIVERSITY
CALCULUS CURRICULUM PROJECT

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Preface

Curriculum revision is generally a process of gradual evolution of scope and sequence, occasionally punctuated by calls for more fundamental changes in content or delivery. The launching of Sputnik precipitated such a call for reform in mathematics education in the 1950's. We find ourselves in the midst of a new period of widespread revitalization efforts in mathematics curriculum and instruction. A forward-looking vision of the entire K-12 mathematics curriculum is outlined in the *Curriculum and Evaluation Standards* of the National Council of Teachers of Mathematics. The Mathematical Sciences Education Board has made an eloquent and urgent case for revitalizing mathematics instruction at all levels in preparation for our country's future workforce needs in *Everybody Counts*. Both of these influential documents recognize the emergence of sophisticated computer and calculator technology as redefining the tools of mathematics education.

Calculus occupies a particularly critical position in mathematics education as the gateway to advanced training in most scientific and technical fields. It is fitting that calculus should receive particular attention as we prepare for the needs of the twenty-first century. The Sloan Conference (Tulane, 1986) and the Calculus for a New Century Conference (Washington, 1987) sounded the call for reform in the calculus curriculum. Now the entire introductory course in calculus is being reexamined under the closest scrutiny that it has received in several years. Through a special funding initiative, the National Science Foundation has made resources available for a variety of calculus curriculum revision efforts to be tried and implemented. The Oregon State University Calculus Curriculum Project is one of these NSF-funded efforts, and this book is one of the major results of the project.

Volume I covers the material for a first year course in calculus of a single variable. (Volume II covers material appropriate for a semester or two quarters of multivariable and vector calculus.) A brief glance at the Table of Contents might suggest that the text does not differ radically from a traditional calculus text in terms of major topics. This is as it should be—calculus reform will not change the importance and vitality of the major ideas of calculus, and any wholesale departure from those ideas should be viewed with great skepticism. What *is possible* is a fresh approach to these important ideas in light of the availability of modern technology. In particular, the technology can invite us to change or adopt new emphases in instruction.

MAJOR THEMES OF THE OREGON STATE PROJECT

Making intelligent use of technology

Computer algebra systems, spreadsheets, and symbolic/graphing calculators are just a few of the readily available technological tools providing students with new windows of understanding and new opportunities for applying calculus. However, technology should not be viewed as a panacea for calculus instruction. This book seeks to take advantage of these new tools, while at the same time alerting the student to their inherent limitations and the care that must be taken to use technology wisely.

While being "technology-aware," the text itself does not assume the availability of any particular machine or software. To do so would invite immediate obsolescence and ignore how quickly technology advances. Rather, the text adopts a language appropriate for the kinds of numerical, graphical, and symbolic capabilities that are found (and will continue to be found) on a wide variety of computer software packages and sophisticated calculators. For example, the language of zooming in on the graph of a function is powerfully suggestive without the need for listing specific keystrokes or syntax.

While technology can provide students new opportunities for understanding calculus, it must be used with care. Numerical computations performed by a machine are subject to magnitude and precision limitations. For example, the calculation of difference quotients is naturally prone to the phenomena of cancellation errors. Machine-generated graphs can also provide misleading information, since graphs consist of a discrete collection of pixels whose locations are computed numerically. Symbolic algebra results need to be interpreted in context. Helping students understand the limitations of technology is a major goal of both the text and the supplement. Students are reminded of the care that must be taken to make intelligent use of technology without becoming a victim of its pitfalls.

In both the text and the student supplements, no specific hardware or software is assumed, so an instructor will need to judge the appropriateness of any particular activity in light of the technology available. However, the exercises activities are designed to be feasible with a very wide variety of available software and hardware. Certainly, a super calculator (symbolic/graphic) has more than enough power to suffice for all the activities, and a simple graphing calculator will be adequate for many of the activities.

Multiple representation approach to functions

The most important concept in all of mathematics is that of *function*, and the function concept is central in calculus. The idea of a function as a process accepting inputs and returning outputs can be captured in a variety of representations—numerically as a table of input-output pairs, graphically as a plot of outputs vs. inputs, and symbolically as a formula describing or modelling the input-output process. The interpretations of the core calculus topics of limits and continuity, differentiation, and integration all have different flavors when approached through different representations. The connections we forge between them enrich our personal concept of function.

All too often, students leave the calculus course with an impoverished mental image of function formed in a context dominated by symbolic forms. This book seeks to take a more balanced three-fold approach to functions. With each new topic or result, an explicit effort is made to interpret the meaning and consequences in a numerical, graphical, and symbolic context. Such an approach does not require the presence of technology, but its availability allows us greater access to numerical and graphical tools, while at the same time reducing the need for heavy emphasis of rote "by hand" symbol manipulation skills.

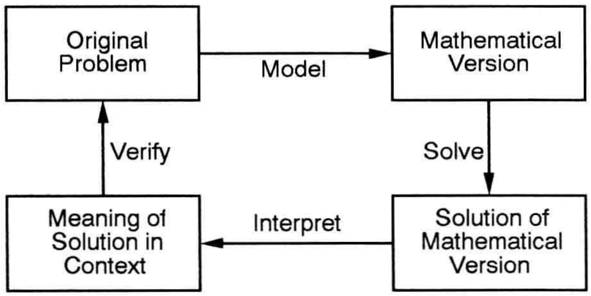
Visualization and Approximation

Two themes that become increasingly important with the availability of technology are visualization and approximation. The ability to obtain a machine-generated graph as a first step instead of a last one can completely turn around our approach to a variety of calculus topics. Graphical interpretation skills become primary. In particular, graphing can be used as a powerful problem solving aid, both in estimating and monitoring the reasonableness of results obtained numerically or symbolically. Whenever possible, explicit mention is made of the visual interpretation of definitions, theorems, and example solutions, often with direct reference to machine-generated graphs.

Much of calculus grew out of problems of approximation, and many of the key concepts of calculus are best understood as limits of approximations. Numerical tools make once exorbitantly tedious calculations into viable computational estimation strategies. Accordingly, approximation and estimation techniques are given a high priority throughout the text.

Problem solving and mathematical modelling

The application of mathematics to solving "real world" problems may be thought of in terms of the process diagrammed below.



Mathematical training all too often deals only with the part of this process labelled "Solve." With the advent of computation devices capable of the rote work of solving, we can properly include training in the mathematical modelling of problems, interpreting the mathematical solutions, and checking the reasonableness of the results. Much effort has gone into embedding the applications of calculus into natural and plausible real world settings.

ABOUT THE MATERIAL IN VOLUME I

Chapter 0 is a brief introduction to the major ideas of calculus, including a discussion of how the notions of infinite processes and approximation arise naturally in the study of real number measurement.

Chapter 1 introduces functions as input-output processes, with the emphasis on multiple representations that recurs throughout the text. From the outset, transcendental functions are treated, including trigonometric, exponential, logarithmic, and inverse trigonometric functions. With respect to terminology and notation, the distinctions between variables, constants, and parameters are highlighted. The modern arrow notation for function assignment processes ($f : x \longmapsto f(x)$) is used both as a means for distinguishing between *constants* and *constant functions*, and for explaining composition of functions.

Chapter 2 discusses limits and continuity. Numerical and graphical approaches receive equal, if not more emphasis than symbolic techniques. The rigorous epsilon-delta definitions are included, but their meanings are explained with reference to their numerical and graphical consequences, rather than a heavy emphasis on proofs. For example, the definition of continuity of a function has a dynamic interpretation in terms of the scaling of a graphing window. The "δ-hunt" for a particular ϵ becomes a search for a certain horizontal scaling, given a vertical scaling. Numerically, ϵ's and δ's can be interpreted as output and input tolerances.

Chapter 3 starts with a review of linear functions, and then uses the important examples of piece-wise linear functions to discuss the notions of *local slope* and *local linearity*. Differentiable functions can then be considered as *approximately* locally linear functions, an idea visually reinforced by zooming in on the graphs of functions. The physical interpretation of derivative as a rate of change is motivated with the problem of estimating a car's speedometer reading using its odometer and a stopwatch. Computing difference quotients (and symmetric difference quotients) are considered as numerically viable techniques for estimating derivative values, and not just an artifact of the formal definition of derivative. Derivative properties and rules are developed and a dictionary of derivative formulas for all the basic algebraic and transcendental functions are included at the end of the chapter.

Chapter 4 emphasizes the use of the derivative as a measurement tool. The chapter begins by discussing the physical interpretation of derivative as a rate of change. A tangent line is considered as the graph of the best linear approximation of a differentiable function at a point (first-order Taylor forms). The use of the derivative to analyze function behavior, critical points, and extrema, and the consequences of the Mean Value Theorem are interpreted from both physical and graphical perspectives. Higher order derivatives are introduced using the physical example of a car's acceleration, and then used in a discussion of concavity, inflection points, and best quadratic approximations (second-order Taylor forms). The discussion of higher order derivatives concludes with an application to cubic splines.

Chapter 5 starts out with a discussion of problem solving in the spirit of Polya and the role of mathematical modelling in solving real world problems. The use of calculus to solve optimization problems are illustrated with examples and exercises drawn from the context of the day-to-day operations of a manufacturing facility. This chapter also treats implicit differentiation, related rates, and parametric and polar equations.

Chapter 6 motivates the idea of definite integral with the example of approximating π using the method of exhaustion. By using piece-wise linear functions as examples, many of the properties of definite integrals are then explored without the necessity of special summation formulas. The idea of a Riemann sum is then motivated as a reasonable approximation technique for more general functions. Antiderivatives are introduced by reversing the problem of determining a car's speedometer reading from its odometer and clock readings to one of determining distance covered from speed and time readings. *Slope fields* (direction fields) are used as a graphical means of approximating the graph of an antiderivative. Noting that $d = rt$ represents the area under the graph of a car's *constant* speed r over time t leads to the more general conjecture that definite integrals can be used to generate antiderivatives. The two Fundamental Theorems of Calculus tie together differentiation and integration. Chapter 6 closes with a discussion of numerical techniques of integration.

Chapter 7 discusses differential equations from a variety of viewpoints. Slope fields are used to visualize solutions to differential equations. The First Fundamental Theorem of Calculus is reviewed for its use in creating antiderivatives. Exponential and logarithmic functions are both re-examined as the solutions of special differential equations. Applications of exponential functions to problems of growth and decay are included. The method of substitution and integration by parts are discussed as the integral counterparts to the chain rule and the product rule in searching for antiderivatives.

Chapter 8 emphasizes the use of the integral as a measurement tool. The role of Riemann sums in modelling measurements involving continuously varying quantities is highlighted over and over again. Geometric examples include the measurement of area, volume, and arc length. Many of the definite integrals encountered in this chapter require the use of machine numerical integration. The use of polar coordinates in integration closes out the chapter.

Chapter 9 continues the discussion of applications of integration to measuring various averages, including centers of mass, and moving averages. The chapter then turns to physical applications such as velocity, force, and work. Improper integrals are introduced, and the chapter concludes with a section on probability measurement.

Chapter 10 discusses sequences and approximations. Long division is used as a familiar example of an infinite process that can produce an infinite sequence of approximations. Several examples of sequences, including recursive and iterative sequences, are examined. Limits are revisited using derivatives in the study of indeterminate forms (L'H ôpital's Rule). Root-finding methods, including the bisection method and Newton's Method, are also included as examples of techniques yielding sequences of approximations. The chapter concludes with a closer look at iterative methods in general and the Fixed Point Theorem.

Chapter 11 starts out with a discussion of the Archimedean property of real numbers and Zeno's paradox to motivate the idea of a series. A series is then defined as the limit of a sequence of partial sums. Tests of convergence include the Nth term test, comparison and limit comparison tests, the integral test, the alternating series test and the root and ratio test. Absolute and conditional convergence are contrasted. After a discussion of power series, including interval and radius of convergence, this chapter concludes with a study of function approximation techniques, in particular, Taylor polynomials and series.

Accompanying Volume I is a *Student Guide to Using Technology in Calculus*. Like the text, this supplement does not assume the use of any particular hardware or software. The introductory chapter of the supplement outlines some of the important factors one must be aware of in order to make intelligent use of technology. The effects of round-off and cancellation errors in numerical computations, the inherent limitations of machine graphics, and the importance of contextual assumptions are some of the issues addressed. The rest of the student guide consists of activities intended to supplement, extend, or reinforce many of the topics in the text. Of course, as the title suggests, the proper use of technology is a central theme throughout the supplement.

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Finally, to Leslie and Colleen, Daniel, Jean, Connor, and Eamon, for tolerating the authors during the temporary insanity that accompanies working on such a project, we dedicate this book.

From the Publisher

It has been said that the only constant in life is change. We see it all around us. From the transformation of seasons ... to the new-found freedoms in Eastern Europe ... to individual growth and development.

The field of mathematics is no exception. Even the notion of *change* itself forms the very foundation of the calculus.

At PWS-KENT Publishing Company we are convinced that the call for reform in calculus instruction is here to stay. It may not alter much in content. It may not usher in a myriad of published product. And it may not appeal to every mathematician. It has received recognition, however, from such venerable institutions as the National Science Foundation, the American Mathematical Society and the Mathematical Association of America. And it has piqued the interest of many dedicated instructors who feel that something new and innovative is needed to revitalize the subject-matter.

Issues and developments in this reform movement have been the focus of several Calculus Reform Workshops which have been jointly sponsored by PWS-KENT, the Oregon State University Calculus Curriculum Project, and the Hewlett-Packard Company (in conjunction with a grant from the NSF). The workshops, whose primary function has been to provide the "grass roots" of the mathematics community with an opportunity to contribute their ideas and concerns on the curriculum, also have contributed greatly to this publication by Thomas Dick and Charles Patton.

As PWS-KENT enters its twenty-sixth year of being "Partners in Education," we hope that the published works of the Oregon State University Calculus Curriculum Project will be recognized as our contribution to an evolving calculus marketplace which has for more than fifteen years been very generous to us.

With this product we invite inquiry, scrutiny, and discovery. For, together, we can seek the security of change.

The Editors

Contents

CHAPTER

0

Introduction: What Is Calculus?

What is calculus? The word itself suggests *calculations*, but to think of calculus as merely a collection of computational techniques does a grave injustice to the subject. Rather, calculus is a network of fundamentally important mathematical *ideas*. These ideas had their origins in attempts to solve particular measurement problems in geometry and physics. Specifically, the problems of measurement of length, area, and volume in geometry and the problems of measuring force, velocity, and acceleration in physics gave birth to the calculus. Today the applications of calculus reach far and wide to quantitative analysis in fields ranging from archaeology to zoology.

There are two major branches of calculus. The *differential calculus* involves measuring the instantaneous rate of change of one quantity relative to the change in another quantity. This *physical* measurement problem corresponds to a *geometric* measurement problem— finding the slope of a graph at a specific point. *Integral calculus* has its origins in the geometric problems of measuring of length, area, and volume. This corresponds to the physical problem of measuring total change from information about rate of change. A realization of the inverse nature of these two problems marked the dawn of the modern age of calculus.

Why is calculus useful? Understanding the dynamics of *change* is a primary goal in the study of any system, whether it be physical, biological, economic, or social. When we analyze processes involving continuous change, calculus can allow us to describe and measure the rate of that change and its total effects. By harnessing the notion of *infinite processes* and their application to approximation, calculus provides both powerful tools and an effective language for describing and measuring change.