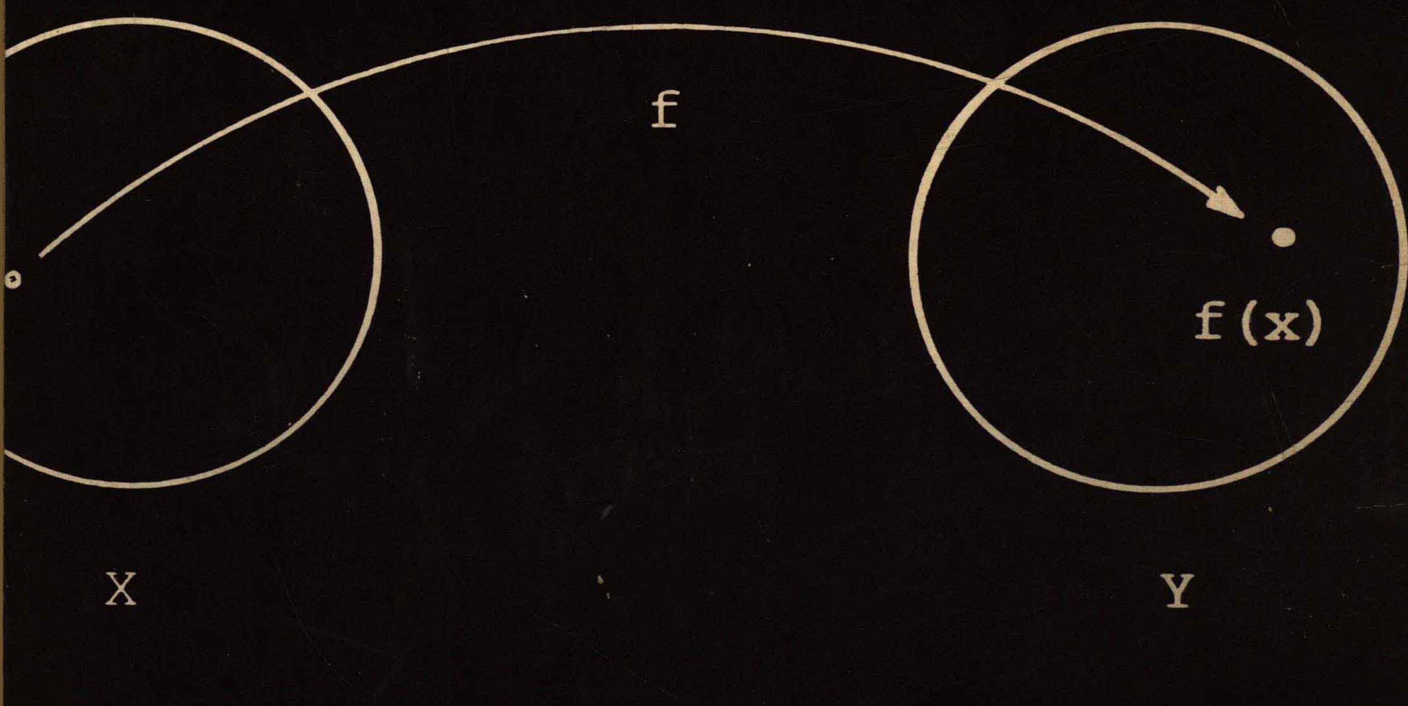
Louis J. Nachman

UNDAMENTAL MATHEMATICS



FUNDAMENTAL MATHEMATICS

Louis J. Nachman
Oakland University

JOHN WILEY & SONS

New York • Santa Barbara • Chichester • Brisbane • Toronto

Library of Congress Cataloging in Publication Data:
Nachman, Louis J.
Fundamental mathematics.

Includes index.
1. Mathematics—1961- I. Title.
QA39.2.N3 510 77-17239
ISBN 0-471-62815-8

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

PREFACE

This textbook subscribes to the educational philosophy that "you learn mathematics by doing mathematics," not by reading about it in a book or by listening to it discussed in a lecture. The author believes that mathematics, even at the precalculus level, is a beautiful interweaving of seemingly different kinds of ideas (at this stage, algebraic and geometric ideas) and should be taught that way. The beginning mathematics student needs many computational skills, and these skills are sometimes neglected in an attempt to "get on with" more formal, definition-theorem-proof mathematics. The beginning student also needs a basic understanding of where the computational techniques came from and what binds them together; this understanding is sometimes neglected in the attempt to develop the basic skills. A precalculus course is not a terminal course in mathematics and thus should probably raise as many questions as it answers. Finally, much of the mathematics at the precalculus level is applicable to real-life situations, and these applications should be a part of a precalculus course.

This textbook attempts to apply all the points of the previous paragraph to a teachable precalculus course. It consists of standard textual material written in a loose, conversational style. Interspersed throughout the text are study problems. These study problems serve several different functions. Some of them are drill problems appearing after examples to give the student an immediate opportunity to check his or her understanding of the skill or technique involved. Virtually every new computational technique is immediately followed by several simple study problem applications of that technique.

Some of the study problems are detailed examples. If a particular skill or application of ideas is complicated or involves several procedures, an example, broken down into steps, where the student supplies the answer to each of the steps, usually follows an example worked out in detail. The stepped examples are then followed by study problems for which the student supplies all the work.

A third kind of study problem involves the discovery of new ideas, techniques, or theorems. The purpose of these study problems is to allow the sudent to see how a mathematician or a user of mathematics reasons from problem to solution and hopefully to help the student realize that mathematics is much more than a collection of formulas, graphs, definitions, and theorems.

The study problems are numbered consecutively within sections. Thus problem 3.24 is the 24th study problem in Section 3 of a particular chapter. Space is provided to answer these problems in the text. Thus, this text is to be read with a pencil and some scrap paper at hand. All the answers to the study problems in a given section will be found following the exercises at the end of that section.

There is more than ample opportunity for the sudent to practice the skills learned and to work with the ideas developed in this text. In addition to the study problems, each section is followed by a set of exercises. The exercises are grouped (more or less) according to the following scheme:

A Computational and drill problems.
B More involved computational or drill problems calling for material from previous sections (sometimes even some thought).

C Applications.

D Problems dealing with the theory underlying the concepts.

Review Exercises, grouped according to the same scheme, appear at the end of each chapter. Answers to selected exercises and review exercises appear in the back of the book.

The "learn mathematics by doing mathematics" philosophy is implemented at two levels in this book. First, and most obvious, are the study problems. The idea is to have the student read the textbook with pencil and paper at hand; to try to force the student to think about the material before he or she does the homework. The second level involves the exercises at the end of each section. Not every exercise has an antecedent example in the text or study problems. The reasoning here is obvious. If there is an example to copy for every problem, the student is only memorizing types of problems, not really learning mathematics. This is not to say that the exercise sets are minefields of new material, but just that there are enough new types of problems so that students will have to use their brains a little to get through them.

By quickly glancing through the book, one can see that a major emphasis is the interrelationship between algebra and geometry. Whenever possible, the geometry of a new idea is used either to motivate or to explain the idea. Hopefully the student will become aware that the interplay between the two kinds of mathematics can be very useful, both for understanding and for applying mathematical concepts.

Some words of warning: This is not a "cookbook" mathematics text. Indeed, the structure of the book makes it very difficult for the student to use it as one. At times the student must work hard to find out what the text is trying to say. It is also not a formal mathematics text. Although definitions are given and theorems are stated, very few formal proofs appear. Instead, the book strives to develop the student's mathematical intuition and informal understanding, a base upon which a more formal and complete understanding can later be built.

Throughout the text, an attempt at introducing practical applications of the mathematics has been made. This is not, however, an applications text. Many applications are explained in the text; others appear only in the exercises.

A precalculus course naturally leads to the calculus. An attempt has been made to include all the basic or fundamental mathematics needed for the calculus in this text. Some of the basic ideas of the calculus have been preintroduced by briefly discussing some of their nonlimit antecedants. Limits (although that name is not used) are touched on in the discussions of rational functions. These preintroductions are meant to set the student's mental stage for the actual discussion of these concepts in a future course. In this way, as well as others, it is hoped that the text will whet the student's mental appetite for more mathematics.

A preliminary version of this book was prepared under a grant from the Alfred P. Sloan Foundation to develop what has now become known as a PSI course in precalculus mathematics. In its present form the textbook is not aimed specifically at self-study programs, although it is probably suitable for such programs with some regularly available tutoring assistance and well-motivated students. The book has been used in standard freshman courses at Oakland University with success. I thank my many colleagues there who have contributed helpful suggestions and comments. In particular, I wish to thank Professor Jerry Grossman who helped read galley proofs and corrected many errors. (All remaining errors are the author's responsibility.)

There are several other people whom I also must thank. Sandra Blagbourne, Caroline Chipman, and Jana DeVore each typed a preliminary version of the text. Patricia Korsog typed the copy for the published preliminary edition. Each of these people worked under extremely trying circumstances. I also thank Ronald St. John of Hamilton Press and especially the ever-patient Gary Ostedt of John Wiley. Last but certainly not least, the patience and understanding of my wife, Pat, and my children have made the writing of this text possible.

Louis J. Nachman

CONTENTS

IV Some Particular Functions

V Exponential and Logarithmic Functions

VI Trigonometric Functions

CHAPTER I

THE REAL NUMBERS

INTRODUCTION

This chapter falls naturally into two parts. The first part, Section 1, is an introduction to the language of set theory. This section deals with notation and terminology that will be used throughout the remainder of the text. Thus, mastery of its content and notation is essential.

The second part of this chapter deals with the real number system and is composed of Sections 2, 3, 4, 5, and 6. In these sections, we present a working definition of the system of real numbers, and we develop a picture of the real number system. The interdependence of algebra and geometry, which is a dominant theme of this book, is then used to discuss the order relations on the set of real numbers and to define the concept of absolute value. Various techniques for working with inequalities and absolute value are developed. These techniques will also be called upon throughout the remainder of the textbook.

The last section, Section 6, is devoted to a specific subset of the set of real numbers, the counting numbers. In this section, several counting techniques are developed. These techniques are then applied to discover a general formula for $(a + b)^n$, for n a positive integer (the binomial theorem).

Throughout this chapter, it is assumed that the reader is familiar with the computational techniques and results contained in a standard high school algebra II course.

SECTION 1

THE LANGUAGE OF SETS

In this section, we introduce the language and notation of elementary set theory. The language developed here will be used throughout the remainder of this book.

Specifically, we briefly discuss the concepts of sets and objects belonging to a set. We define what is meant by a subset and when two sets are considered to be equal. We discuss pictures of sets (called Venn diagrams), the empty set, the union and intersection of sets, and the complement of a set.

After completing this section you should be able to

1. read and write set builder notation and other notation for sets
2. tell when one set is a subset of another and when two sets are equal
3. find the union and intersection of sets
4. find the complement of set A in set X if A is a subset of X.

SETS AND NOTATION

The idea of a *set* is basic to much of the work in mathematics. We can think of a set as a bunch, family, class, deck, or collection of objects which are called *elements* of the set. When possible, we denote a set by listing the elements of the set inside curly braces.

Example. $A = \{1,2,3,4,5,6\}$ is the set of numbers 1,2,3,4,5,6. The numbers 1,2,3,4,5, and 6 are the elements of the set.

Example. B = {George Washington} is the set whose only element is George Washington.

Example. C = {A,2,3,4,5,6,7,8,9,10,J,Q,K} is the set of cards in a suit in a standard bridge deck.

Capital letters are usually used to name sets. If A is the name of some set and a is an element of set A, we denote this symbolically by

$$a \in A$$

Notation	$a \in A$ is read "a is an element of A" or "a belongs to A."

Example. $2 \in C$ from above
$5 \in A$
George Washington $\in B$.

Note that Millard Fillmore is not in set B. We use the notation $\notin$ to denote "*is not an element of*" or "*does not belong to.*"

Example. M. Fillmore $\notin B$
$17 \notin A$.

*1.1. Let $A = \{*, \Delta, 0, 3\}$. Which of the following sentences are true?

a. $* \in A$ ______
b. $* \notin A$ ______
c. $0 \in A$ ______
d. $4 \in A$ ______
e. $\Delta \in A$ ______
f. $\Delta \notin A$ ______
g. $3 \notin A$ ______
h. $-1 \notin A$ ______
i. $A \notin A$ ______

1.2. Let $B = \{2,4,6,8,\ldots,20\}$. What do you think the three dots mean? ______

1.3. Let B be the set in problem 1.2 above. Which of the following sentences are true?

a. $2 \in B$ ______
b. $2 \notin B$ ______
c. $3 \in B$ ______
d. $3 \notin B$ ______
e. $10 \in B$ ______
f. $12 \in B$ ______
g. $16 \in B$ ______
h. $18 \notin B$ ______

The three dots notation is a convenient way of saying "continue the pattern." It can be used for long, finite lists as above or for nonfinite lists as below.

1.4. Let $C = \{3,6,9,12,\ldots\}$. What do the three dots mean? ______

1.5. Let C be the set in problem 1.4 above. Which of the following are true?

a. $3 \in C$ ______
b. $3 \notin C$ ______
c. $4 \in C$ ______
d. $4 \notin C$ ______
e. $18 \notin C$ ______
f. $18 \in C$ ______
g. $21 \in C$ ______
h. $300 \notin C$ ______

Some of the sets we discuss in this course will have many elements in them (many are not finite). It will not always be possible to list every element of these sets. One way around this problem is the *three dots notation* used above. We can use this notation when there is a definite sequence of objects with a detectable pattern. See if you can use this notation in the following:

1.6. Let X be the set of positive whole numbers. Use the three dots notation to write the set X. $X =$ ______

1.7. Let V be the set of squares of the positive whole numbers from 1 through 20. Use the three dots notation to write the set V. $V =$ ______

Not all sets lend themselves to the three dots notation. For such sets, we have another notation, *set builder notation.*

> **Notation** $\{x \mid P\}$ is read "the set of all x's that satisfy property P" or "the set of all x such that P." This is called set builder notation.

Example. $\{x \mid x \text{ is a positive even whole number}\}$ is a set builder notation for the set $\{2,4,6,8,10,\ldots\}$.

Example. $\{x \mid x^2 = 9\}$ is a set builder notation for the set $\{-3,3\}$.

Example. $\{(x,y) \mid x \neq y\}$ is the set of all ordered pairs of numbers x and y that have the property that $x \neq y$. So, for example, $(2,4) \in \{(x,y) \mid x \neq y\}$ but $(-3,-3) \notin \{(x,y) \mid x \neq y\}$.

*Answers to study problems appear at the end of each section.

1.8. Let $X = \{x \mid x \text{ is a whole number greater than } 2\}$. Circle the numbers below that are elements of X.

$$4 , ½ , 2 , -3 , 6.2 , 0 , -1 , 3 , 10 , 5$$

1.9. Let $Y = \{(x,y) \mid y = 2x\}$. Then $(-1,-2) \in Y$, since $-2 = 2(-1)$. Which of the following are true?

a. $(-6,-12) \in Y$ ______ c. $(½,1) \in Y$ ______
b. $(11,21) \in Y$ ______ d. $(0,0) \in Y$ ______

1.10. Let $Z = \{(x,y,z) \mid y = x^2 \text{ and } z = 0\}$. Which of the following are true?

a. $(0,0,0) \in Z$ ______ c. $(-1,4,0) \in Z$ ______
b. $(1,-1,0) \in Z$ ______ d. $(3,9,1) \notin Z$ ______

SUBSETS

We now turn to a discussion of the various relationships that exist between sets. Here are two examples of one of these relationships.

1.11. If we let $\mathbb{N} = \{1,2,3,4,...\}$ and we let $E = \{2,4,6,8,...\}$, is every element of $\mathbb{N}$ an element of E? Is every element of E an element of $\mathbb{N}$? ______________________________

1.12. Let $A = \{x \mid x \text{ is a positive number}\}$, and let $B = \{y \mid y \text{ is a number greater than } 1\}$. How are the two sets, A and B, related? ______________________________

If A and B are two sets, and if every element of A is also an element of B, then we say *A is a subset of B* and write $A \subseteq B$.

Definition 1.1 If A and B are sets, then *A is a subset of B* (written $A \subseteq B$) if whenever $x \in A$, x must also be in B.

Example. $\{1,2,3\} \subseteq \{1,2,3,4\}$ because every element of $\{1,2,3\}$ is also an element of $\{1,2,3,4\}$. However, $\{1,2,3,4\} \nsubseteq \{1,2,3\}$ because $4 \in \{1,2,3,4\}$ but $4 \notin \{1,2,3\}$.

Example. If $\mathbb{N} = \{1,2,3,...\}$ and $\mathbb{Z} = \{...,-2,-1,0,1,2,...\}$ then $\mathbb{N} \subseteq \mathbb{Z}$. Is $\mathbb{Z} \subseteq \mathbb{N}$?

Example. $\{2,3,2\} \subseteq \{2,3,4\}$ because every element of $\{2,3,2\}$ is also an element of $\{2,3,4\}$. The fact that 2 is listed twice in $\{2,3,2\}$ does not prevent $\{2,3,2\}$ from being a subset of $\{2,3,4\}$.

We can draw a picture, such as that in Fig. 1.1, which represents the sentence "$A \subseteq B$."

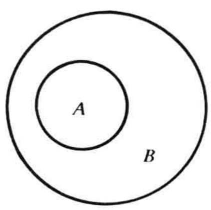


Figure 1.1

Sketches of sets, like that in Fig. 1.1, are called *Venn diagrams.* Note that every element of set A is also an element of set B

1.13. Suppose $A \subseteq B$ and $x \in A$. Do we know anything else about x? If so, what? ____________________

____

1.14. Suppose $A \subseteq B$ and $x \in B$. Do we know anything else about x? If so, what? ____________________

____

1.15. Let $A = \{2,4,6,8\}$, $B = \{2,4\}$, and $C = \{1,2,3,4,5,6,7,8\}$. Which of the following are true?

a. $A \subseteq B$ ______ c. $A \subseteq C$ ______ e. $B \subseteq C$ ______

b. $B \subseteq A$ ______ d. $C \subseteq A$ ______ f. $C \subseteq B$ ______

1.16. In the Venn diagram in Fig. 1.2, each of the labeled regions represents a set. Write down all the subset relations determined by the picture.

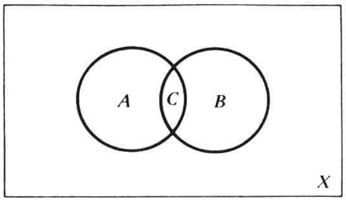


Figure 1.2

____

____

____

____

EQUALITY OF SETS

We now know that $A \subseteq B$ when every element of A is also an element of B. We will also need some way of deciding when two sets are equal. For instance, if $A = \{x \mid x \text{ is a one-digit positive whole number divisible by } 3\}$, then A can be written $\{3,6,9\}$ or $\{9,3,6\}$ or $\{9,6,3\}$, etc. Although these three sets are not written in the same order, they do have the same elements. We will take this concept, that two sets have the same elements, as our definition of equality of sets.

> **Definition 1.2** If A and B are sets, then $A = B$ if every element of B is an element of A and every element of A is an element of B.

Example. $\{3,5,7,9\} = \{3,7,5,7,9\}$ because every element of the first set is an element of the second set and every element of the second set is an element of the first set. Note that the repetition of the 7 in the second set is not relevant.

Example. $\{1,2,3,4,...\} = \{x \mid x \text{ is a positive whole number}\}$.

1.17. Let $A = \{1,2,3,4\}$. Which of the following are equal to A?

a. $\{2,1,3\}$
b. $\{2,3,4,1\}$
c. $\{1,2,3,4,5\}$
d. $\{4,2,1,3\}$
e. $\{1\}$
f. $\{1,3,1,2,1,4\}$

1.18. Let $X = \{x \mid x$ is a whole number that has a whole number square root$\}$. Which of the following sets is equal to X?

a. $\{1,3,5,7,\ldots\}$
b. $\{1,4,9,16,\ldots\}$
c. $\{1,9,25,\ldots\}$
d. $\{0,1,4,9,16,\ldots\}$

1.19. Suppose $A \subseteq B$. What else would you need to know in order to be sure that $A = B$? ______________

____

1.20. Use the $\subseteq$ notation to rewrite Definition 1.2. $A = B$ if ______________

____

To review briefly, we say $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$. Neither the order of the elements nor the number of times an element appears is important when deciding whether or not two sets are equal. We now turn to methods of putting sets together to form new sets.

UNIONS AND INTERSECTIONS

If A and B are two sets, we can form two new sets from A and B. These new sets are called A *union* B ($A \cup B$) and A *intersect* B ($A \cap B$).

These two new sets are shown in Fig. 1.3. Intuitively $A \cup B$ is formed by combining A and B to form a

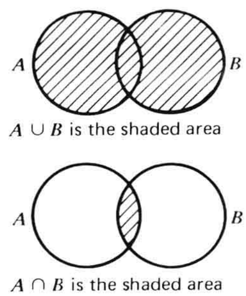


Figure 1.3

larger set consisting of all those elements that are either in A or in B. $A \cap B$ is formed by taking all elements that are common to both A and B.

1.21. Let $A = \{1,2,3\}$ and $B = \{2,4,6,8\}$. Then $\{1,2,3,4,6,8\}$ and $\{2\}$ are two sets related to A and B. Which one is $A \cup B$? ______________

Is the other one $A \cap B$? ______________

1.22. Let $X = \{a,b,c,d\}$ and $Y = \{b,d,e,g\}$. Find $X \cup Y$ and $X \cap Y$.

$X \cup Y =$ ______________ $X \cap Y =$ ______________

We can write a formal definition for the union and intersection of two sets.

Definition 1.3 If A and B are sets, then
$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.
$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Example. $F = \{x \mid x$ is a positive whole number divisible by $5\}$ and $T = \{x \mid x$ is a positive whole number divisible by $2\}$.

$T \cup F = \{2,4,5,6,8,10,12,14,15,16,\ldots\}$

$T \cap F = \{10,20,30,40,\ldots\}$.

Example. Let $D = \{x \mid x$ is a card in a standard bridge deck$\}$.

Let $R = \{x \mid x \in D$ and x is red$\}$

Let $A = \{x \mid x \in D$ and x is an ace$\}$.

$R \cup A = R \cup \{A \text{ (clubs)}, A \text{ (spades)}\}$. $R \cap A = \{A \text{ (hearts)}, A \text{ (diamonds)}\}$.

1.23. Let $A = \{2,4,6,8,10,\ldots\}$ and $B = \{3,6,9,12,\ldots\}$. Use the three dots notation to write out $A \cup B$ and $A \cap B$.
$A \cup B =$ ______________________________
$A \cap B =$ ______________________________

1.24. Let D be the deck of cards in the example above. Let $C = \{x \mid x \in D$ and x is a club or x is an ace$\}$ and $E = \{x \mid x \in D$ and x is a heart or x is a king$\}$. Find the following:
$C \cup E =$ ______________________________
$C \cap E =$ ______________________________

1.25. If x is an element of both set A and set B, then x is an element of the ______________________ of A and B.

1.26. If x is either in set A or in set B (or in both), then x is an element of the ______________________ of A and B.

1.27. By definition, $A \subseteq B$ if ____

1.28. If $A \subseteq B$ and $B \subseteq A$, then we say A ____ B.

THE EMPTY SET

1.29. Let $A = \{2,4,6,8,\ldots\}$ and $B = \{1,3,5,7,9,\ldots\}$. List the elements of $A \cap B$. ______________________
____.

Of course, set A and set B in problem 1.29 have no elements in common; their intersection is an *empty set.*

Definition 1.4 The set with no elements is called the *empty set* or the *null set* and is symbolized by the Greek letter ϕ (phi).

Before we explain why we say "the" empty set, try these problems.

1.30. ϕ represents the set that has ______________________________

1.31. Which of the following represent the empty set?
a. $\{x \mid x$ is greater than 0 and x is negative$\}$
b. $\{x \mid x$ is now king of the U.S.A.$\}$
c. $\{x \mid x = x^2\}$
d. $\{x \mid x$ is a person 9 feet tall$\}$

The following is a brief explanation of the fact that *all* empty sets are equal. Let A be any set. Because there are no elements of ϕ that are not in A, we will agree that $\phi \subseteq A$.

Using this line of reasoning, if ϕ_1 and ϕ_2 are both empty sets, then $\phi_1 \subseteq \phi_2$ and $\phi_2 \subseteq \phi_1$. But then $\phi_1 = \phi_2$. Thus, all empty sets are equal, and we can talk about *the* empty set rather than *an* empty set.

COMPLEMENTS OF SETS

If we are discussing two sets A and X, and $A \subseteq X$, then we will sometimes want a name for the part of X that is not in A, the part of X shaded in Fig. 1.4.

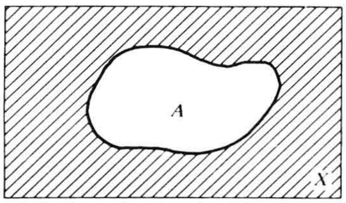
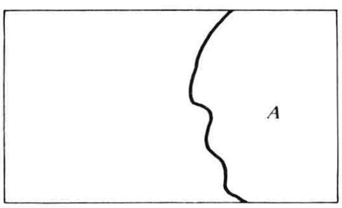


Figure 1.4

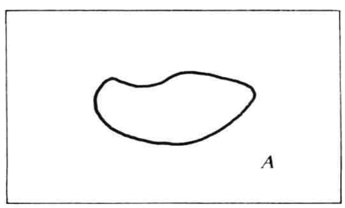
This part of X is called the *complement of A in X,* and we denote it by X-A.

1.32. In each of the following Venn diagrams, shade the complement of A.

a.



b.



1.33. Let $\mathbb{N} = \{1,2,3,4,...\}$. For each of the following, find the complement of the given set in $\mathbb{N}$.
a. $E = \{2,4,6,8,...\}$, $\mathbb{N}$-E = ______________________________
b. $T = \{10,11,12,...\}$, $\mathbb{N}$-T = ______________________________
c. $S = \{1,4,9,16,25,...\}$, $\mathbb{N}$-S = ______________________________

The formal definition of the complement of A in X is given by

Definition 1.5 If $A \subseteq X$, then the complement of A in X, denoted X-A, is given by

$$X\text{-}A = \{x \mid x \in X \text{ and } x \notin A\}.$$

When we talk about the complement of a set, it is important to be aware of what set we are taking the complement in, as the next problem demonstrates.

1.34. Let $A = \{2,4,6\}$, $B = \{2,4,6,8\}$, and $C = \{2,4,6,8,10\}$.
  a. Find the complement of A in B. ____________
  b. Find the complement of A in C. ____________
  c. Are these two sets the same? ____________

Now that you have completed this section, you should be familiar with the following words and notation:

- set
- element
- . . . three dots notation
- $\{x \mid P\}$ set builder notation
- $\in$ is an element of
- $\notin$ is not an element of
- $\subseteq$ subset
- $=$ equality of sets
- ϕ the empty set
- $\cup$ union of sets
- $\cap$ intersection of sets
- X-A the complement of A in X

You should be able to

- read and write set builder notation
- tell when one set is a subset of another and when two sets are equal
- find the union and intersection of sets
- find the complement of a set

EXERCISES I/1

A 1. Use either set builder notation or listing to write the following sets.

  a. X is the set of all college students under 12 years old.

  b. B is the set of x's such that x is greater than 3 but x is not equal to 6.

  c. The set of all pairs (x,y) such that x is greater than 7 and y is less than 7.

d. The set of all pairs (x,y) the sum of whose numbers is greater than -10.

e. The set of all numbers between (but not equal to) -2 and 1.

2. Translate the following into words.

a. $M = \{m \mid m > 0\}$

b. $S = \{x \mid x^2 + 2 = 7\}$

c. $P = \{p \mid -10 \leqslant p \leqslant 10 \text{ and } p^2 \neq 16\}$

d. $T = \{(x,y) \mid x = y \text{ and } x \geqslant 0\}$

3. Let $X = \{1,2,3\}$. Let $Y = \{x \mid x \text{ is a positive whole number}\}$. Let $Z = \{z \mid z \text{ is a real number}\}$. Write out all the subset relations that you can find involving sets X, Y, and Z.

4. Let $X = \{2,4,6,8\}$, $Y = \{3,6,9,12\}$, and $Z = \{1,2,3,4\}$. List the following sets:

a. $X \cup Y$

b. $X \cap Y$

c. $Y \cup Z$

d. $Y \cap Z$

e. $X \cap X$

f. $X \cap Z$

g. $(X \cup Y) \cup Z$

h. $X \cup (Y \cup Z)$

i. $X \cap (Y \cap Z)$

j. $(X \cap Y) \cap Z$

k. $X \cap (Y \cup Z)$

l. $(X \cap Y) \cup (X \cap Z)$

5. Let $X = \{1,2,3,4,\ldots,20\}$. Let $A = \{x \mid x \in X \text{ and } x \text{ is even}\}$, $B = \{x \mid x \in X \text{ and } x \text{ is a multiple of } 3\}$, $C = \{x \mid x \in X \text{ and } x \text{ is less than } 10\}$, $D = \{x \mid x \in X \text{ and } x \text{ is odd}\}$, and $E = \{x \mid x \in X \text{ and } x \text{ is a multiple of } 5\}$. Find the following:

a. $A \cup B$

b. $C \cap D$

c. $A \cap D$

d. $A \cap E$

e. $A \cup D$

f. $C \cup D$

g. $C \cap D$

h. $B \cap C$

i. $E \cup A$

6. Let $\mathbb{Z}$ be the set $\{\ldots -3,-2,-1,0,1,2,3,\ldots\}$. Find the complements of each of the following sets in $\mathbb{Z}$.

a. $\{x \mid x \in \mathbb{Z} \text{ and } x \text{ is between } -2 \text{ and } 4 \text{ (including } -2 \text{ and } 4)\}$

b. $\{x \mid x \in \mathbb{Z} \text{ and } x \text{ is greater than } 2 \text{ or } x \text{ is less than } -5\}$

c. $\{x \mid x \in \mathbb{Z} \text{ and } x \text{ is greater than or equal to } -11\}$

d. $\{x \mid x \in \mathbb{Z} \text{ and } x \text{ is between } -7 \text{ and } -9 \text{ (not including } -7 \text{ and } -9)\}$

7. Write out the following sets using either set builder notation or the three dots notation.

a. The set of even, nonnegative whole numbers.

b. The set of ordered pairs (x,y) for which $y = x^2 - 4$.

c. The set of positive fractions whose numerators are integers and whose denominators are all 2.

d. The set of whole numbers.

e. The set of rational numbers (fractions whose numerators and denominators are integers) between 4 and 11 inclusive.

8. Let $X = \{1,2,3,\ldots\}$. Let $E = \{2,4,6,8,\ldots\}$, $T = \{3,6,9,\ldots\}$ and $F = \{5,10,15,20,\ldots\}$. Using either the three dots notation or set builder notation, write out the following sets.