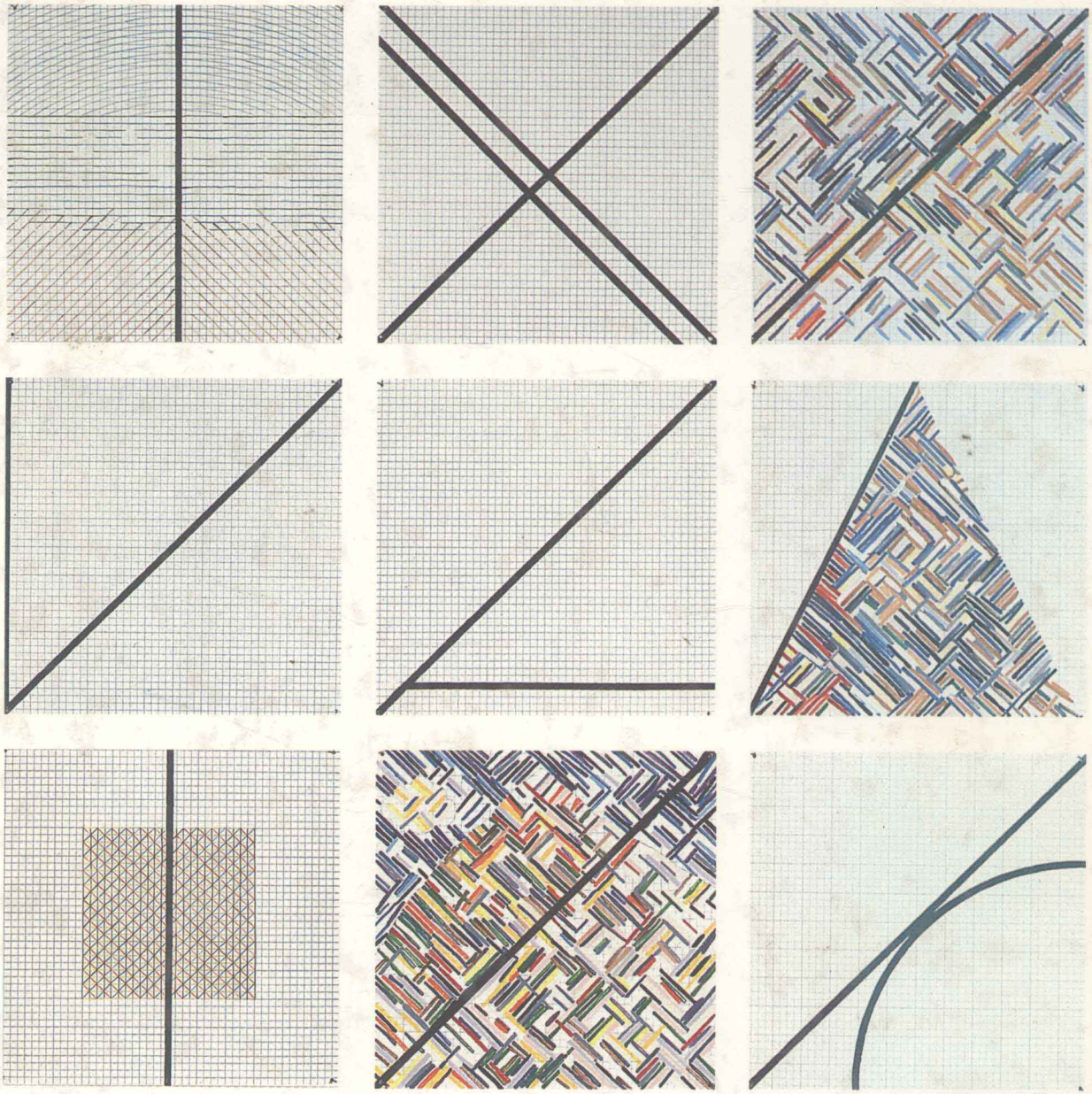

CALCULUS WITH APPLICATIONS

to the Management, Life, and Social Sciences

SPENCE / VANDEN EYNDEN

CALCULUS WITH APPLICATIONS
to the Management, Life, and Social Sciences

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TO THE STUDENT

If you want further help with this course, you may want to obtain a copy of the *Student's Solutions Manual* that accompanies this textbook. This manual provides detailed step-by-step solutions to the odd-numbered exercises in the textbook and can help you study and understand the course material. Your college bookstore either has this manual or can order it for you.

Cover and spine: Details, *Rhapsody* by Jennifer Bartlett. Photo Courtesy Paula Cooper Gallery.

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Portions of this book have been adapted from *Finite Mathematics* by Daniel Gallin.

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PREFACE

Calculus With Applications to the Management, Life, and Social Sciences is intended for use in a brief calculus course that emphasizes mathematical applications and models. Such courses have become commonplace at most American colleges and universities during the past fifteen years, a development that reflects the increased use of quantitative concepts and techniques in many disciplines outside the mathematical and physical sciences.

Flexibility

Calculus With Applications contains ample material for a four-credit course lasting one semester or two quarters. Consequently, instructors will have considerable flexibility in designing a course to meet the needs of their students.

Prerequisites

Although it is assumed that the reader has completed the equivalent of one and a half years of high school algebra, essential algebraic concepts are reviewed when necessary. In particular, Chapter 1 presents the essential ideas about functions that are required for the remainder of the book. For students with strong algebra backgrounds, this chapter can be covered quickly or even omitted. Additional algebra topics are contained in an optional appendix that can be used by instructors who desire more complete coverage. The need for review, however, has been balanced against the risk of boredom. We prefer to avoid the discussion of algebra for its own sake in favor of treating mathematics that is more readily applicable to the disciplines for which the book is written.

APPROACH

The mathematics presented in this book is developed intuitively, yet without loss of mathematical precision. Above all we have tried to provide students with conceptual understanding rather than a collection of mechanical procedures. For example, we use graphs of functions extensively to reinforce the analytic concepts under discussion. In addition, we include a wide variety of applications which will motivate students to learn mathematics as well as to see its power and utility in other disciplines.

Exponential and Logarithmic Functions

We have chosen to disperse the presentation of exponential and logarithmic functions throughout the book, rather than to confine it to a single chapter. In our experience, students who are exposed to these functions throughout the calculus develop a much deeper understanding of them than when the functions are discussed primarily in one brief chapter. Moreover, the early introduction of the exponential and logarithmic functions allows for more varied examples and applications than could be given using only rational functions. Our desire to introduce exponential and logarithmic functions as soon as possible has led us to present the chain rule prior to the rules for differentiating products and quotients. This enables us to use the exponential and logarithmic functions as examples when discussing the product and quotient rules.

Calculators

We assume that students will have access to a calculator when working exercises, especially those involving natural logarithms and values of the exponential function e^x. Nevertheless, most exercises are designed so that the necessary computations can be performed by hand, and tables giving the values of $\ln x$ and e^x are included.

KEY FEATURES

Applications

Special emphasis has been placed on applications to business, economics, and the life and social sciences in special sections called ''Mathematics in Action.'' These sections present realistic uses of mathematics rather than contrived or artificial examples. At the discretion of the instructor, they may be omitted, however, as no other parts of the text depend on them. Other sections, marked ''optional'' in the table of contents, contain mathematics that can be skipped without loss of continuity. The extent to which the application sections and the optional sections are covered will depend on the length of the course and the nature of the students being taught.

Examples and Exercises

Calculus With Applications contains nearly 300 numbered examples that are carefully chosen to demonstrate both the mathematical techniques under consideration and their usefulness in solving real-world problems. A large number of exercises are included in the book. The exercises within each section range from easy to difficult, and many contain realistic applications of the mathematical ideas presented.

Practice Problems

Over 200 practice problems occur throughout the book. These are designed to test a student's understanding of the mathematical content being developed by providing an immediate opportunity to work a problem involving the concepts

and techniques being discussed. As a further help to the student, both answers (at the end of each section) and *complete solutions* (at the back of the book) to practice problems are provided.

Chapter Reviews

Each chapter concludes with a comprehensive chapter review that lists the new terms and important formulas introduced in that chapter. These reviews also contain exercises that are intended as a practice chapter test for students. Although these exercises may sometimes require more than an hour to complete, they cover all the principal ideas in the chapter and should enable students to test their understanding prior to an examination.

Format

The format of this book is intended to help students recognize important results and techniques. Terminology being defined appears in boldface. Boxes are used to highlight theorems and procedures for emphasis and ease of reference. A second color is also used functionally in the artwork and in the text (for example, to distinguish the graph of a function from a tangent line or to annotate calculations).

COURSE ORGANIZATION

Calculus With Applications can be subdivided into seven major subjects: functions of one variable (Chapter 1); differential calculus (Chapters 2–4); integral calculus (Chapters 5-6); multivariable calculus (Chapter 8); trigonometry (Chapter 9); and infinite series (Chapter 10). In Chapter 7, concepts from integral calculus are applied to probability. The diagram on the next page shows the logical dependence of the chapters.

SUPPLEMENTS

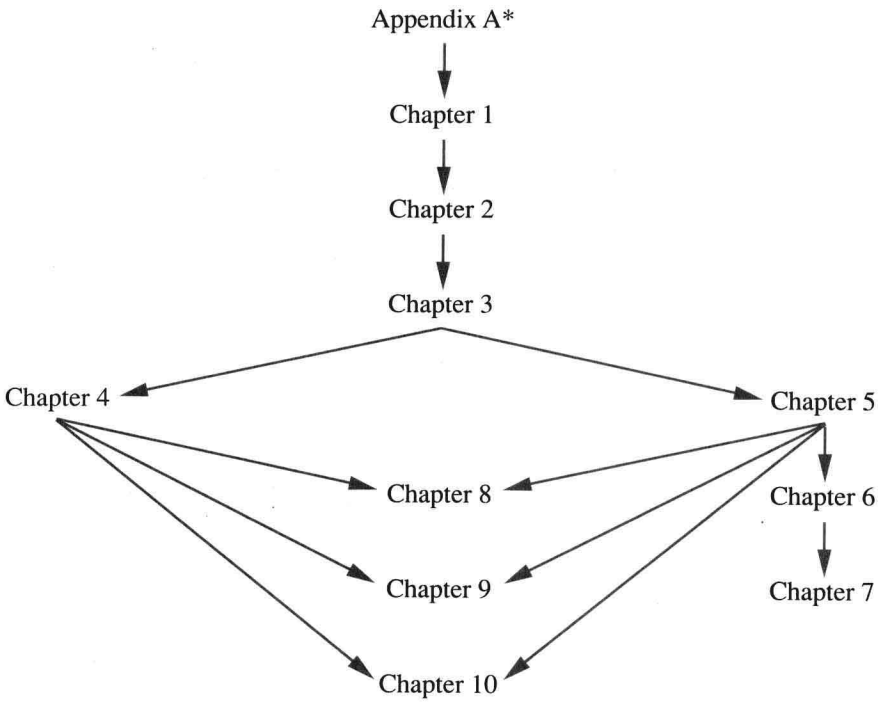
Manuals

The following resources were prepared by John I. Hill, Vanessa F. Miller, Linda Ann Spence, and Joan Vanden Eynden, of Illinois State University:

The **Instructor's Guide and Solutions Manual** gives a lengthy set of test questions for each chapter, organized by section, plus answers to all the questions. It also provides complete solutions to all of the even-numbered text exercises.

The **Instructor's Answer Manual** gives the answers to every text exercise, collected in one convenient location.

The **Student's Solutions Manual,** available for purchase by students, provides detailed, worked-out solutions to all of the odd-numbered text exercises, plus a chapter test for each chapter. Solutions to the chapter test questions are given at the back of the book.



*Logical dependence of the chapters. The appendix is optional.

The **Short Calculus Workbook,** prepared by Walter Turner of Western Michigan University, is designed to help students master the concepts involved in this course. Worked-out examples are presented so that students may follow the solutions step by step. These examples are followed by problems progressing in level of difficulty; only part of the solution is presented, and the student is asked to complete the rest. The **Workbook** supplements the problems presented in the assigned textbook and provides many additional worked examples.

For those courses requiring a brief introduction to linear algebra, a **Primer for Linear Algebra,** by Stephen Demko, Georgia Institute of Technology, is a valuable supplement to this or any calculus text.

Visuals

A set of **overhead transparencies** showing charts, figures, and portions of examples is available and can be used to accompany lectures. The transparencies are especially useful in large classroom situations.

Computerized Testing

The Scott, Foresman/Little, Brown Test Generator for Mathematics enables instructors to select questions by section or chapter or to use a ready-made test for each chapter. Instructors may generate tests in multiple-choice (IBM^R/Macintosh^R versions) or open-response formats, scramble the order of questions, and produce multiple versions of each test (up to 9 with Apple II^R and up to 25 with IBM^R and Macintosh^R). The system features a preview option that allows instructors to view questions before printing, to regenerate variables, and to replace or skip questions.

Software

Computer Applications for Finite Mathematics and Calculus by Donald R. Coscia is a softbound textbook packaged with two diskettes (in Apple II and IBM-PC versions) with programs and exercises keyed to the text. The programs allow students to solve meaningful problems without the difficulties of extensive computation. This book bridges the gap between the text and the computer by providing additional explanations and exercises for solution using a microcomputer. The chapters on "Limits and Continuity," "Differential Calculus," and "Integral Calculus," are appropriate for use with the Spence/Vanden Eynden book.

Graphing Functions: An Introduction to the Electronic Spreadsheet by Samuel W. Spero is a workbook for students that will be accompanied by the 1-2-3 compatible electronic spreadsheet, VP-Planner Plus for the IBM-PC and compatibles.

Because one cannot learn calculus without graphing functions, it is important that a graphing tool be made available to students. The **Graphing Functions** workbook not only instructs the student in the use of a very powerful, computer-based graphing tool, it also introduces him or her to the electronic spreadsheet. The electronic spreadsheet is the preferred computational tool in business and industry.

The workbook is closely linked to the material in the Spence/Vanden Eynden text and can be profitably used from Chapter 1 on.

RELATED BOOKS IN THE SERIES

- *Finite Mathematics* by Spence and Vanden Eynden
- *Applied Mathematics for the Management, Life, and Social Sciences* by Spence and Vanden Eynden. This book is a combined version of *Finite Mathematics* and *Calculus With Applications,* and is intended for use in a one-year sequence of finite mathematics and calculus.

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Harald M. Ness *University of Wisconsin, Fond du Lac*
John Gresser *Bowling Green State University*

We hope to have created a text that instructors can teach from and students can learn from with both success and enjoyment. We welcome your feedback and comments.

Lawrence E. Spence
Charles Vanden Eynden

Index of Applications

Behavioral and Social Science

General Interest

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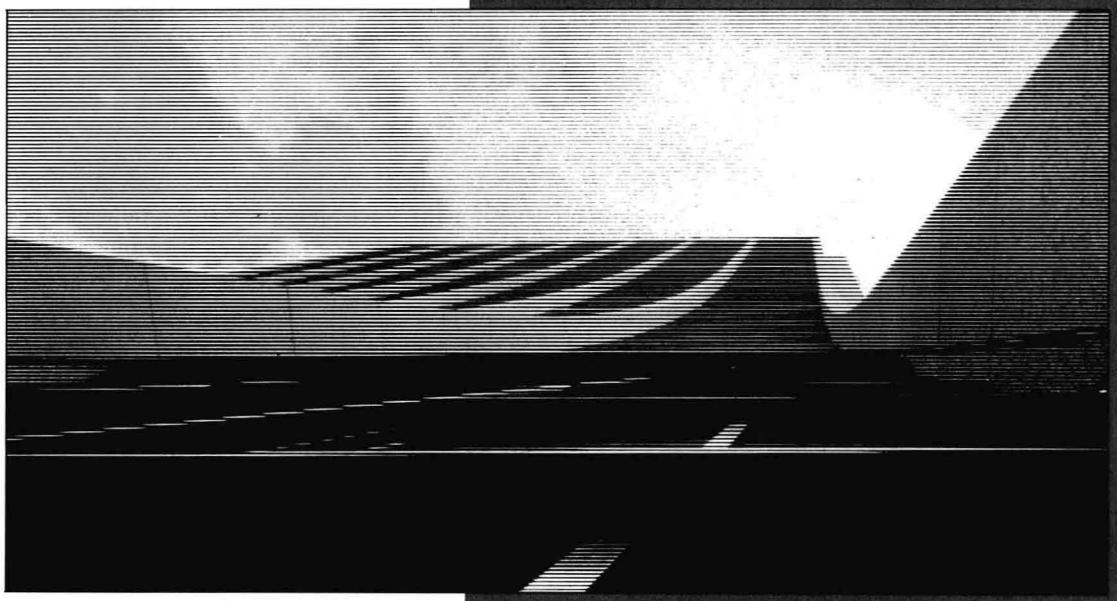

FUNCTIONS

In this chapter we will introduce the principle types of functions that will be encountered in our study of calculus: polynomial, rational, exponential, and logarithmic functions. We will also discuss ways in which functions can be combined to produce new functions.

The simplest possible relationship between two quantities is the linear one; yet this is often a good approximation to reality. We will begin our study of functions by investigating linear relationships.

1.1 Graphing in the Euclidean Plane

When Sarah Marchant's husband died early in 1985 she used some of the insurance money to buy IBM stock, hoping that it would increase in value. After several years she began to wonder how her investment was doing. She was able to find most of the reinvestment reports she had received. These listed the selling price of the stock at various times as follows.

Date	*Price*
3–31–87	150 1/8
9–30–85	123 7/8
3–29–85	127
6–28–85	123 3/4
9–30–86	134 1/2
12–31–85	155 1/2
6–30–86	146 1/2

These seemed to her to be a jumble of numbers and dates until she arranged the reports in chronological order, then plotted the stock prices against time as in Figure 1.1. From this picture she could see that the general trend of the price was upward for the past two years, although there was a downturn in most of 1986. Even so, the stock was worth approximately 20 percent more than when she bought it. When she called her broker for advice, she already had an overview of the stock's performance since her purchase.

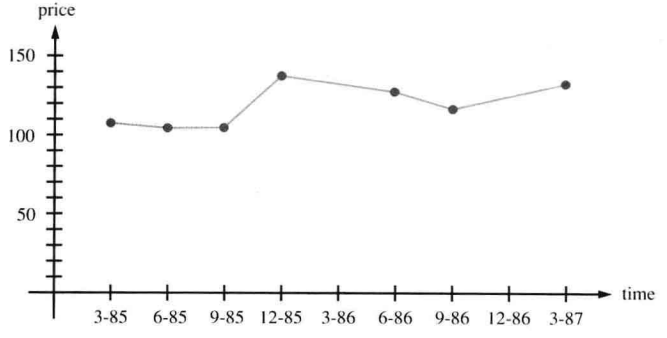

FIGURE 1.1

THE REAL NUMBER LINE

In this example time is measured horizontally and stock price vertically along **real number lines**. To introduce a coordinate system on a given straight line: (i) designate a particular point O on the line as the **origin** of the system; (ii) select one of the two directions on the line to be the **positive** direction; and (iii) choose a **unit length** on the line for measuring distances. With such a coordinate system established, each real number x can be represented as a unique point on the line. To plot the value x, we start at the origin and move x units along the line, taking into account the sign (positive or negative) of x. As an illustration, several values are plotted as points on the number line of Figure 1.2. The positive direction is chosen (by convention) to the right, as indicated by the arrow. In this way every real number corresponds to exactly one point, and conversely every point corresponds to exactly one number, called its **coordinate**.

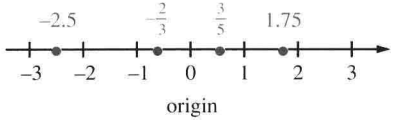

FIGURE 1.2

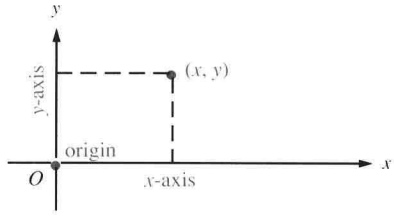

FIGURE 1.3

RECTANGULAR COORDINATES IN THE PLANE

To establish a coordinate system in the plane we choose two perpendicular lines, or **coordinate axes.** By convention these axes are horizontal and vertical, and are called the ***x*-axis** and ***y*-axis** respectively. We introduce a coordinate system on each axis in such a way that the lines share a common **origin** O at their point of intersection. (See Figure 1.3.) The positive direction is usually taken to the right on the x-axis and upward on the y-axis. The unit length may or may not be the same on the two axes.

The location of a point P is now specified by an **ordered pair** (x, y) of real numbers. To plot the ordered pair (x, y), we start at the origin and move x units horizontally and then y units vertically, taking into account the signs of x and y. Figure 1.4 shows an xy-coordinate system in the plane, with several points plotted for illustration.

For example, the ordered pair (3, 2) corresponds to point A in the figure, obtained by starting at the origin and moving 3 units to the right, then 2 units upward. In the same way, every ordered pair (x, y) determines a unique point