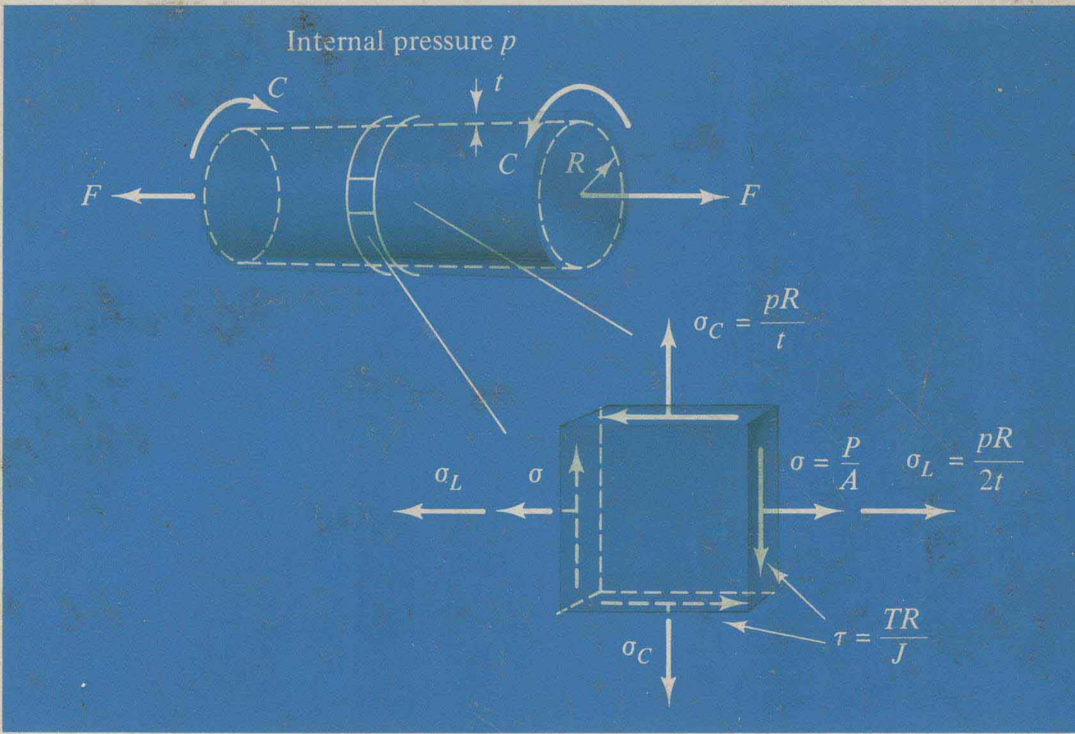
STATICS AND STRENGTH OF MATERIALS



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PREFACE

This book is designed for use in a first undergraduate course covering the subject matter traditionally referred to as statics and strength of materials. A combined statics and strength of materials course has been offered at The Ohio State University for a number of years. This text is an outgrowth of my experiences, and to some degree those of my former colleagues, in teaching this course.

The subjects of statics and strength of materials have much in common in that they are both concerned primarily with bodies in equilibrium. Consequently, equilibrium has been made the central theme of this book. This has made it possible to present a highly unified treatment of the statics of rigid and deformable bodies.

Chapters 1 through 4 are devoted to subject matter covered in traditional statics courses. This material provides a complete treatment of vector statics, and serves as a background for later studies in dynamics. Although the material is presented from a vector viewpoint, I have not hesitated to drop formal vector methods wherever their use is not warranted. Consequently, scalar-geometric methods are used for most two-dimensional and for some simple three-dimensional problems.

A thorough treatment of vectors and vector algebra is presented in Chapter 2, with emphasis upon applications to statics. The concepts of moments and couples are also introduced in this chapter. Equilibrium is treated in Chapter 3, followed by a study of statically equivalent force systems in Chapter 4. Chapter 4 also includes a discussion of distributed forces and the related concepts of center of gravity, centroids, and area moments of inertia.

The consideration of equilibrium before the study of statically equivalent force systems and resultants is a major departure from tradition, but offers several advantages. Most importantly, it exposes the student to meaningful problems early in the course without first building up a backlog of abstract work regarding the properties of force systems. Early work with forces and moments as they relate to meaningful physical problems gives the student a better feeling for these quantities and their physical effects. Once the principles of equilibrium have been presented, the study of statically equivalent force systems follows as a natural consequence of the need to know how to handle distributed forces. I believe that this places the importance of static equivalence in better perspective than when it is considered before equilibrium. Furthermore, statically equivalent force systems can then be defined in terms of their effect upon equilibrium. This very clearly brings out the limited nature of static equivalence, which is important for the work in later chapters involving deformable bodies.

The work on deformable bodies is contained in Chapters 5 through 11. With equilibrium being the common thread throughout, the difference between problems involving rigid and deformable bodies lies fundamentally in the degree of sophistication required in the mathematical modeling of the problem. Accordingly, the determination of stresses and deformations is approached from the viewpoint that these are indeterminate problems for which the material properties and geometry of the deformations must be taken into account in order to obtain a solution.

The concepts of stress and strain are introduced in Chapter 5, along with a discussion of material properties. Chapters 6 through 10 deal with axially-loaded members, torsion, bending, combined loadings, and buckling, respectively. Both elastic and inelastic responses are considered. A brief, but complete, discussion of the common methods of experimental strain and stress analysis is given in Chapter 11.

Every attempt has been made to present the material in a consistent and orderly fashion. The fundamentals are presented as succinctly as possible, consistent with good understanding. On the other hand, considerable discussion has been devoted to the meaning and interpretation of the fundamentals and the procedures for applying them to engineering problems. As a further aid to the student, there are numerous illustrative problems with complete explanations designed to reinforce the concepts presented. There are approximately 800 exercise problems, with answers to the even-numbered problems provided. While my concerns lay primarily with making the material easily understood by beginning students, individual instructors will find that they have considerable latitude to expand upon subjects and to add their own flavoring to the presentation.

At the time of writing, the transition between the traditional US-British system of units and the Système International d'Unités (SI) is in progress. Accordingly, both systems are used to an approximately equal extent. This has made necessary several compromises to avoid confusion. For example, the practice recommended in SI of using a space instead of a comma to set apart groups of three digits on either side of the decimal point is also used for quantities expressed in US-British units. In problems using SI, bodies are sometimes described in terms of their weight or specific weight, although, strictly speaking, they should be described in terms of their mass or mass density, since mass, and not force, is the fundamental quantity in SI. A complete discussion of units of measure is included in Chapter 1.

This book contains sufficient material for a five-semester-hour course. For courses of fewer credits, it will be necessary to delete certain material. The content of combined courses in statics and strength of materials apparently varies widely from school to school. Consequently, the material has been organized in such a way that topics can be easily deleted to fit course content. In particular, discussions of the inelastic response of members is placed in separate sections, and the exercise problems are placed at the end of the appropriate section instead of at the end of the chapters. Aside from the emphasis upon equilibrium, the work on rigid and deformable bodies has not been intermixed. Thus, this book should serve equally well as a text for separate courses in statics and strength of materials.

A project of this magnitude cannot be completed alone. I am deeply indebted to Ms. Josephine French for typing the several drafts. My dear friend and former graduate student, Dr. Leonard Sung, checked the solutions to all the example problems and assisted with the solutions of the exercise problems. Assistance with the exercise problems was also provided by Mr. Shah Malik, Mr. Jim Lester, and Dr. Kuang-shi Ju. Professor Terry Richard offered numerous helpful comments and suggestions on the chapter on Experimental Strain and Stress Analysis. The photographs, except where acknowledged otherwise, were taken by Mr. Scott Vibberts. Special thanks are due my anonymous reviewers, whose comments and suggestions were most helpful. The patience and guidance of all those I worked with at Prentice-Hall are greatly appreciated.

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CONTENTS

chapter one INTRODUCTION

1.1 SCOPE

Let us begin with a brief overview of where we are going and what we hope to accomplish in the following chapters.

In this text we shall be concerned with the determination of the forces acting upon material bodies and their mechanical response to these forces. By mechanical response, we shall generally mean the deformation of a body and the stresses produced within it. Our interest in such problems stems from the fact that they pervade science and technology and are of direct concern to engineers, designers, and scientists.

For instance, structural engineers must be able to determine the loads acting upon a building and to design the building so that it responds to these loads in an acceptable manner. The same is true for other structures such as nuclear reactors, bridges, highways, machine parts, airplanes, pipelines, storage tanks, and electrical transmission lines. Designers of electrical machinery must account for forces produced by the magnetic and electrical fields present, and orthopedic surgeons require knowledge of the forces to which the human body is subjected in order to better repair bone fractures and to design artificial limbs and replacement parts for bones and joints. Mechanical engineers must determine the forces acting upon and within machines in order to properly design gears, bearings, and other machine elements. Agricultural engineers are faced with the problem of determining the effects of the forces to which fruits, vegetables, and grains are subjected during harvesting, handling, and storage.

These are but a few examples of problems involving the determination of forces acting upon physical systems and their response to these forces. How many other examples can you think of? Obviously, problems like these are innumerable.

Many of the problems we have mentioned are complex and cannot be treated in all generality in an introductory text such as this. Consequently, it will be necessary to restrict the types of problems to be considered.

First, we shall consider only static problems involving bodies at rest, or more precisely, bodies in equilibrium. A large number of the problems encountered in practice fall into this category. The behavior of bodies in motion is treated in texts on dynamics and vibrations.

Second, we shall consider only solid bodies. Gasses and liquids will not be considered per se, although we shall consider the loadings they impart to a solid body, such as a container or dam.

Third, although actual structures and machines are usually complex, they often consist of simple component parts such as rods, shafts, beams, and columns. We shall consider only the response of these basic structural elements and simple combinations thereof. This will enable us to solve many problems that are important in their own right and will provide the fundamentals necessary for consideration of more complex problems.

We have now identified in general terms the class of problems to be considered in this text. But what information will be needed to solve these problems? This question can probably best be answered by considering the steps involved in the solution of an engineering problem.

The first step in any engineering analysis is to clearly identify and focus attention upon the particular system or subsystem of interest. For example, suppose that we wish to find the forces that a driver exerts upon the seat of an automobile. Should we consider the driver? The seat? The automobile? The combination of driver, seat, and automobile? Or just what? A clear definition and statement of the problem usually help resolve this question.

After identifying the system of interest, the next step is to formulate an idealized model of the problem. The idea is to simplify the problem as much as possible by considering only those factors that have an important bearing upon the quantities to be determined, while neglecting those which do not. For instance, the weight of a floor beam might be only a very small fraction of the total load it supports and could, therefore, be neglected when determining the deflection (sag) of the beam.

Once a model has been decided upon, it is analyzed mathematically by using whatever physical laws are applicable. The results obtained are then examined to see if they make good sense and correspond to physical reality. A further check of the validity of the model may be obtained by performing critical experiments, or by comparing the results with those obtained from analyses based on more sophisticated models. If the model is found to be deficient, it is modified and the process repeated until satisfactory results are obtained.

What does the preceding discussion tell us about the things we must know in order to solve statics problems? Obviously, we must know how to construct appropriate models of problems, we must know the physical laws which apply, and we must know how to describe the quantities of interest in a form amenable to mathematical analysis.

The remaining sections of this chapter are devoted to a brief discussion of the basic concepts and physical laws that apply to statics problems. This will be followed in Chapter 2 by a study of how to describe certain quantities of interest, forces in particular, in a convenient mathematical form.

Once this background material has been mastered, we shall be in a position to solve some engineering problems. There will be two phases to our work. The first involves the determination of all the forces acting upon the system of interest; the second involves the determination of the response to these forces. Chapters 3 and 4 are devoted to the first phase. Here we shall develop and learn to apply the mathematical conditions for equilibrium and, at the same time, begin to develop some insight into problem modeling. Equilibrium is the single most important topic to be considered in this text, and the results presented in these chapters will be used throughout the remainder of our work.

Chapters 5–11 are devoted to the determination of the response of load-carrying members to applied loadings. In particular, we shall consider procedures for analyzing a member to see if it can fulfill its intended purpose and for designing a member so that it can safely support a given loading. The mechanical properties of common engineering materials will also be discussed, insofar as they relate to the problems of interest.

1.2 BASIC CONCEPTS AND PRINCIPLES

The study of the forces acting upon physical systems and their response to these forces forms the branch of science known as *mechanics*. Mechanics is one of the oldest branches of science, and some of its principles, such as the principle of the lever, probably came into play during man's earliest attempts to cope with his environment.

Although simple principles of mechanics were undoubtedly used in prehistoric times, it is unlikely that they were understood. The study of mechanics dates to the time of Aristotle (384–322 BC) and Archimedes (287–212 BC). Despite this early start, a satisfactory formulation of the principles of mechanics was not available until Sir Isaac Newton postulated the three laws of motion which are now the basis of most engineering applications of mechanics. These laws were presented in his *Philosophiae Naturalis Principia Mathematica* (The Mathematical Principles of Natural Science) published in 1686. The modern theories of relativity and quantum mechanics show Newton's laws to be inexact. However, the innumerable problems that have been solved successfully using these laws testify to the fact that their precision is extremely high, except in problems involving subatomic particles or velocities approaching the speed of light.

It will not be one of our objectives to trace in detail the history of the developments in mechanics. Those who are interested will find a number of good books available on this important and fascinating subject.

There are three main divisions of mechanics: *statics*, *kinematics*, and *dynamics*. Statics is concerned with systems in equilibrium and with the force interactions that operate to establish equilibrium. Kinematics is the study of the geometry of motion. Dynamics is concerned with the relationships between force interactions and the motions they produce. As we have already mentioned, we shall be concerned only with the statics of solid bodies.

Basic concepts. Mechanics is based upon the concepts of force, mass, length, and time. Although each of us has a degree of familiarity with these concepts as a result of our physical intuition and experiences, these concepts cannot truly be defined. Rather, we give a rough indication of their physical significance and leave their actual description to be determined from the laws and postulates that describe the relationships between them. Thus, we may speak of the mass of an object as being a measure of the quantity of matter in it, but this statement can hardly be construed as being a precise definition of mass.

Our intuitive concepts of mass, length, and time will be adequate for problems to be considered in this text, particularly since time does not enter into statics problems and mass enters only indirectly. Force, however, plays a dominant role in statics and merits further comment.

The concept of force. The concept of force in mechanics provides a simple and convenient way to describe the very complex physical interactions between systems that alter or tend to alter the motion or state of rest of the systems. The key word here is interaction, which implies that two systems always participate in the creation of a force.

For example, when we stretch a rubber band, there is a force interaction between the band and our hands. This interaction causes the band to stretch, and it also produces the effect that we experience as a resistance to this stretching. The situation is illustrated schematically in Figure 1.1, where the arrows $\mathbf{F}_1$ represent the effect of the force interaction on our hands and the arrows $\mathbf{F}_2$ represent its effect on the band. According to Newton's third law, force interactions have an equal and opposite effect on the interacting systems. Thus, the effects $\mathbf{F}_1$ and $\mathbf{F}_2$ shown in Figure 1.1 are of equal magnitude and opposite direction. In other words, whatever tendency there is for our hands to stretch the band, the band has the same tendency to resist the stretching and hold our hands together.

Although the word force is properly used to denote an interaction between systems, it is more commonly used to denote the individual effects of the interaction. Thus, the individual effects $\mathbf{F}_1$ and $\mathbf{F}_2$ shown in Figure 1.1 would each be called a force. We shall also use the word force in this context, but in so doing it is

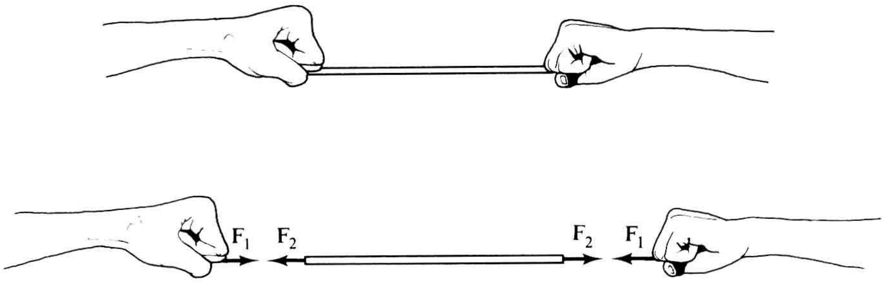
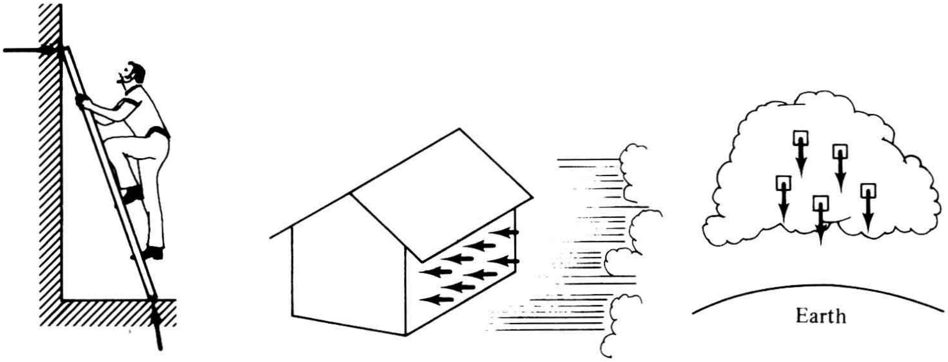


Figure 1.1



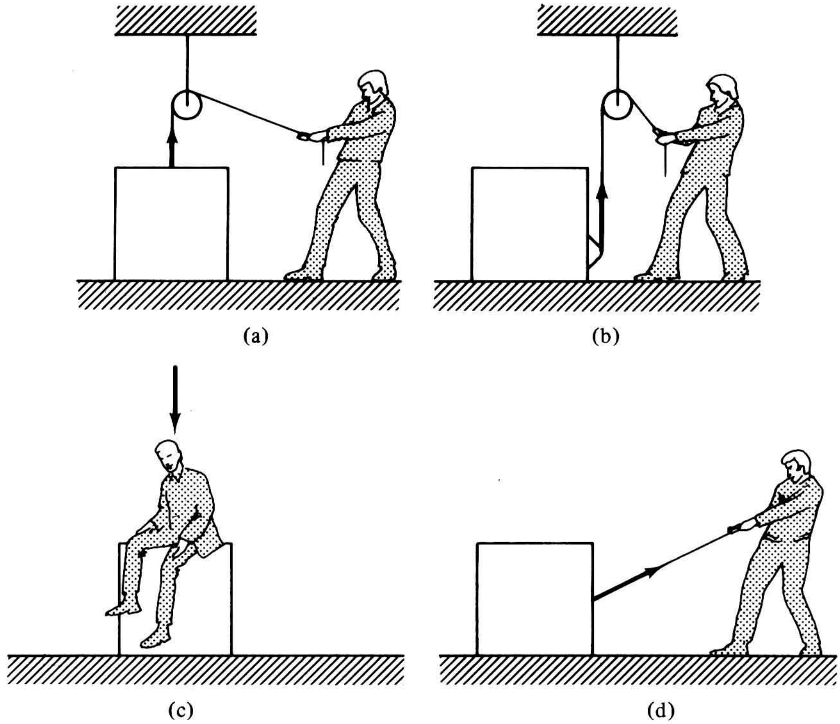
(a) (b) (c)

Figure 1.2

essential to remember that we are talking about only one aspect of an interaction between two systems.

Force interactions arise when bodies come into direct physical contact, but they can also occur between systems that are separated from one another. Gravitational, electrical, and magnetic force interactions are of this latter type. Forces may act essentially at a point, as in the case of a ladder resting against a wall [Figure 1.2(a)], or they may be distributed over an area, as in the case of the wind loading on the side of a building [Figure 1.2(b)]. Some forces, such as the gravitational attraction between an object and the earth [Figure 1.2(c)], are distributed throughout a volume.

The physical effect of a force depends upon its *magnitude*, *orientation*, *sense*, and *point of application*, and these four factors must be specified in order to completely describe the force. For example, the behavior of the block shown in Figure 1.3(a) clearly depends upon the magnitude of the force exerted upon it by the man via the rope and pulley. If the force is too small, the block will remain at rest, but if the man pulls hard enough, the block can be lifted. The block can also be lifted if the rope is attached to a different point, as in Figure 1.3(b), but the block will rotate as it lifts. If the sense of the force is reversed [Figure 1.3(c)], the block will only be pressed more firmly against the surface upon which it rests.



(a) (b) (c) (d)

Figure 1.3

Finally, if the orientation of the force is changed, as in Figure 1.3(d), the block can be slid, but not lifted. In each of these cases, the force produces a different physical effect.

Forces have one other very important property: They combine according to the rules for vector addition. This is one of the fundamental postulates of mechanics, and it is based upon experimental evidence.

Quantities that have magnitude, orientation, and sense, and which combine according to the rules of vector addition, are known in mathematics as *vectors*. Thus, force interactions are vector quantities and can be represented by directed line segments (arrows), as we have been doing. The significance of this result is twofold. First, the vector character of forces will be important to us in developing a physical feeling for them and the actions they produce. Second, all the rules of vector algebra can be used in mathematical operations involving forces. This will be helpful in the derivation of basic equations and in the solution of problems. Vectors and vector algebra will be discussed in Chapter 2.

Weight and mass. The force exerted on a body because of its gravitational interaction with the earth is called the *weight* of the body. As will be shown in Chapter 4, the magnitude W of the weight of a body with mass m is given by the relation

$$W = mg \tag{1.1}$$

where g is the so-called *acceleration of gravity*. The value of g at or near the surface of the earth is 32.2 feet/second^2 (9.81 metres/second^2).

The distinction between weight and mass is widely misunderstood, mostly because of the common usage versus the standard technical usage of these terms. For example, grocery store scales are calibrated in units of force, usually pounds and ounces, and we speak of 5 pounds of potatoes as being a certain quantity of the vegetable. What we really mean, however, is the mass of potatoes which experiences a force of magnitude 5 pounds due to the gravitational attraction of the earth.

The mass of an object can be determined by balancing it against the standard unit of mass, the kilogram, on a balance scale (Figure 1.4). (The kilogram will be discussed in Section 1.3.) From Eq. (1.1), we find that the weight of the standard kilogram is $W = (1\text{ kg})\,g$ and the weight of the mass m is $W = mg$. When the scale is balanced, the tendency of each of these forces to rotate the scale arm about the fulcrum will be equal, and we have $(1\text{ kg})\,ga = mgb$, or $m = a/b$ kilograms. Since g does not appear in this result, the scale really measures mass instead of weight. This is why balance scales are used in commerce.

Even though a balance scale measures mass, it can be calibrated to read in force units. The grocery scale is an example. Herein lies the source of much of the confusion between mass and weight. From the balance condition for the scale, we find that the weight W of the object is equal to $g(a/b)$ units of force. Since g varies from place to place, the calibration of the scale in force units would be valid only at a particular location. Any change in the weight of the object resulting from a change in g would not be detected by the scale because it actually measures mass.

Force can be measured with a spring scale (Figure 1.5). Experiments show that the elongation of a properly constructed spring is proportional to the force applied to it. The spring scale is based on this principle. If a standard kilogram is suspended from the scale, the elongation of the spring will correspond to g units of force.

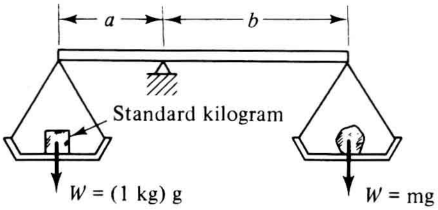


Figure 1.4

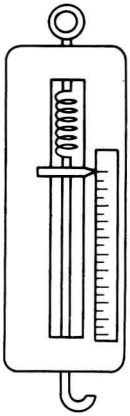


Figure 1.5