

# Mathematics

For Elementary Teachers

SIXTH EDITION

A CONTEMPORARY APPROACH



Gary L. Musser • William F. Burger • Blake E. Peterson

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Sixth Edition

# MATHEMATICS FOR ELEMENTARY TEACHERS

*A Contemporary Approach*

Gary L. Musser  
*Oregon State University*

William F. Burger

Blake E. Peterson  
*Brigham Young University*



John Wiley & Sons, Inc.

To:

*Marge, my mother, for her continuing encouragement; Irene, my wife, for her constant support; Greg, my son, for being a great father; Maranda, my granddaughter, for her enthusiasm for learning; and Mary, Bill Burger's daughter, for the joyful times she shared with Bill.*

G.L.M.

*Shauna, my beautiful wife, for her unwavering love and encouragement; Quinn, Joelle, Taren, and Riley, my four children, for bringing me great joy and happiness as well as being my built-in laboratory; Dad, for the legacy of service and teaching he left behind; and Mom, for her continued, never-ending support.*

B.E.P.

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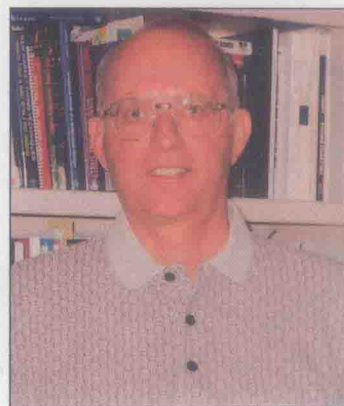
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## About the Authors

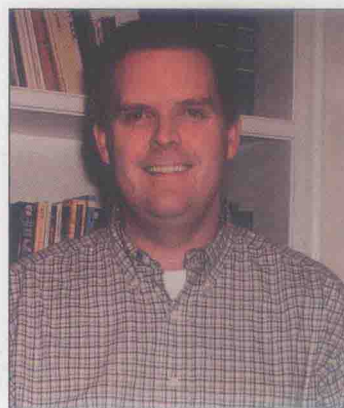
**Gary L. Musser** is currently Professor Emeritus from Oregon State University. He earned both his B.S. in Mathematics Education in 1961 and his M.S. in Mathematics in 1963 at the University of Michigan and his Ph.D. in Mathematics (Radical Theory) in 1970 at the University of Miami in Florida. He taught at the junior and senior high, junior college, college, and university levels for more than 30 years. He served his last 24 years teaching prospective teachers in the Department of Mathematics at Oregon State University. While at OSU, Dr. Musser developed the mathematics component of the elementary teacher program. Soon after Professor William F. Burger joined the OSU Department of Mathematics in a similar capacity, the two of them began to write the first edition of this book. Professor Burger passed away during the preparation of the second edition, and later Professor Blake E. Peterson was hired at OSU. Professor Peterson joined Professor Musser as a coauthor of the fifth edition.



Professor Musser has published 40 papers in many journals, including the *Pacific Journal of Mathematics*, *Canadian Journal of Mathematics*, *The Mathematics Association of America Monthly*, the NCTM's *The Mathematics Teacher*, the NCTM's *The Arithmetic Teacher*, *School Science and Mathematics*, *The Oregon Mathematics Teacher*, and *The Computing Teacher*. In addition, he is a coauthor of two other college mathematics books: *College Geometry—A Problem-Solving Approach with Applications* and *Mathematics in Life, Society, and the World*. He also coauthored the K–8 series *Mathematics in Action*. He has given more than 65 invited lectures/workshops at a variety of conferences, including NCTM and MAA conferences, and was awarded 15 federal, state, and local grants to improve the teaching of mathematics.

While Professor Musser was at OSU, he was awarded the university's prestigious College of Science Carter Award for Teaching. He is currently living in sunny Las Vegas, where he continues to write, do research, ponder the mysteries of the stock market, and enjoy his granddaughter, the sunshine of his life.

**Blake E. Peterson** is currently an Associate Professor in the Department of Mathematics Education at Brigham Young University. He was born and raised in Logan, Utah, where he graduated from Logan High School and Utah State University in secondary mathematics education. After graduation, he took his new wife, Shauna, to southern California, where he taught at Chino High School for two years. In addition to teaching general math and geometry, he coached basketball and football. In 1988, he began graduate school at Washington State University, where he later completed a M.S. and Ph.D. in pure mathematics.



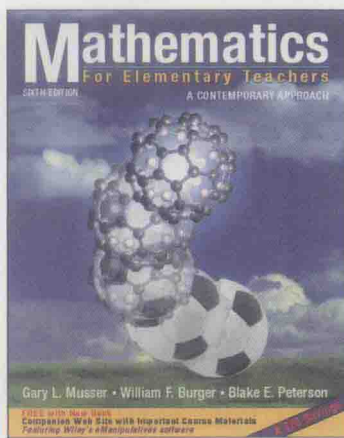
After completing his Ph.D., Dr. Peterson was hired as a mathematics educator in the Department of Mathematics at Oregon State University in Corvallis, Oregon, where he taught for three years. It was at OSU that he met Gary Musser. He has since moved his wife and four children to Provo, Utah, to assume his position at Brigham Young University. As a professor, his first love is teaching, for which he has received a College Teaching Award in the College of Science. He has also designed the "Mathematics Teaching with Technology" and "Mathematics Teaching and the Classroom" courses at Brigham Young University.

Dr. Peterson has published papers in *Rocky Mountain Mathematics Journal*, *The American Mathematical Monthly*, *The Mathematical Gazette*, *Mathematics Teacher*, *Mathematics Magazine*, and *Mathematics Teaching in the Middle School*. His current research interests are the mathematical dialogue that occurs during teacher collaborations. In addition to teaching and writing, Dr. Peterson has done consulting for the College Board, is the president of the Utah Association of Mathematics Teacher Educators, is an associate editor of the journal *School, Science and Mathematics Education*, and is a board member of the Utah Council of Teachers of Mathematics.

Aside from his academic interests, Dr. Peterson enjoys spending time with his family, playing basketball, and working in the yard.







## A Glowing Soccer Ball

Can a soccer ball glow? If it is a Carbon-60 molecule in the right setting, it can. In 1985, Richard Smalley, Harold Kroto, and Robert Curl discovered the third form of pure carbon called fullerenes. The other two forms of pure carbon are diamond and graphite. A fullerene, also called a Bucky Ball, consists of 60 carbon atoms arranged in the shape of a soccer ball. A unique feature of a fullerene, as compared to the other forms of pure carbon, is that it is more complex and the 60 atoms form a hollow sphere consisting of hexagons and pentagons on the surface. The name of fullerene, or Bucky Ball, comes from the name of the inventor of the geodesic dome, Buckminster Fuller.

So how does a Bucky Ball glow? In 1999, a group of researchers from University of California at Los Angeles and University of California at Santa Barbara added attachments to Bucky Balls that modified their electronic structure and caused them to emit white light. The surprising feature about this discovery is that up to this point, Bucky Balls were known for the ability to absorb light. There are other organic materials that can be easily engineered to emit green, orange, or yellow light, but white light has been quite difficult to generate. The fact that a molecule known for absorbing light has been engineered to emit white light makes this result even more astonishing.

Are there any practical applications for such a discovery? Whether it is dashboard displays in cars or cell phones, companies are always searching for ways of efficiently making their products useful in the light or dark. Several companies are researching ways of using organic materials for their products. It is not yet known if the glowing Bucky Ball will be an efficient way of meeting these needs.



## SPOTLIGHTS ON TECHNOLOGY

**Chapter 1****eManipulatives***Counterfeit Coin**Tower of Hanoi***Spreadsheets***Consecutive Integer Sum**Sum of the Odds***Chapter 2****eManipulatives***Venn Diagrams**Multibase Blocks***Spreadsheets***Base Converter**Function Machines and Tables***Chapter 3****eManipulatives***Number Line***Chapter 4****eManipulatives***Base Blocks: Addition**Base Blocks: Subtraction***Spreadsheets***Scaffold Division***Calculator***Finding quotients and remainders***Chapter 5****eManipulatives***Sieve of Eratosthenes***Spreadsheets***Euclidean***Calculator***GCF**GCF using Euclidean algorithm***Chapter 6****eManipulatives***Equivalent Fractions**Comparing Fractions**Adding Fractions**Dividing Fractions***Calculator***Fraction equality**Converting improper fractions to mixed numbers**Cross-multiplication of fraction inequality**Adding fractions on a fraction calculator**Simplifying on a fraction calculator***Chapter 7****eManipulatives***Percent Gauge***Calculator***Converting fractions to percents**Solving percent problems**Finding discounts**Compound interest***Chapter 8****eManipulatives***Chips Plus**Chips Minus***Calculator***Integer computation on a scientific calculator**Negative Sign Key**Conversions from standard to scientific notation***Chapter 9****eManipulatives***Balance Beam Algebra**Function Grapher***Spreadsheets***Cubic***Calculator***Using a fraction calculator to find sums and differences of rational numbers**Using a fraction calculator to find products of rational numbers**Comparing fractions with negative numerators or denominators**Using calculators to find square roots**Using calculators to find roots of real numbers**Using exponent key to calculate real exponents***Chapter 10****eManipulatives***Histogram**Scatterplots***Spreadsheets***Circle Graph Budget**Standard Deviation***Calculator***Finding the mean of a data set**Finding the standard deviation***Chapter 11****eManipulatives***Simulation***Spreadsheets***Coin toss**Roll the dice***Calculator***Using factorial key to count permutations of  $n$* *Calculating  ${}_nP_r$* *Calculating  ${}_nC_r$* **Chapter 12****eManipulatives***Tessellations**Slicing Solids***Geometer's Sketchpad***Name That Quadrilateral**Triangle Angle Sum***Chapter 13****eManipulatives***Geoboard**Pythagorean Theorem***Geometer's Sketchpad***Rectangle Area**Same Base, Same Height, Same Area**Parallelogram Areas**Dynamic Pythagorean Theorem**Triangle Inequality***Chapter 14****eManipulatives***Congruence***Geometer's Sketchpad***Tree Height**Circumcircle**Midquad***Chapter 15****eManipulatives***Coordinate Geoboard***Geometer's Sketchpad***Slope**Perpendicular Lines***Chapter 16****eManipulatives***Translation Transformations***Geometer's Sketchpad***Size Transformation*

**W**elcome to a world of mathematical understanding that we hope you will find stimulating, rewarding, enlightening, and fun. We salute you for choosing teaching as a profession and hope that your experiences with this book will help prepare you to be the best possible teacher of mathematics that you can be. We have presented this elementary mathematics material from a variety of perspectives so that you will be more able to address the broad range of learning styles that you will encounter in your future students. This book also encourages prospective teachers to gain an understanding of the underlying concepts of elementary mathematics while maintaining an appropriate level of mathematical precision.

We have also sought to present this material in a manner consistent with the recommendations in (1) *A Call for Change: Recommendations on the Mathematical Preparation of Teachers*; prepared by the Mathematical Association of America's Committee on the Mathematical Education of Teachers; (2) *The Mathematical Education of Teachers*; prepared by the Conference Board of the Mathematical Sciences; and (3) the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics*, *Professional Standards for Teaching Mathematics*, and *Principles and Standards for School Mathematics*. In addition, we have received valuable advice from many of our colleagues around the United States through questionnaires, reviews, focus groups, and personal communications. We have taken great care to respect this advice and to ensure that the content of the book has mathematical integrity and is accessible and helpful to the variety of students who will use it. As always, we look forward to hearing from you about your experiences with this text.

GARY L. MUSSER  
BLAKE E. PETERSON

## Content Features

**Number Systems** Insofar as possible, number topics are covered sequentially to parallel their development in the school curriculum. Fractions and integers are each treated as extensions of whole numbers. Rational numbers are developed briskly as extensions of both the fractions (by adjoining their opposites) and the integers (by adjoining their reciprocals). The mathematical structure of an ordered field continues to serve to unify this presentation. The important applications of statistics and probability serve as a capstone to the study of number systems.

**Approach to Geometry** Geometry is organized from the point of view of the five-level van Hiele model of a child's development in geometry. After studying shapes and measurement, geometry is approached more formally through Euclidean congruence and similarity, coordinates, and transformations. The Epilogue provides an eclectic approach by solving geometry problems using a variety of techniques.

## Underlying Themes

**Problem Solving** An extensive collection of problem-solving strategies is progressively developed; these strategies can be applied to a generous supply of problems in the exercise/problem sets. The depth of problem-solving coverage can be varied by the number of strategies selected throughout the book and by the problems assigned.



**Deductive Reasoning** The use of deduction is promoted throughout the book. The approach is gradual, with later chapters having more multistep problems. In particular, the last sections of Chapters 14, 15, and 16 and the Epilogue offer a rich source of interesting theorems and problems in geometry.

**Technology** The Math Explorer calculator, which has been upgraded to the TI-34 II, is used in many schools. These calculators are used to illustrate its capability to do long division with remainder, fraction calculations, and more. A graphing calculator is also illustrated at a few relevant junctures. The eManipulatives, which were on the CD in the fifth edition, have been expanded and integrated throughout the book. Many of these activities are electronic versions of the traditional manipulatives, while others expose students to some useful, modern software applications that are learning situations as well as problem-solving environments. In addition, dynamic Web site-centered activities are provided for students to solve problems using spreadsheets and dynamic geometry software. Webmodules are available on these latter two topics to encourage students to extend their knowledge.

## Additional Topics

Since reviewers were split concerning where the following topics should appear in the text, they are placed near the end of the book to allow for maximum flexibility.

Topic 1, “Elementary Logic,” may be used anywhere in a course.

Topic 2, “Clock Arithmetic: A Mathematical System,” uses the concepts of opposite and reciprocal and hence may be most instructive after Chapter 6, “Fractions” and Chapter 8, “Integers” have been completed. This section also contains an introduction to modular arithmetic.

Topic 3, “Introduction to Graph Theory,” develops the ideas of vertex-edge graphs or networks as a method to model and visualize certain problem situations. Because of the visual nature of the graphs, this topic may fit best at the end of Chapter 12. The topic, however, is independent enough to fit almost anywhere in the course.

## Course Options

The material in this book has been organized to allow for a wide variety of courses. At Oregon State University, all preservice elementary teachers have been required to take 12 quarter-hours of mathematics. This book is used for the first 9 quarter-hours, a one-year sequence. Each week, students attend 3 one-hour lectures and a 1-hour lab in which materials from the *Student Resource Handbook* are covered. This book, together with *A Guide to Problem-Solving*, is also used in a 3 quarter-hour junior-level course in problem solving for elementary teachers. This course is devoted entirely to developing the students’ facilities in applying Pólya’s four-step process and the strategies introduced at the beginning of each chapter.

Since many schools have special mathematical requirements, the following are suggested chapters and topics to fit any particular course.

Basic course: Chapters 1–7.

Basic course with logic: Topic 1, Chapters 1–7.

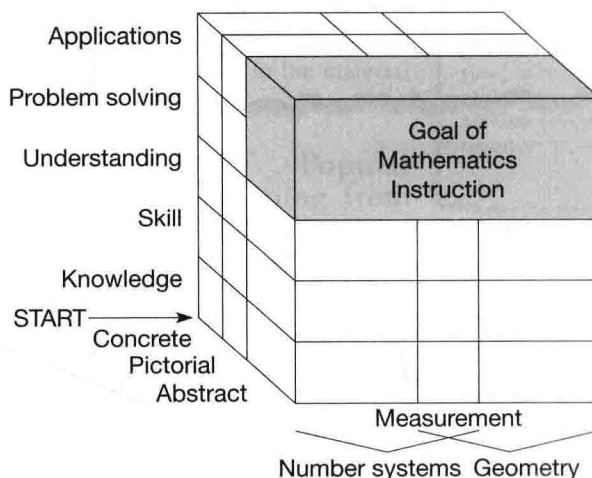
Basic course with informal geometry: Chapters 1–7, 12.

Basic course with introduction to geometry and measurement: Chapters 1–7, 12, 13.



## Pedagogical Features

The general organization of the book was motivated by the following mathematics learning cube:

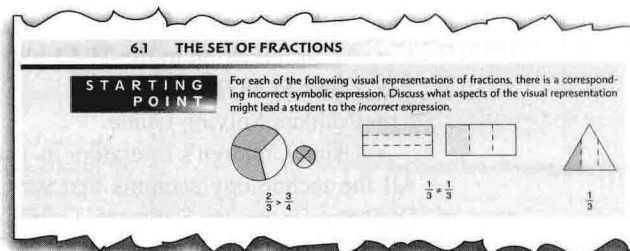


The three dimensions of the cube—cognitive levels, representational levels, and mathematical content—are integrated throughout the textual material as well as in the problem sets and chapter tests. Problem sets are organized into exercises (to support knowledge, skill, and understanding) and problems (to support problem solving and applications).


We have developed new pedagogical features to implement and reinforce the goals discussed above and to address the many challenges in the course.

## Summary of Changes in the Sixth Edition

- **Starting Points** have been added to the beginning of each section. These Starting Points can be used in a variety of ways. First, they can be used by an instructor at the beginning of class to have students engage in some novel thinking and/or discussion about forthcoming material. Second, they can be used in small groups where students discuss the query presented. Third, they can be used as an advanced organizer homework piece where a class begins with a discussion of what individual students have discovered.




- **Spotlights on Technology** appear throughout the book at points where use of technology enriches the presentation. The technology used includes activities from our expanded eManipulative activities, spreadsheet activities, Geometer's Sketchpad activities, and calculators including both TI-34 II and a graphing calculator. Many of these rich activities can be accessed through our Web site at John Wiley ([www.wiley.com/college/musser](http://www.wiley.com/college/musser)).




**Spotlight on Technology** The rolling of two dice and recording the sum of the number of dots on their faces can be simulated using a spreadsheet. Refer to the dynamic spreadsheet *Roll the Dice*, in the Spreadsheet webmodule, which contains a dice-rolling spreadsheet for you to work with. Use the spreadsheet to simulate rolling two dice 200 times. How close are the results of this experiment to the theoretical probabilities in Table 11.2?

www.wiley.com/college/musser



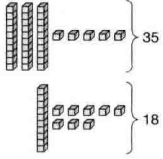
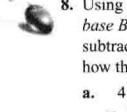
**Spotlight on Technology** To better understand the properties of the various quadrilaterals, it is helpful to be able to see these properties in action. The Geometer's Sketchpad® webmodule activity, *Name That Quadrilateral*, displays seven different quadrilaterals in the shape of a square. However, each quadrilateral is constructed with different properties. Some have right angles, some have congruent sides, and some have parallel sides. By dragging each of the points on each of the quadrilaterals, you can determine the most general name of each quadrilateral. See if you can name all seven of the quadrilaterals.

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**Spotlight on Technology** The Chapter 4 eManipulative activity, *Base Blocks: Subtraction*, utilizes the comparison approach to model subtraction. By placing blue blocks and red blocks next to each other, you can compare, match up, and remove blocks until only the difference remains. The problem  $35 - 18$  is modeled by beginning with the blocks shown at the right. Perform  $35 - 18$ ,  $321 - 43$ , and  $234 - 158$  on the eManipulative. What process of moving the blocks is analogous to “borrowing” in the standard subtraction algorithm?

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**8.** Using the Chapter 4 eManipulative activity, *Multi-base Blocks—Subtraction*, model the following subtraction problems using base ten blocks. Sketch how the base ten blocks would be used.

a.  $\begin{array}{r} 413 \\ - 57 \\ \hline \end{array}$       b.  $\begin{array}{r} 625 \\ - 138 \\ \hline \end{array}$

• **eManipulative Activities** are integrated throughout the book in Spotlights in Technology as well as the problems and exercises. The activities, problem solving and exploratory in nature, are designed to develop concepts central to each chapter by allowing students to interact with virtual manipulatives. These virtual manipulatives are similar to physical manipulatives frequently used to teach mathematics (for elementary through secondary) and for pre-service students. Examples include the geoboard, base ten blocks, black and red chips, and pattern blocks. Many of the problems in the problem sets have been designed to take advantage of the technology. Icons are used to identify the technology used to solve each problem.

• **A Companion Web Site** has been developed to provide a rich bank of resources for both instructors and students. The expanded Web site contains the following material:

- The eManipulative Activities
- An Introduction to Spreadsheets
- An Introduction to Geometer's Sketchpad
- An Introduction to Logo
- An Introduction to the TI-83 plus Graphing Calculator
- Links to NCTM Standards
- Problem Solving Guide
- Using Children's Literature in Teaching Mathematics

All the technology sections that were at the end of the fifth edition book (LOGO, Dynamic Geometry Software, Graphing Calculators) are now available on our John Wiley Web site.

**Four new cumulative tests** are available on the Web site covering material up to the end of Chapters 4, 9, 11, and 16.

• **Updated Contents**

- **Chapter 8** now includes the chip model for multiplication.
- **Chapter 10** has been updated with new data.
- **Chapter 11** has been revised to moderate the pace of the coverage.
- An **Introduction to Graph Theory** has been added as a topic near the end of the book.

Handbook Algebra

Name \_\_\_\_\_

**Add in Any Order**

You can add in any order and get the same sum.

$2 + 1 = 3$        $1 + 2 = 3$

Use and to add. Write each sum. Color to match.

1     

$4 + 1 = \underline{\quad}$        $1 + 4 = \underline{\quad}$

2     

$3 + 2 = \underline{\quad}$        $2 + 3 = \underline{\quad}$

3     

$3 + 0 = \underline{\quad}$        $0 + 3 = \underline{\quad}$

**Talk About It • Reasoning**

What happens to the sum when you change the order of the numbers you are adding? Use and to prove your answer.

Chapter 2 • Using Addition

17

- **Reflections from Research** marginal notes have been added and updated.
- **Student Page Snapshots** have been updated. Each chapter has a page from an elementary school textbook relevant to the material being studied.

## Summary of Popular Features Continuing from the Fifth Edition

- **Problem solving strategies** are integrated throughout the book. Each chapter introduces a new Problem Solving Strategy; a comprehensive list is included at the beginning of each chapter. Following the chapter opening vignette, each chapter, beginning with Chapter 2, contains a relevant Initial Problem that introduces a new strategy.
- **Mathematical Structure** reveals the mathematical ideas of the book. Main Definitions, Theorems, and Properties in each section are highlighted by boxes for quick review.
- **Exercise / Problem Sets** are separated into Part A (all answers are provided in the back of the book and all solutions are provided in our supplement—*Hints and Solutions for Part A Problems*) and Part B (answers are only provided in the *Instructor's Resource Manual*). Also, exercises and problems are distinguished so that students can learn how they differ.

### Reflection from Research

Students who only view fractions like  $\frac{3}{4}$  as “three out of 4 parts” struggle to handle fraction multiplication problems such as  $\frac{2}{3}$  of  $\frac{9}{10}$ . Students who can more flexibly view  $\frac{3}{4}$  as “three fourths of one whole or three units of one fourth” can better solve multiplication of two proper fractions (Mack, 2001).

### NCTM Standard 2000 Number and Operations Grades 6–8

All students should use factors, multiples, prime factorization, and relatively prime numbers to solve problems.

and Part B (answers are only provided in the *Instructor's Resource Manual*). Also, exercises and problems are distinguished so that students can learn how they differ.

- **Problems for Writing/Discussion** are included at the end of each problem set as well as at the end of each chapter review.
- **NCTM Standards 2000** are called out in the margins at relevant points in the text. The inside front cover contains the essentials of the NCTM Principles and Standards for School Mathematics.
- **Historical vignettes** open each chapter and introduce ideas and concept central to each chapter.

**Problem Solving Strategies**

1. Guess and Test
2. Use a Variable
3. Draw a Picture
4. Look for a Pattern
5. Make a List
6. Solve a Simpler Problem
7. Draw a Diagram
8. Use Direct Reasoning
9. Use Indirect Reasoning

**Strategy:**  
**Use Indirect Reasoning**

Occasionally, in mathematics, there are problems that are not easily solved using direct reasoning. In such cases, indirect reasoning may be the best way to solve the problem. A simple way of viewing indirect reasoning is to consider an empty room with only two entrances, say *A* and *B*. If you want to use direct reasoning to prove that someone entered the room through *A*, you would watch entrance *A*. However, you could also prove that someone entered through *A* by watching entrance *B*. If a person got into the room and did not go through *B*, the person had to go through entrance *A*. In mathematics, to prove that a condition, say “*A*,” is true, one assumes that the condition “not *A*” is true and shows the latter condition to be impossible.

**Initial Problem**

The whole numbers 1 through 9 can be used once, each being arranged in a  $3 \times 3$  square array so that the sum of the numbers in each of the rows, columns, and diagonals is 15. Show that 1 cannot be in one of the corners.

2			5
1			6
3			7
4			8
			9

**DEFINITION**

**Addition of Whole Numbers**

Let *a* and *b* be any two whole numbers. If *A* and *B* are disjoint sets with  $a = n(A)$  and  $b = n(B)$ , then  $a + b = n(A \cup B)$ .

**THEOREM**

Let *a*, *m*, and *n* be any whole numbers where *m* and *n* are nonzero. Then

$$a^m \cdot a^n = a^{m+n}$$

**PROPERTY**

**Commutative Property for Whole-Number Addition**

Let *a* and *b* be any whole numbers. Then

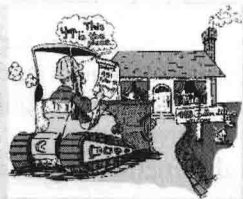
$$a + b = b + a$$



- **Mathematical Morsels** end every section with an interesting historical tidbit. They are a reward for completing a section!
- **People in Mathematics** highlights many of the giants in mathematics throughout history as well as others who have contributed to mathematics in various ways.

### MATHEMATICAL MORSEL

When learning the names of two-digit numerals, some children suffer through a "reversals" stage, where they may write 21 for twelve or 13 for thirty-one. The following story from the December 5, 1990, *Grand Rapids* [Michigan] Press stresses the importance of eliminating reversals. "It was a case of mistaken identity. A transposed address that resulted in a bulldozer blunder. City orders had called for demolition on Tuesday of a boarded-up house at 451 Fuller Ave. S.E. But when the dust settled, 451 Fuller stood untouched. Down the street at 415 Fuller S.E., only a basement remained."



### People in Mathematics



**Emmy Noether (1882–1935)**

Emmy Noether was born and educated in Germany and was graduated from the University of Erlangen, where her father, Max Noether, taught mathematics. There were few professional opportunities for a woman mathematician, so Noether spent the next eight years doing research at home and teaching for her increasingly disabled father. Her work attracted the attention of the mathematicians Hilbert and Klein, who invited her to the University of Göttingen—at that time one of the world's leading centers of mathematics. Initially, Noether's lectures were announced under Hilbert's name, because the university refused to admit a woman lecturer. Conditions improved, but in 18 years at Göttingen, she was routinely denied the promotions that would have come to a male mathematician of her ability. When the Nazis came to power in 1933, she was dismissed from her position. She emigrated to the United States and spent the last two years of her life at Bryn Mawr College. In 1983, the French mathematician Jean Dieudonné wrote that Emmy Noether "was by far the best woman mathematician of all time, and one of the greatest mathematicians of the twentieth century."



**David Hilbert (1862–1943)**

David Hilbert, professor of mathematics at the University of Göttingen, surveyed the spectrum of unsolved problems in 1900, and selected 23 for special attention. He felt these 23 were crucial for progress in mathematics in the coming century. Hilbert's vision was proved to be prophetic. Mathematicians took up the challenge, and a great deal of progress resulted from attempts to solve "Hilbert's problems." Hilbert himself made important contributions to the foundations of mathematics and attempted to prove that mathematics was self-consistent. He became one of the most influential mathematicians of his time, yet he once remarked that when he read or heard new ideas in mathematics, they seemed "so difficult and practically impossible to understand," until he worked the ideas through for himself.

- **Chapter Review** is located at the end of each chapter together with a Chapter Test.
- **Epilogue**, following Chapter 16, provides a rich eclectic approach to geometry.
- **Logic and Clock Arithmetic** are discussed near the end of the book.

## Companion Web Site/Supplements Package

### Supplements for the Instructor

**Instructor's Resource Manual** This manual contains the following: (a) chapter-by-chapter discussion of the text material; (b) student "expectations" (objectives) for each chapter; (c) answers for all Part B exercises and problems; (d) answers for all the even-numbered problems in the Problem Solving Guide.

**Computerized Test Bank** The test bank contains true/false, multiple-choice, and open-ended questions.

**eGrade** This online assessment system contains a large bank of skill-building problems and solutions. Instructors can now automate the process of assigning, delivering, grading, and routing all kinds of homework, quizzes, and tests while providing students with immediate scoring and feedback on their work. Wiley eGrade "does the math" . . . and much more. For more information, visit [www.wiley.com/college/egrade](http://www.wiley.com/college/egrade).

- **Instructor's Companion Web Site** at [www.wiley.com/college/musser](http://www.wiley.com/college/musser)

- Instructors have access to all student Web site features
- PowerPoint Slides: more than 190 PowerPoints including figures from the text and several generic masters such as for dot paper, grids, and other formats
- Test Bank: contains true/false, multiple-choice, and free-response questions

## Supplements for the Student

**Student Resource Handbook** This handbook is designed to enhance student learning as well as to begin to model effective classroom practices. Since many instructors are working with students to create a personalized journal, this edition of the handbook is three-hole-punched for easy customization.

—*Prepared by Karen Swenson and Marcia Swanson, two exceptional mathematics educators.*

ISBN: 0471236799

### FEATURES INCLUDE:

- **Warm Ups:** Short problem solving activities.
- **Hands On Activities:** Activities that help develop initial understandings at the concrete level.
- **Two Dimensional Manipulatives:** Cutouts are provided on cardstock.
- **Exercises:** Additional practice for building skills in concepts.
- **Mental Math:** Short activities to help develop better mental math skills.
- **Self-Test:** New 10-item tests in a variety of formats designed to assess student knowledge of key areas.
- **Solutions:** Solutions to all items in the handbook to enhance self-study.
- **Resource Articles:** Up-to-date references from journals for elementary teachers to help provide a connection to the classroom.
- **Directions in Education:** Specially written articles that provide insights into major issues of the day, including the Standards of the National Council of Teachers of Mathematics.

**Hints and Solutions Manual for Part A Problems** This manual can be used to help students develop problem solving proficiency in a self-study mode.

—*Developed by Lynn Trimpe, Roger Maurer, and Vikki Maurer of Linn-Benton Community College.*

ISBN: 0471236780

### FEATURES INCLUDE:

- **Hints:** These are provided to give students a start on all Part A problems in the text.
- **Additional Hints:** For more challenging problems, a second hint is provided.
- **Complete Solutions to Part A Problems:** Carefully written out solutions are provided to model one correct solution.

- **Student Companion Web Site at**  
[www.wiley.com/college/musser](http://www.wiley.com/college/musser)

**A Guide to Problem Solving** This online resource contains more than 200 creative problems keyed to the strategies in the textbook.

—*Prepared by Don Miller, who was a professor of mathematics at St. Cloud State University.*

### FEATURES INCLUDE:

- **Opening Problem:** An introductory problem to motivate the need for a strategy.
- **Solution/Discussion/Clues:** A worked-out solution of the opening problem together with a discussion of the strategy and some clues on when to select this strategy.
- **Practice Problems:** A second problem that uses the same strategy together with a worked-out solution and two practice problems.
- **Mixed Strategy Practice:** Four practice problems that can be solved using one or more of the strategies introduced to that point.

• **Additional Practice Problems and Additional Mixed Strategy Problems:**

Sections that provide more practice for particular strategies as well as many problems for which students need to identify appropriate strategies.

**eManipulative Activities Online** eManipulatives are integrated in each chapter and are keyed to “Spotlight on Technology” exercises throughout the text. These activities are designed to develop concepts central to the chapter and many of the activities mirror physical manipulatives. The goal is to engage the learner in a way that will lead to a more in-depth understanding of the concept.

—*Prepared by Lawrence O. Cannon, E. Robert Heal, and Richard Wellman of the Department of Mathematics and Statistics at Utah State University. This project is supported by the National Science Foundation.*

**The Geometer’s Sketchpad Activities and Tutorial** The *Geometer’s Sketchpad* Activities are keyed to “Spotlight on Technology” sections throughout the text. These dynamic activities allow you to work through selected problems using Java Sketchpad™ exercises, which were created from the Geometer’s Sketchpad’s award-winning learning environment. In addition, an online tutorial is designed to introduce students to *The Geometer’s Sketchpad*®, an effective learning tool that helps future teachers broaden their mathematical understanding and the use of technology in the classroom.

—*Prepared by Armando Martinez-Cruz, California State University, Fullerton.*

**Spreadsheet Activities and Tutorial** “Spotlight on Technology” activities are keyed to specific online spreadsheet activities. A tutorial is designed to introduce students to the use of spreadsheets. Examples illustrate the use of spreadsheets and the tutorial contains exercises and problems.

—*Prepared by Keith Leatham, Portland State University.*

**Children’s Literature Tutorial** This section consists of an introduction, suggestions on how to integrate children’s literature into the classroom, and a list of book titles, which is annotated by math topic and includes questions, a checklist of how to evaluate books, and a general reference list.

—*Prepared by Joan Cohen Jones, Eastern Michigan University.*

**Technology Section** This section contains an Introduction to Programming in LOGO: Turtle Geometry and an Introduction to Graphing Calculators.

**Links to National Council of Teachers of Mathematics and Other Helpful Math Sites.**





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