



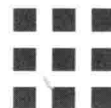
G. R. Lindfield // J.E.T. Penny

Third Edition

Numerical Methods

using MATLAB®



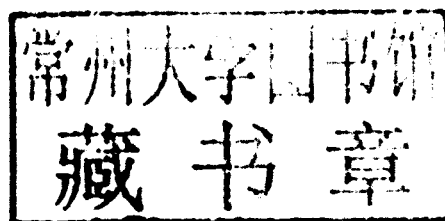


Numerical Methods Using MATLAB[®]

Third Edition

G.R. Lindfield

J.E.T. Penny



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Academic Press is an imprint of Elsevier



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225 Wyman Street, Waltham, MA 02451, USA
The Boulevard, Langford Lane, Kidlington, Oxford, OX5 1GB, UK

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Library of Congress Cataloging-in-Publication Data

Lindfield, G.R. (George R.) Numerical methods using MATLAB[®] / G.R. Lindfield, J.E.T. Penny. — 3rd ed.
p. cm.

Penny's name appears first on the earlier edition.

Includes bibliographical references and index.

ISBN 978-0-12-386942-5 (pbk.)

1. Numerical analysis—Data processing. 2. MATLAB. I. Penny, J.E.T. (John E.T.) II. Title.

QA297.P45 2012

518.0285'53—dc23

2012015199

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

For information on all Academic Press publications
visit our website at <http://store.elsevier.com>

Printed in the United States of America

12 13 14 15 16 10 9 8 7 6 5 4 3 2 1

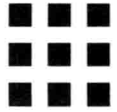
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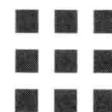
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To our wives, Zena Lindfield and Wendy Penny, and our now adult children, Helen and Katy, and Debra, Mark and Joanne, for their patience and support. Also to our various cats, who have walked over, and even slept on, the computer keyboard!



Preface

The third edition of *Numerical Methods Using MATLAB*[®] is an extensive development of the first and second editions of this book. All MATLAB scripts and functions have been checked and revised to ensure that they are executable in the current version of MATLAB, version 7.13.

Our primary aim in this text is unchanged from previous editions; it is to introduce the reader to a wide range of numerical algorithms, explain their fundamental principles, and illustrate their application. The algorithms are implemented in the software package MATLAB, which is constantly being enhanced and provides a powerful tool to help with these studies.

Many important theoretical results are discussed, but it is not intended that a detailed and rigorous theoretical development in every area be provided. Rather, we wish to show how numerical procedures can be applied to solve problems from many fields of application, and that the numerical procedures give the expected theoretical performance when used to solve specific problems.

When used with care, MATLAB provides a natural and succinct way of describing numerical algorithms and a powerful means of experimenting with them. However, no tool, irrespective of its power, should be used carelessly or uncritically.

This text allows the reader to study numerical methods by encouraging systematic experimentation with some of the many fascinating problems of numerical analysis. Although MATLAB provides many useful functions, this text also introduces the reader to numerous useful and important algorithms and develops MATLAB functions to implement them. The reader is encouraged to use these functions to produce results in numerical and graphical form. MATLAB provides powerful and varied graphics facilities to give a clearer understanding of the nature of the results produced by the numerical procedures. Particular examples are given throughout the text to illustrate how numerical methods are used to study problems, including applications in the biosciences, chaos, neural networks, engineering, and science.

It should be noted that this introduction to MATLAB is relatively brief and is meant as an aid to the reader. It can in no way be expected to replace the standard MATLAB manual or textbooks devoted to MATLAB software. We provide a broad introduction to the topics, develop algorithms in the form of MATLAB functions, and encourage the reader to experiment with these functions, which have been kept as simple as possible for reasons of

clarity. These functions can be improved, and we urge readers to develop those that are of particular interest to them.

In addition to a general introduction to MATLAB, the text covers the solution of linear equations and eigenvalue problems; methods for solving nonlinear equations; numerical integration and differentiation; the solution of initial value and boundary value problems; curve fitting, including splines, least squares, and Fourier analysis; and topics in optimization such as interior point methods, nonlinear programming, and genetic algorithms. Finally, we show how symbolic computing can be integrated with numerical algorithms. Specifically in this third edition, in Chapter 1 we have added descriptions and given examples of some functions recently added to MATLAB and have included a discussion of handle graphics with examples. Chapter 4 now includes a section on Lobatto's method for integration and the Kronrod extension. Chapter 8 has been extensively revised and includes a description of the continuous genetic algorithm, Moller's scaled conjugate gradient method, and methods for solving constrained optimization problems.

The text contains many worked examples, practice problems (many of which are new to this edition), and solutions. We hope we have provided an interesting range of problems.

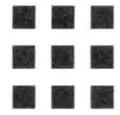
The text is suitable for undergraduate and postgraduate students and for those working in industry and education. We hope readers will share our enthusiasm for this area of study. For those who do not currently have access to MATLAB, this text provides a general introduction to a wide range of numerical algorithms and many useful and interesting examples and problems.

For readers of this book, additional materials, including all .m file scripts and functions listed in the text, are available on the book's companion site: www.elsevierdirect.com/9780123869425. For instructors using this book as a text for their courses, a solutions manual is available by registering at the textbook site: www.textbooks.elsevier.com.

We would like to thank the many readers from all over the world who provided helpful comments, which have enhanced this edition. We also acknowledge the valuable assistance given to us by our colleague, David Wilson, in guiding us in the restructuring of Sections 7.5, 7.6, and 7.7.

We would be pleased to hear from readers who note errors or have suggestions for improvements. Also, we would like to thank key Elsevier staff, including Patricia Osborn, Acquisitions Editor; Kathryn Morrissey, Editorial Project Manager; Joe Hayton, Publisher; Fiona Geraghty, Editorial Project Manager; Kristen Davis, Designer; and Marilyn Rash, Project Manager.

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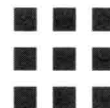
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An Introduction to MATLAB[®]

MATLAB[®] is a software package produced by The MathWorks, Inc. (www.mathworks.com) and is available on systems ranging from personal computers to supercomputers, including parallel computing. In this chapter we aim to provide a useful introduction to MATLAB, giving sufficient background for the numerical methods we consider. The reader is referred to the MATLAB manual for a full description of the package.

1.1 The MATLAB Software Package

MATLAB is probably the world's most successful commercial numerical analysis software package, and its name is derived from the term “matrix laboratory.” It provides an interactive development tool for scientific and engineering problems and more generally for those areas where significant numeric computations have to be performed. The package can be used to evaluate single statements directly or a list of statements called a script can be prepared. Once named and saved, a script can be executed as an entity. The package was originally based on software produced by the LINPACK and EISPACK projects but currently includes LAPACK and BLAS libraries which represent the current “state-of-the-art” numerical software for matrix computations. MATLAB provides the user with

1. Easy manipulation of matrix structures
2. A vast number of powerful built-in routines that are constantly growing and developing
3. Powerful two- and three-dimensional graphing facilities
4. A scripting system that allows users to develop and modify the software for their own needs
5. Collections of functions, called toolboxes, that may be added to the facilities of the core MATLAB. These are designed for specific applications, for example, neural networks, optimization, digital signal processing, and higher-order spectral analysis.

It is not difficult to use MATLAB, although to use it with maximum efficiency for complex tasks requires experience. Generally MATLAB works with rectangular or square arrays of data (matrices), the elements of which may be real or complex. A scalar quantity is thus a matrix containing a single element. This is an elegant and powerful notion but it can present the user with an initial conceptual difficulty. A user schooled in such languages as C++ or Python is familiar with a pseudo-statement of the form $A = B + C$ and can immediately interpret it as an instruction that A is assigned the sum of values of the numbers stored in