IEEE 1980 Frontiers of Engineering in Health Care

September 28-30, 1980

IEEE 1980 FRONTIERS OF ENGINEERING IN HEALTH CARE

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MESSAGE FROM THE PRESIDENT OF THE IEEE ENGINEERING IN MEDICINE AND BIOLOGY SOCIETY

This national and international conference is the highlight of our activities as a technical and professional society of the IEEE. This conference is our second one, and we anticipate this conference becoming an annual event and contributing to the emergence of engineers in health care delivery. As the organizers and sponsor of this conference, we have made every possible effort to have it serve as a focal point for the latest developments involving technology in health care. Our technical committees, publication editors, and administrative committee members (AdCom) have planned for the past year to bring together the relevant research efforts, the clinical developments, and the industrial applications. Key people have been invited to organize the sessions and workshops. Many invited papers have been included to ensure the proper balance in the sessions. Tutorials and workshops have been included as part of the regular conference program. The full conference papers are published in this Conference Proceedings, and the abstracts of each paper were published in the September issue of our EMBS Transactions.

It is a gratifying moment to see our EMBS Society meet and work together at our own annual conference. May I welcome you to our conference, and I hope that your interaction with the other participants will lead to a more effective communication for our Society.

I would like to express my gratitude to those who have labored to bring this program together; and, in particular, to Lee Ostrander, Ph.D., for being the program chairman and organizer.

Mort Schwartz, Ph.D.
President

MESSAGE FROM THE PROGRAM CHAIRMAN

The Second Annual Conference of IEEE/EMBS continues with the goals of the first, i.e. to meet the changing needs of EMBS members. The theme is Frontiers of Engineering in Health Care.

The emphasis in the conference is on up-to-date technical advances that will influence and/or are now influencing the design, safe and effective use, and management of medical devices and systems. The topics covered represent professional interests within EMBS, an organization directed to the solution of medical and biological problems and located within a larger electrical and electronics engineering "umbrella" organization, which is the IEEE. As the work of EMBS members evolves so does the content of the Conference.

The Proceedings contains papers relating to the technical presentations at the Conference and describing current work and results in biomedical engineering. Abstracts for Conference workshops and tutorials are also included.

A Conference Proceedings is the work of many people. I would like to acknowledge the help of the authors who contributed papers of good quality suitable for this Proceedings. I would also like to extend special thanks to the organizers listed in the Program Committee who defined the technical focus within sessions and who in many cases invited participants.

Lee E. Ostrander, Ph.D.
Program Chairman

IEEE Engineering in Medicine and Biology Society

The Engineering in Medicine and Biology Society of the IEEE (IEEE/EMBS) is an association of 7,000 members concerned with the application of engineering science and methodology to biology, medicine, and health care delivery systems.

EMBS is a Society within the umbrella framework of the Institute of Electrical and Electronics Engineers (IEEE) offering identification with the world's largest professional engineering organization of 180,000 members. Activities of the IEEE/EMBS include:

Publications - The IEEE Transactions on Biomedical Engineering is a monthly publication of reviewed articles reporting original research and application and development, short communications to disclose new ideas, and tutorials and reviews. The EMBS Newsletter is published quarterly and contains news and events of current interest to biomedical engineering professionals.

Conferences - Conference Proceedings are widely distributed and indexed through IEEE and may be purchased at member's prices from IEEE. The Society also cosponsors and/or cooperates in other national and regional biomedical conferences.

Technical and Professional Committees - EMBS Committees organize conference sessions, workshops and special activities on behalf of the Society. Technical Committees include: bioelectric phenomena, clinical engineering, medical instrumentation, prosthetic and sensory aids, signal processing and information handling, transducers and devices, and biomaterials. Professional Committees include: awards, biomedical coordination, education, ethics, membership, professional activities, publications, standards, government affairs, and industrial relations. In addition, EMBS participates, through appointed delegates, in other national bodies such as ANSI and NFPA as well as in broad based IEEE Technical Committees addressing such issues as energy, ocean engineering, environmental quality, man and radiation, and social implications of technology.

Regional Councils and Chapters - Society members have the opportunity to exchange technical and professional information with colleagues in their same geographic area through meetings and activities of 7 EMBS Regional Councils and 33 Chapters. Membership in these geographically organized subdivisions is an automatic component of Society Membership.

Membership in IEEE/EMBS is open to all qualified persons in grades designated student, senior member, fellow, and affiliate. Biomedical Professionals who wish to join EMBS but not join the IEEE umbrella organization may do so as affiliate members of EMBS. Affiliate members are accorded the opportunities of participation in all EMBS programs and activities as planned and administered by the EMBS elected Administrative Committee (AdCom).

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NEW METHODS APPLICABLE TO SOLUTIONS OF THE FORWARD ECG PROBLEM

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SUMMARY

This paper presents a variety of methods and test results related to increasing the speed and accuracy of the forward calculations of ECG surface potentials. The information should be of interest to those intending systematically to model the ECG process either in research studies or clinical applications.

INTRODUCTION

Our laboratory is currently working towards an extended capability for modeling the ECG with the long-range goal of designing an efficient, semi-automatic system that will permit the torso model configuration to be tailored in many important respects to an individual subject under study. A propagating heart model to generate time-varying equivalent generator sources will be incorporated in the simulation to complete the model configuration. This paper reports on the progress made in certain aspects of this program. Specifically, methods are described which (1) greatly increase the speed of solving the forward problem using established iterative techniques, (2) effectively remove the constraint imposed by having to work with a particular set of heart-lead transfer coefficients from a restricted set of locations in the myocardium, and (3) efficiently triangularize a torso geometry from a relatively unrestricted set of surface point coordinates. In addition, practical information is presented concerning the choice and computation of matrix coefficients of the iterative solution.

One significant difficulty present when dealing with realistic heart-torso models is the usual restriction on the number of source locations in the myocardium for which heart-lead transfer coefficients are available. The traditional approach has been to limit sharply the number of locations, as done by Holt et al. (1) and by Miller and and Geselowitz (2). These restrictions may lead to unrealistic models and are unnecessary. In particular, a propagating heart model should include in excess of 250,000 cells for satisfactory spatial resolution. This leads to a similar number of source locations, each of which may produce a source with components in all three (3) coordinate directions. Lumping large numbers of these sources together to obtain equivalent sources at a relatively few locations may be necessary for some applications, such as the inverse solutions of Holt et al. (1), but for the forward solution we have been able to overcome this limitation by two technical improvements. One is the development of a method for calculating forward solutions, and hence transfer coeficients, at greatly increased speeds. The second is the development of "source independent" transfer coefficients.

FORWARD SOLUTION FROM STORED COEFFICIENT MATRIX

The first of these methods to be described, is a technique for greatly increasing the speed with which the forward solution can be calculated from a given set of sources. The iterative method, either the Gelernter-Swihart (3) or Barr et. al. (4) formulation, requires a solution of the form:

$$\vec{F} = [\alpha_{ij}]\,\vec{R} \qquad (1)$$

in which the F's form a column matrix of the forcing terms, and the R's form a column matrix of the response terms. The alpha matrix contains the spatial factors due to distance effects modified by internal and surface conductivity boundaries. Until now, in large problems, it has been considered necessary to recalculate all elements of the matrix during each iteration when solving for the responses. For the UVM homogeneous torso presently being investigated, the triangular subdivisions yield a matrix containing in excess of 4 million elements. This would necessitate 4 million calculations (typically solid angles or approximations thereto) for each iteration. It presently requires 15-25 iterations to meet a convergence criterion of 0.01% change in the total charge magnitude from iteration to iteration. In the past, the requirement to recompute this many coefficients was considered a burden necessary due to the high cost of resident memory and high-speed random access mass storage. These excessive calculations, however, are no longer essential. Our method, to be called the "Stored Matrix" method, takes advantage of expanding technology to create and store the spatial terms of the matrix equation.

The Stored Matrix method utilizes the parallel data handling capabilities of a modern mini-computer system. This capability is used to read data asynchronously from disk into an array in resident memory, while processing of a dot produce occurs, concurrently, using a second array. Upon completion of the dot product and data transfer, the roles of the two arrays may be exchanged. We have implemented this technique utilizing a 67.4 mega-byte moving head disk drive on a PDP-11/55 and approximations to the alphas in Eq. (1) using an inverse distance squared relationship. The numerical results were of course identical, since the coefficients (alphas) were identical. The Stored Matrix solution using the disk drive, however, provided a reduction in the computation time by a factor of 19.5. Fig. 1 shows a comparison of the timing for the conventional vs. the Stored Matrix methods when using the alpha "inverse distance squared" approximation and disk drive described above. As can be seen, gains beyond those already realized are still possible with this technique by the reduction of the row dot product times.

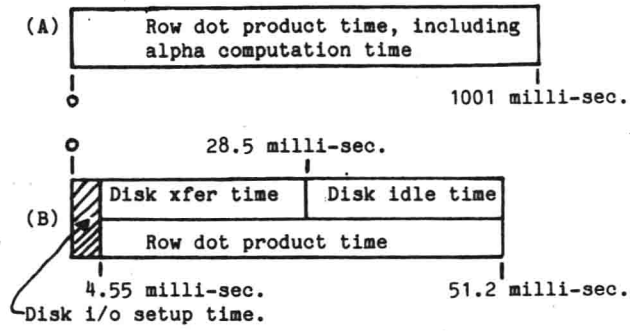

Figure 1. Timing of the Traditional (A) vs. the Stored Matrix (B) method for the Forward Solution

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An additional major advantage of this method stems from the fact that more complex calculations may be utilized in the calculation of the alpha matrix, since the added expense is incurred only once. The use of solid angle, or averaged solid angle calculations, takes from 5 to 10 times the computation required for the inverse distance squared approximation mentioned above. This greatly alters the timing relationship in Fig. 1 inasmuch as the time for the Stored Matrix method does not change while the traditional method takes a correspondingly longer time.

SOURCE INDEPENDENT HEART LEAD TRANSFER COEFFICIENTS

The second development, complementary to the one described above, has yielded a technique for completely eliminating the need for a set of fixed location transfer coefficients. That is, in calculating the surface potentials from a set of sources defined at discrete time intervals, one can sum up the source effects at a given surface sub-area as a forcing function, and solve the forward problem afresh during each time subdivision. Alternatively, one can precalculate a set of heart-lead transfer coefficients and simply assign dipole strengths and orientations with which to multiply each transfer coefficient. In this method, three (3) sets of transfer coefficients from each source location are required - one for each orthogonal dipole component.

Here, we describe a third alternative to be referred to as "Source-Independent" transfer coefficients. These depend only on the torso and not on the source locations. Conceptually, all that is involved is the inversion of the alpha matrix in Eq. (1). The elements of the inverted matrix can be interpreted as transfer coefficients from each subdivision of the torso and inhomogeneity interfaces. Due to the size and nature of the alpha matrix, it does not lend itself to direct inversion techniques. If, however, the forcing matrix in Eq. (1) is replaced with one that contains a single element of unity value, with all other elements equal to zero, it can be seen that the solution to this specialized forward problem yields the ith column of the inverse matrix. By successive calculations, moving the unity forcing term each time, the inverse matrix can be found. For example, the first calculation would have the form:

$$(2) \qquad \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = [\alpha_{ij}] \vec{R}$$

with solution:

$$(3) \qquad [\alpha_{ij}]^{-1} [\alpha_{ij}] \vec{R} = [\alpha_{ij}]^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = [\alpha_{j1}]$$

The term, $[\alpha_{ij}]$, is the first column of the inverse matrix. Once the complete inverse is found, the responses can be calculated from a single matrix multiplication of the inverse times the forcing terms:

$$(4) \qquad \vec{R} = [\alpha_{ij}]^{-1} \vec{F}$$

It should be noted that the use of the usual forward solution technique, allows the additional specification or deflation necessary to account for the singularity inherent in the alpha matrix.

If one compares this method with the transfer coefficient method, using a 2000 sub-division example, and considers that three (3) coefficients are required at each source location, the two methods are comparable if 670 (2000/3) source locations are employed. Considering that the heart is 3-dimensional, this result implies a grid of source locations spaced at intervals corresponding to the cube root of 670. This results in approximately 8-9 source locations in each coordinate direction of the 3-dimensional model. This number corresponds, roughly, to a spacing of 1 cm between equivalent source locations. While for many applications, this represents sufficient accuracy, there are numerous drawbacks in addition to the accuracy problem. Decisions about the lumping of the cellular sources may be changed for a variety of reasons; the experiments in a given torso related to heart size, orientation, wall thickness, location, etc. may all require new sets of transfer coefficients.

The "Source Independent" transfer coefficient technique can also be compared to the proceedure in which the forward solution is calculated for any set of sources as needed. In the specific case when one is working with the propagating model, there are substantial drawbacks using the latter method. If the T-wave is to be included, and the model incorporates the equivalent of 5 milli-second steps (a minimum for modeling the QRS), the entire QRS-T interval involves about 80 sets of calculations for just one experiment. Under these assumptions, the break-even point, in comparision with the "Source-Independent" method, occurs at 25 experiments with the propagating model. Considering all the changes that one might make in the heart geometry itself, its position and orientation in relation to the torso, and the various combinations one might wish to explore in the initial activation, action potential forms, and pathological states, the solving of the complete forward solution at each step of every experiment appears to be the least efficient scheme of the three.

A further advantage to the "Source-Independent" transfer coeficient scheme is its transportability. We plan to make all the inverse matrices we calculate available on tape to any interested investigator. They can thus be used indefinitely in the same fashion as a book of tables of physical data, with the added advantage of automatic data acquisition and entry. Our imediate goal is the calculation of the "Source-Independent" transfer matrix for the UVM torso model (5) (6) for which extensive documentation exists. With both the anatomical data and the associated inverse matrix, an investigator with access to a computer and magnetic tape drive will be in a position to carry out a wide variety of model studies that may be of interest.

The "Source-Independent" transfer coefficient matrix method has been tested on a simple torso model having 90 subdivisions to form the approximated surface. The results of using this matrix and the traditional method for calculating the surface potentials from a given source have been compared and found to yield virtually identical results.

SEMI-AUTOMATIC TRIANGULARIZATION OF THE TORSO SURFACE

The methods discussed above are generally applicable to any extended study involving multiple solutions of the forward problem. The third topic to be considered here is the automatic acquisition and computer entry of surface coordinate information and its conversion to a form suitable for the calculations of the forward problem. The instrumentation required to measure a sufficient number of torso surface points rapidly and efficiently under clinical conditions is in the initial stages of development, but will not be discussed here.

The portion of the program to be described, assumes the existence of a set of torso surface coordinates lying in horizontal layers similar to those in a cross-sectional anatomical atlas (7) or produced by a CAT scan. The proceedure translates these surface coordinates into a set of covering triangles which can then be used as sub-division information for the forward problem. The layer spacings are roughly divided into groups, with greater spacings associated with greater distances from the heart (and thus the sources). The layer spacing within each group need only be roughly equal and the distribution of points around each layer need only be roughly uniform. While we do not have definitive information about a minimal subarea (triangle) size in the regions where the potentials are changing most rapidly, it is clear that making all subarea triangles equal to those where the greatest resolution is needed, is wasteful. In the discussion to follow, a systematic method for shifting from one set of triangle sizes to another is described.

The principal contribution of the method proposed, is that it removes the need for careful alinement of the point locations between layers, or for identical numbers of points in each layer grouping, It also accommodates expansions and contractions of the inter-electrode perimeter distances that naturally follow from the body contours. The factor that makes this problem one of considerable significance is the effect of the relative shapes of the approximating triangles. Swihart (8) has suggested, and our experience confirms, that it is desirable, and, in the extreme necessary, that the triangles be as close to equilateral as possible. This factor affects both the accuracy of the resulting solutions and in extreme cases can cause the solutions not to converge at all. The ideal subdivision of the surface would consist of equilateral triangles, which implies triangles having vertices alternately on the upper and lower horizontal layers (Fig. 2). Any method for subdividing the torso should attempt to approximate this condition.

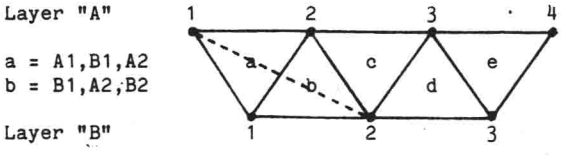

Figure 2. Adjacent Torso Layer Triangularization

The method of forming the triangles from the points on adjacent horizontal layers, starts with files containing the coordinates of points on the two layers in a sequence corresponding to a progression around the layers in one direction. Triangle formation starts with the choice of a starting triangle which contains two adjacent points on one layer ("A"), see Fig. 2, and a third point on layer "B". These three vertices are then recorded as a subarea (A1,B1,A2). The algorithm now eliminates the first vertex (A1) and shifts the remaining vertices to the left. The new third vertex is chosen from the layer which previously contained only one vertex. Thus the new triangle becomes (B1,A2,B2). As each triangle is created, the angles at the three vertices are tested for large deviations from the equilateral condition. Currently the triangle is considered "bad" if any one of the three angles (α_1 , α_2 , or α_3) is greater than 90 degrees. A type # "n" error is defined as an angle greater than 90 degrees at the nth vertex (see Fig. 3), n = 1, 2, or 3. When a type 1

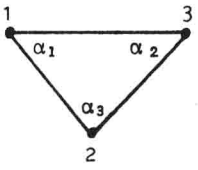

Figure 3. General Triangle Configuration.

error is detected, this triangle and the next are considered in a "look ahead" scheme to determine the most favorable way of connecting the four vertices contained therein. In 2 dimensions, this would amount to choosing between one or the other diagonal in the trapezoid that has the four points as its vertices. Consider a triangle (B1,A1,B2) in Fig. 2. This would be flagged as a bad triangle because α_1 is greater than 90 degrees. The 4 vertices A1, B1, A2, and B2 would than be considered and the triangle pair (A1,B1,A2) and (B1,A2,B2) would be selected. A type 2 error is not easily corrected without added points and, if it appears frequently, it can be an indicator that the original choice of surface points should be altered. The appearance of a type 3 error is handled in a manner similar to that used for a type 1 error. In this case, however, the current triangle and the previous triangle are considered in a "look behind" scheme. Again the manner of connecting the 4 vertices which yields the best overall result is retained.

In addition to the consideration of the angles, it was deemed appropriate to vary the surface point spacing over the torso surface. In so doing, the quantization of the surface could be made sufficiently fine in the regions closest to the heart generators, while allowing a coarse representation in regions distant from the sources. To accomplish this end, certain layers were identified as "transition" layers. These layers represent places where the number of points was reduced (or increased) by a factor of 2. On these particular layers, the smaller number of points represent physically measured locations. In Fig. 4, layer "B" represents a transition layer. The odd numbered points in this layer are physically measured points. These points are used to connect with the adjacent layer, "C", containing the fewer number of points. To connect to the other adjacent layer, "A", additional points on "B" are required. These are provided by the bisection of the line connecting the physically measured points in the transistion layer. Thus the surface remains closed and the pattern regular despite the discontinuity in the number of points per layer.

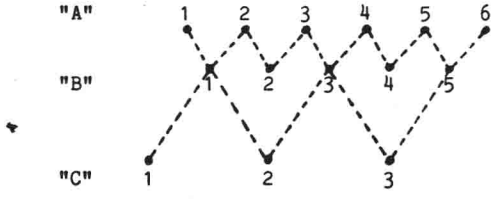

Figure 4. Transition Layer Surface Points in Two dimensions.

It should be noted that the algorithm for determining the triangularization is not dependent on the "transition layer" concept. It was considered desirable, however, to use this type of approach in the digitizing scheme to maintain a fully closed surface with both a minimum number of subdivisions and angle errors. With our current data set, the UVM

torso has 2008 subdivisions for the skin boundary. For this surface, this technique results in 68 triangles with uncorrectable angle errors; 24 type # 1, 14 type # 2 and 30 type # 3.

Generating the Spatial Coefficients

Investigators interested in solving the forward problem will find the literature incomplete with respect to practical factors and algorithms helpful in making needed decisions prior to calculating the alpha's of Eq. (1). In the potential method, Barr et al. (4), the diagonal terms are simply 2π; this is exact once the surface is triangularized. The off-diagonal terms can be dealt with by a variety of approximations. In the sphere example of their paper, they appear to have used for α_{ij}, the spherical area of each, jth, subdivision, divided by the distance between the centroids of subareas i and j, times the cosine of the angle between the distance vector and the sphere normal at the centroid of sub-area, j. This coefficient is an approximation to the precise term, the solid angle subtended at the centroid of subarea i by subarea j. The solid angle of the spherical subareas is extremely difficult to calculate precisely and if one attempts to utilize their subdivisions with plane triangles to approximate the surface, gaps in the surface will appear. This problem, as noted earlier in this paper, tends to occur where the subdivision fineness changes. We have filled in these gaps with additional triangles while maintaining the Barr et al. (4) subdivisions, and redid the solution using exact solid angles subtended by the plane areas. Excellent results were obtained, considering the coarseness of the approximation to the surface. The maximum deviation from the theoretical results were within 5% in most cases.

To calculate the solid angle subtended at the ith centroid by subarea j, a formula has been given by Barnard et al. (9), the derivation of which is not referenced. We had pursued this problem independently and derived a somewhat simpler formula than theirs. Both, however, are based on the same elementary textbook theorem of solid geometry which states that:

$$\Omega = \beta_1 + \beta_2 + \beta_3 - \pi \tag{5}$$

where Ω is the solid angle of a spherical triangle, and β_1, β_2, and β_3 are the internal spherical angles.

Our derivation takes the following form. Given three vertices, A, B, and C, of a plane triangle and point O, the point from which the solid angle is to be calculated, assume a sphere centered at O and passing through B. Let the projections of the lines AB and CB on the sphere be A'C and C'B, the latter forming angle β_1 of Eq. 5 on the sphere surface at point B. The same angle is formed by the straight tangent lines at B pointing in the directions A'B and C'B. Since the angle between the tangents is the same as the angle between the planes formed by OAB and OCB with normals $\underset{\sim}{N}_1$ and $\underset{\sim}{N}_2$ or, equivalently, between the normals $\underset{\sim}{N}_1$ and $\underset{\sim}{N}_2$ themselves, $\underset{\sim}{N}_1$ and $\underset{\sim}{N}_2$ can be found from the cross product operations $\underset{\sim}{BO} \times \underset{\sim}{AB}$ and $\underset{\sim}{BO} \times \underset{\sim}{CB}$ respectively. The desired internal angle is then found by vector algebra as

$$\beta = \cos^{-1}\left(\frac{\underset{\sim}{N}_1 \cdot \underset{\sim}{N}_2}{|\underset{\sim}{N}_1|\,|\underset{\sim}{N}_2|}\right) \tag{6}$$

This calculation, which is seen to be somewhat simpler than that of Barnard et al. (9), must be repeated for each internal angle. The sum of these minus is always positive. The sense of the outward pointing solid angle is found by taking the sign of the result of an inner product of an outward normal, $\underset{\sim}{N}$, with a vector OA or OB or OC. In most cases $\underset{\sim}{N}$ must be found by a systematic cross-product operation on the triangle sides. Hence the subtended solid angle equals $(\beta_1 + \beta_2 + \beta_3 - \pi) \times S$ where S is the sign and β_1, β_2, and β_3 are the three internal angles. To take care of the sign, Barnard et al. (9) suggest finding the unit normal at the centroid, taking its inner product with a vector along the radius from O to the centroid of the triangle, normalizing the product to a magnitude of unity, and multiplying the solid angle by this computed +/- 1. Again, this requires more computation than does our proceedure since the localization of the centroid, the generation of the vector to the centroid from O, and the two normalization operations are unnecessary.

There are many considerations affecting the choice of coefficients in making a forward calculation. Gelernter and Swihart used Coulomb's Law for point charges as the off-diagnonal terms, these are singular at zero distance and are inaccurate generally at close spacing, such as might appear in a parallel plate capacitor problem. This situation was examined in detail by Rush et al. (10) who first published the non-singular integral form expressions for the coefficients. These are, in a sense, analogous to the solid angle coefficients of Barr et al.

Still further precision might be anticipated if the integral coefficients from the centroids were averaged over the triangle. Tests of this thesis were carried out by Rush et al. (10) who calculated that integral coefficients tended to be less accurate than the point-form Coulomb Law equation in many practical cases whereas the averaging scheme was somewhat better. Barnard et al. (11) pointed out that the average integral coefficients for the two methods, with appropriate choice of form, led to alpha matrices which were transposes of each other.

In the interest of finding average coefficients for triangles of arbitrary size and shape, we sought triangle subdivision algorithms which could be conveniently programmed and with areas easily found. This search has led us to derive a simple theorem of plane geometry for the purpose. From the three sets of vertex coordinates of a triangle, the centroid is found by averaging each x, y and z coordinate over the three vertices. Geometrically, the centroid is obtained as the intersection of the lines connecting the vertices to the mid-points of the opposite sides. Lines connecting these midpoints to each other divide the original triangle into four triangles of equal area, one fourth the original. It is a simple matter to repeat the process for each of the four triangles to get sixteen of equal area and so on. This process leads systematically to triangular subdivisions of equal area, in powers of four, which exactly tesselate the boundaries of the original triangle. This scheme might be used, alternatively, to perform both the integration and averaging numerically, thereby avoiding the solid angle calculations in favor of repeated use of the point-form formulas. Evaluations of this proceedure and its resulting coefficients in comparison with other methods discussed have not yet been made.

Comparisons between the charge and potential methods have not yet considered the following significant point. The former method first calculates charge densities or charges on the triangular subdivisions in the process of satisfying boundary

conditions. The potentials are then obtained in a second step using the Coulomb Law potential formula. While this appears to be a disadvantage with respect to the latter approach, it is a more general and realistic solution for the torso potentials. Specifically, we have found it useful to interpolate potentials between centroids using Coulomb's Law, rather than relying on purely mathematical interpolations from the centroid potentials. This method gives continuous potentials over the surface by direct calculation, as compared to the discrete potentials at the centroids found by the Green's Theorem approach of Barr et al. (4). The continuous potentials, however, are still an approximation since the charge densities are assumed constant on each subarea. The entire analysis and testing of surface potentials at non-centroid points remains to be accomplished, so far as we are aware.

A further consideration with respect to the potential calculation is the combination of the Coulomb's Law interpolation technique with the "Source Independent" transfer coefficients. In general the potential for the charge formulation is given by:

$$\vec{\Phi}_e = [\beta]\, q_s + \vec{\Phi}_i \qquad (7)$$

were $\vec{\Phi}_e$ is the potential at a given set of electrodes, $[\beta]$ is the Coulomb's Law coefficient matrix, $\vec{q}_s$ is the vector of boundary charges found via the charge technique, and $\vec{\Phi}_i$ is a vector of the infinite medium potentials at the electrodes due to the sources in question.

If we let the response terms, $\vec{R}$, in Eq. 4 become $\vec{q}_s$, and the forcing terms, $\vec{F}$, become q_o, we can rewrite Eq. 4 for the charge formulation. Note that q_o is equivalent to the the impressed field due to the sources. Eq. (4) becomes:

$$\vec{q}_s = [\alpha_{ij}]^{-1} \vec{q}_o \qquad (8)$$

If we substitute this into Eq. 7, we get:

$$\vec{\Phi}_e = [\beta]\, [\alpha_{ij}]^{-1} \vec{q}_o + \vec{\Phi}_i \qquad (9)$$

and let:

$$[\gamma] = [\beta]\, [\alpha_{ij}]^{-1} \qquad (10)$$

then:

$$\vec{\Phi}_e = [\gamma]\, \vec{q}_o + \vec{\Phi}_i \qquad (11)$$

It should be noted that the $[\gamma]$ matrix can be precalculated and is independent of the source term. Further, only the actual number of electrodes on which the potentials are desired need be considered. That is, $[\gamma]$ is a matrix of dimension N X M were N is the number of electrodes and M is the number of surface subdivisions. Thus, the final matrix multiplication step in the Source Independent method typically involves a matrix multiplication significantly reduced in size from that implied by the dimensions of Eq. (1).

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