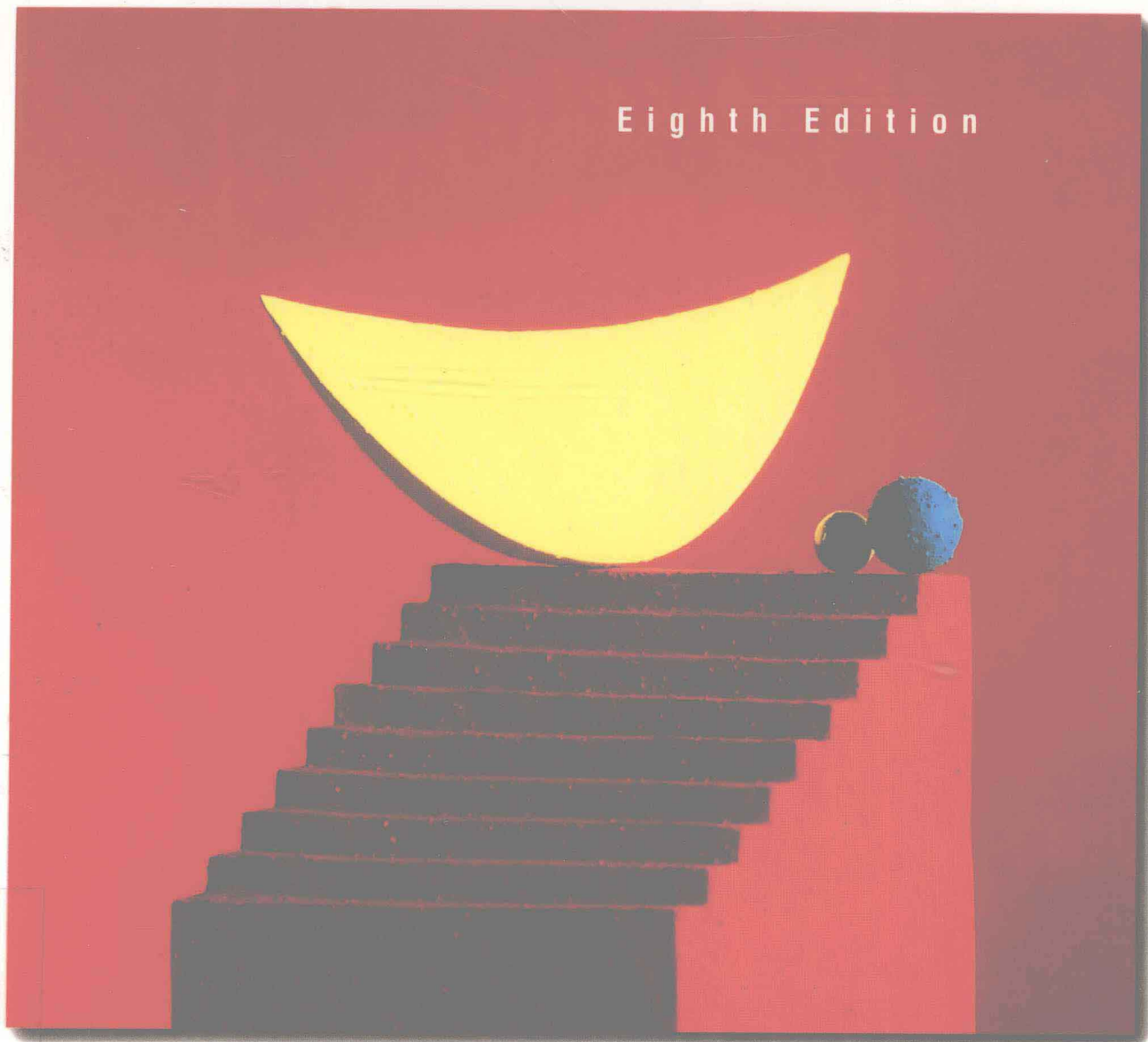


Calculus

for Business, Economics, Life Sciences, and Social Sciences

Eighth Edition



Raymond A. Barnett / Michael R. Ziegler / Karl E. Byleen



CALCULUS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES,
AND SOCIAL SCIENCES

EIGHTH EDITION

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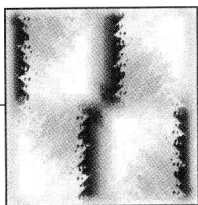
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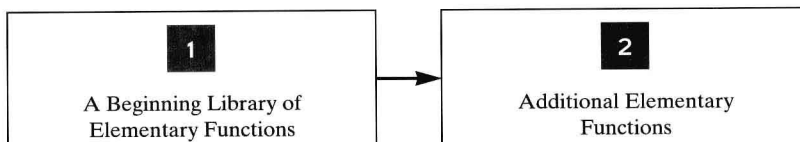
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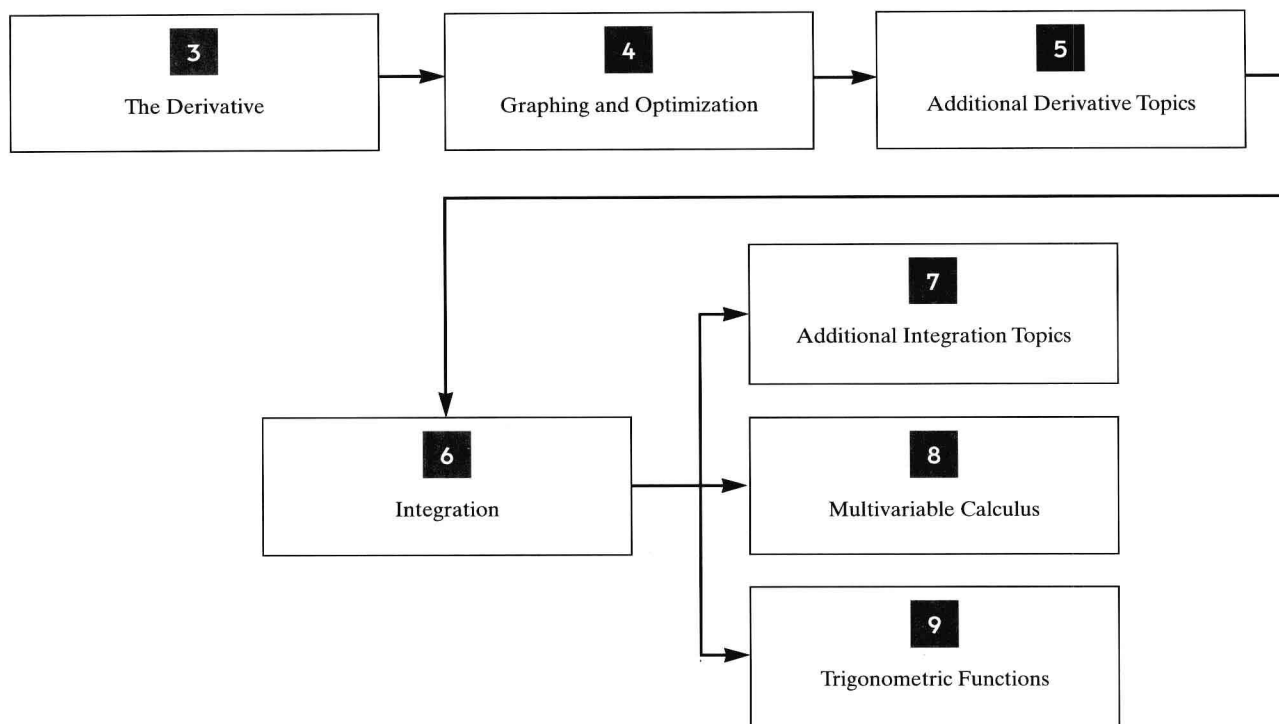


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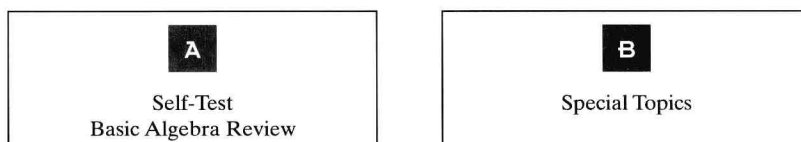
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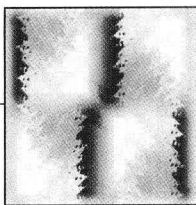
PART TWO CALCULUS



APPENDIXES



*Selected topics from Part One may be referred to as needed in Part Two or reviewed systematically before starting Part Two.



PREFACE

The eighth edition of *Calculus for Business, Economics, Life Sciences, and Social Sciences* is designed for a one-term course in calculus and for students who have had $1\frac{1}{2}$ –2 years of high school algebra or the equivalent. The choice and independence of topics make the text readily adaptable to a variety of courses (see the Chapter Dependency Chart on the preceding page). It is one of eight books in the authors' college mathematics series.

Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. Fundamental to a book's growth and effectiveness is classroom use and feedback. Now in its eighth edition, *Calculus for Business, Economics, Life Sciences, and Social Sciences* has had the benefit of having a substantial amount of both.

■ EMPHASIS AND STYLE

The text is **written for student comprehension**. Great care has been taken to write a book that is mathematically correct and accessible to students. Emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. Most derivations and proofs are omitted except where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

■ EXAMPLES AND MATCHED PROBLEMS

Over 290 completely worked examples are used to introduce concepts and to demonstrate problem-solving techniques. Many examples have multiple parts, significantly increasing the total number of worked examples. Each example is followed by a similar **matched problem for the student to work** while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

■ EXPLORATION AND DISCUSSION


Every section contains **Explore–Discuss** problems interspersed at appropriate places to encourage the student to think about a relationship or process before a result is stated, or to investigate additional consequences of a development in the text. **Verbalization** of mathematical concepts, results, and processes is encouraged in these Explore–Discuss problems, as well as in some matched problems, and in some problems in almost every exercise set. The Explore–Discuss material also can be used as in-class or out-of-class **group activities**. In addition, at the end of every chapter, we have included two special **chapter group activities** that involve several of the concepts discussed in the chapter. Prob-

blems in the exercise sets that require verbalization are indicated by color problem numbers.

■ EXERCISE SETS



The book contains over 4,000 problems. Many problems have multiple parts, significantly increasing the total number of problems. Each exercise set is designed so that an average or below-average student will experience success and a very capable student will be challenged. Exercise sets are mostly divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) levels.

■ APPLICATIONS

A major objective of this book is to give the student substantial experience in **modeling and solving real-world problems**. Enough applications are included to convince even the most skeptical student that mathematics is really useful (see the Applications Index inside the back cover). Worked examples involving applications are identified by . **Almost every exercise set contains**

application problems, usually divided into business and economics, life science, and social science groupings. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems taken from professional journals and books. No specialized experience is required to solve any of the applications.

■ TECHNOLOGY

The generic term **graphing utility** is used to refer to any of the various graphing calculators or computer software packages that might be available to a student using this book. (See the description of the software accompanying this book later in this Preface.) Although **access to a graphing utility is not assumed**, it is likely that many students will want to make use of one of these devices. To assist these students, **optional graphing utility activities** are included in appropriate places in the book. These include brief discussions in the text, examples or portions of examples solved on a graphing utility, problems for the student to solve, and a **group activity that involves the use of technology** at the end of each chapter. Beginning with the group activity at the end of Chapter 1, and continuing throughout the text, **linear regression** on a graphing utility is used at appropriate points to illustrate **mathematical modeling with real data**. All the optional graphing utility material is clearly identified by either  or  and can be omitted without loss of continuity, if desired.

■ GRAPHS

All graphs are computer-generated to ensure mathematical accuracy. Graphing utility screens displayed in the text are actual output from a graphing calculator.

■ ADDITIONAL PEDAGOGICAL FEATURES

Annotation of examples and developments, in small color type, is found throughout the text to help students through critical stages (see Sections 1-1 and 3-2). **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-1 and 3-4). **Boxes** are used to highlight important definitions, results, and step-by-step processes (see Sections 1-1 and 3-2). **Caution** statements appear throughout the text where student errors often occur (see Sections 3-2 and 4-1). **Functional use of color** improves the clarity of many illustrations, graphs, and developments, and guides students through certain critical steps (see Sections 1-1 and 3-2). **Boldface type** is used to introduce new terms and highlight important comments. **Chapter review** sections include a review of all important terms and symbols and a comprehensive review exercise. **Answers to most review exercises**, keyed to appropriate sections, are included in the back of the book. Answers to all other odd-numbered problems are also in the back of the book.

■ CONTENT

The text begins with the development of a library of elementary functions in Chapters 1 and 2, including their properties and uses. We encourage students to investigate mathematical ideas and processes **graphically** and **numerically**, as well as **algebraically**. This development lays a firm foundation for studying mathematics both in this book and in future endeavors. Depending on the syllabus for the course and the background of the students, some or all of this material can be covered at the beginning of a course, or selected portions can be referred to as needed later in the course.

The material in Part Two (Calculus) consists of **differential calculus** (Chapters 3–5), **integral calculus** (Chapters 6–7), **multivariable calculus** (Chapter 8), and a brief discussion of differentiation and integration of **trigonometric functions** (Chapter 9). In general, Chapters 3–6 must be covered in sequence; however, certain sections can be omitted or given brief treatments, as pointed out in the discussion that follows (see the Chapter Dependency Chart on page viii).

Chapter 3 introduces the **derivative**, covers the **limit properties** essential to understanding the definition of the derivative, develops the **rules of differentiation** (including the chain rule for power forms), and introduces **applications** of derivatives in business and economics. The interplay between graphical, numerical, and algebraic concepts is emphasized here and throughout the text.

Chapter 4 focuses on **graphing** and **optimization**. The first three sections cover continuity and first-derivative and second-derivative graph properties, while emphasizing **polynomial graphing**. **Rational function** graphing is covered in Section 4-4. In a course that does not include graphing rational functions, this section can be omitted or given a brief treatment. Optimization is covered in Section 4-5, including examples and problems involving end-point solutions.

The first three sections of Chapter 5 extend the derivative concepts discussed in Chapters 3 and 4 to **exponential and logarithmic functions** (including the general form of the chain rule). This material is required for all the remaining chapters. **Implicit differentiation** is introduced in Section 5-4 and applied to **related rate problems** in Section 5-5. These topics are not referred to elsewhere in the text and can be omitted.

Chapter 6 introduces **integration**. The first two sections cover **antidifferentiation** techniques essential to the remainder of the text. Section 6-3 discusses some applications involving **differential equations** that can be omitted. Sections 6-4 and 6-5 discuss the **definite integral** in terms of **Riemann sums**, including **approximations** with various types of sums and some **simple error estimation**. As before, the interplay between the graphical, numeric, and algebraic properties is emphasized. These two sections also are required for the remaining chapters in the text.

Chapter 7 covers **additional integration topics** and is organized to provide maximum flexibility for the instructor. The first section extends the **area** concepts introduced in Chapter 6 to the area between two curves and related applications. Section 7-2 covers three more **applications** of integration, and Sections 7-3 and 7-4 deal with additional **techniques of integration**. Any or all of the topics in Chapter 7 can be omitted.

The first five sections of Chapter 8 deal with **differential multivariable calculus** and can be covered any time after Section 5-3 has been completed. Section 8-6 requires the **integration** concepts discussed in Chapter 6.

Chapter 9 provides brief coverage of **trigonometric functions** that can be incorporated into the course, if desired. Section 9-1 provides a review of basic trigonometric concepts. Section 9-2 can be covered any time after Section 5-3 has been completed. Section 9-3 requires the material in Chapter 6.

Appendix A contains a **self-test** and a **concise review of basic algebra** that also may be covered as part of the course or referred to as needed. As mentioned above, Appendix B contains additional topics that can be covered in conjunction with certain sections in the text, if desired.

■ SUPPLEMENTS FOR THE STUDENT

1. **A Student Solutions Manual and Visual Calculus** by Garret J. Etgen and David Schneider is available through your book store. The manual includes detailed solutions to all odd-numbered problems and all review exercises. *Visual Calculus* by David Schneider contains over twenty routines that provide additional insight into the topics discussed in the text. Although this software has much of the computing power of standard calculus software packages, it is primarily a teaching tool that focuses on understanding mathematical concepts, rather than on computing. These routines incorporate graphics whenever possible to illustrate topics such as secant lines; tangent lines; velocity; optimization; the relationship between the graphs of f, f', f'' ; and the various approaches to approximating definite integrals. All the routines in this software package are menu-driven and very easy to use. The software will run on DOS or Windows platforms.
2. **A Graphing Calculator Manual** by Carolyn L. Meitler contains examples illustrating the use of graphics calculators to solve problems similar to those discussed in the text. The manual follows the chapter organization of the text, making it easy to find examples in the manual illustrating appropriate calculator solution methods for problems in the text. The manual includes keystrokes for the TI-82, TI-83, TI-85, and TI-86 calculators. However, the examples and techniques can be used with any graphing utility.

3. The **PH Companion Website**, designed to complement and expand upon the text, offers a variety of teaching and learning tools, including links to related websites, practice work for students, and the ability for instructors to monitor and evaluate students' work on the website. For more information, contact your local Prentice Hall representative.
www.prenhall.com/barnett

■ SUPPLEMENTS FOR THE INSTRUCTOR

For a summary of all available supplementary materials and detailed information regarding examination copy requests and orders, see page xvi.

1. **PH Custom Test, a menu-driven random test system** for either Windows or Macintosh is available to instructors. The test system has been greatly expanded and now offers **on-line testing**. Carefully constructed algorithms use random-number generators to produce different, yet equivalent, versions of each of these problems. In addition, the system incorporates a unique **editing function** that allows the instructor to create additional problems, or alter any of the existing problems in the test, using a full set of mathematical notation. The test system offers **free-response, multiple-choice, and mixed exams**. An almost unlimited number of quizzes, review exercises, chapter tests, midterms, and final examinations, each different from the other, can be generated quickly and easily. At the same time, the system will produce answer keys, student worksheets, and a gradebook for the instructor, if desired.
2. A **Test Item File**, prepared by Laurel Technical Services, provides a hard copy of the test items available in PH Custom Test.
3. An **Instructor's Resource Manual** provides over 100 transparency masters and all the answers not included in the text. This manual is available to instructors without charge.
4. A **Student Solutions Manual and Visual Calculus** by Garret J. Etgen and David Schneider (see Supplements for the Student) is available to instructors.
5. A **Graphing Calculator Manual** by Carolyn L. Meitler (see Supplements for the Student) is available to instructors. The manual contains all the necessary information for a student with no previous experience with a graphing calculator, eliminating the need for the instructor to prepare materials related to calculator usage. In particular, separate appendixes for the TI-82, TI-83, TI-85, and TI-86 graphing calculators contain detailed instructions, including calculator-specific keystrokes, for performing the various operations required to effectively use each of these calculators to solve problems in the text. Furthermore, the methods illustrated for these calculators are easily adapted to other graphing utilities. The manual is very effective both for a class where all students purchase the same calculator and in a setting where students are using a variety of different calculators—an important consideration as more and more students arrive at college having already purchased a graphing calculator.

6. The **PH Companion Website**, designed to complement and expand upon the text, offers a variety of interactive teaching and learning tools, including links to related websites, practice work for students, and the ability for instructors to monitor and evaluate students' work on the website. For more information, contact your local Prentice Hall representative.
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■ ERROR CHECK

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error-free. For any errors remaining, the authors would be grateful if they were sent to: Michael R. Ziegler, 509 W. Dean Court, Fox Point, WI 53217; or, by e-mail, to: michaelziegler@execpc.com

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R. A. Barnett

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K. E. Byleen

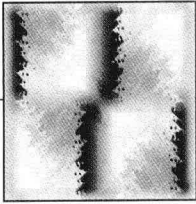
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CALCULUS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES,
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PART ONE

A LIBRARY OF ELEMENTARY FUNCTIONS

