



*A Publication of the  
International Society on General Relativity and Gravitation*

# General Relativity and Gravitation

*One Hundred Years After  
the Birth of Albert Einstein*

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## Volume 1

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Edited by

**A. Held**

*Institute of Theoretical Physics  
University of Berne  
Berne, Switzerland*

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**A. Held**

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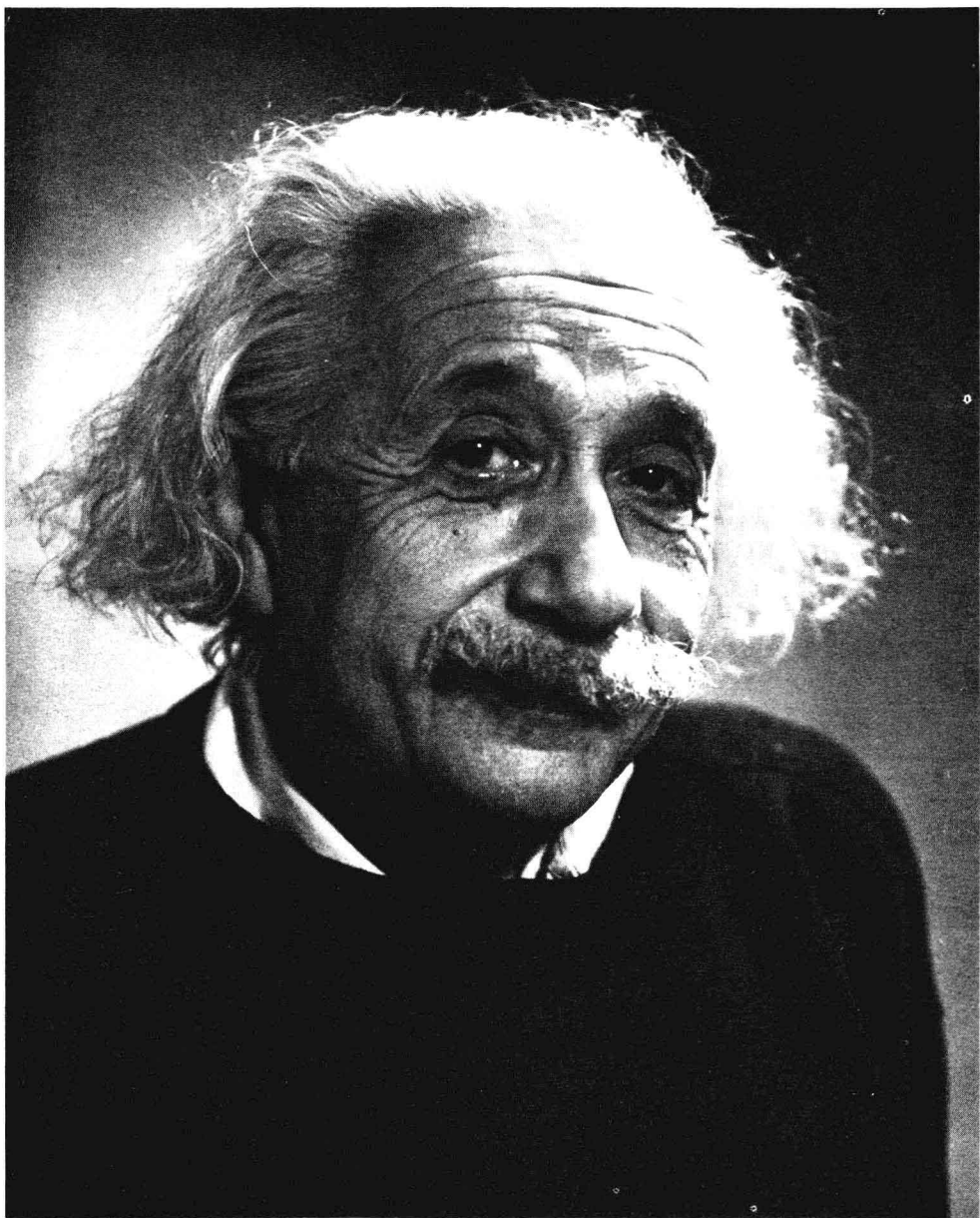
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# General Relativity and Gravitation

*One Hundred Years After  
the Birth of Albert Einstein*

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Volume 1



ALBERT EINSTEIN

# Contributors

- S. Benenti**, Institute of Mathematical Physics, University of Torino, Torino, Italy  
**P. Bergmann**, Department of Physics, Syracuse University, Syracuse, New York  
**D. Brill**, Department of Physics and Astronomy, University of Maryland, College Park, Maryland  
**Y. Choquet-Bruhat**, Department of Mechanics, University of Paris, VI, Paris, France  
**P. C. W. Davies**, Department of Mathematics, King's College, London, England  
**S. Deser**, Department of Physics, Brandeis University, Waltham, Massachusetts  
**R. d'Inverno**, Faculty of Mathematical Studies, University of Southampton, Southampton, England  
**S. Ferrara**, Laboratori Nazionali di Frascati, Frascati, Italy  
**M. Francaviglia**, Institute of Mathematical Physics, University of Torino, Torino, Italy  
**H. F. Goenner**, Institute for Theoretical Physics, University of Göttingen, Göttingen, Germany  
**J. N. Goldberg**, Department of Physics, Syracuse University, Syracuse, New York  
**F. W. Hehl**, Institute for Theoretical Physics, University of Cologne, Cologne, Germany  
**H. Hora**, Department of Theoretical Physics, University of New South Wales, Kensington-Sydney, Australia  
**J. Isenberg**, Applied Mathematics Department, University of Waterloo, Waterloo, Ontario, Canada  
**P. S. Jang**, Theoretical Physics Institute, University of Alberta, Edmonton, Alberta, Canada  
**A. Komar**, Department of Physics, Yeshiva University, New York, New York  
**Y. Ne'eman**, Department of Physics and Astronomy, Tel Aviv University, Ramat Aviv, Israel  
**J. Nester**, Department of Physics, University of Saskatchewan, Saskatoon, Saskatchewan, Canada  
**J. Nitsch**, Institute for Physics, University of Cologne, Cologne, Germany  
**J. Stachel**, Einstein Project, Institute for Advanced Study, Princeton, New Jersey  
**A. H. Taub**, Department of Mathematics, University of California, Berkeley, California

- C. Teitelboim**, Institute for Advanced Study, Princeton, New Jersey
- A. Trautman**, Institute of Theoretical Physics, Warsaw University, Warsaw, Poland
- P. van Nieuwenhuizen**, Institute for Theoretical Physics, State University of New York, Stony Brook, New York
- P. Von der Heyde**, Institute for Theoretical Physics, University of Cologne, Cologne, Germany
- J. W. York, Jr.**, Department of Physics and Astronomy, University of North Carolina at Chapel Hill, North Carolina

# Foreword

The year 1979 is the one-hundredth anniversary of Albert Einstein's birth. The International Society for General Relativity and Gravitation, which has yet to celebrate the tenth anniversary of its birth, has decided to mark the Einstein Centenary by offering to the community of scientists this collection of articles.

Albert Einstein was a towering personality in theoretical physics. The general theory of relativity was perhaps his greatest achievement, and it is certainly the one that most people associate with his name. Einstein also was a philosopher and a social activist who hated passionately war and every form of violence. Our organization was founded as an international society because the modest number of relativists throughout the world made a single organization the most rational way for promoting our common aims, as well as to emphasize the international nature of the search for scientific knowledge and, by our common efforts, perhaps to make some small contribution toward the cooperation and friendship of all peoples. We believe that we are acting in the spirit of the man whom we are honoring by dedicating these volumes to his memory.

Peter G. Bergmann  
*President*



# Preface

Together with the decision to produce these volumes came the realization that one cannot hope to cover adequately all of the significant activities in general relativity even in two books devoted primarily to review articles. Quantitatively the research now covers too wide a front for that; qualitatively no one individual, even when writing on a specific topic, can hope to do justice to all the different attitudes toward the subject that exist within the community of relativists. The editors of these volumes have tried to present a work that represents a good number of the facets of the field and to some extent we feel that we have succeeded although there do remain obvious gaps.

Without doubt the problem considered most pressing by the majority of today's relativists is the apparent incompatibility between the two major physical theories of the twentieth century, namely relativity and quantum theory. As a reflection of this concern these volumes offer a number of articles dealing with quantization and general relativity. Within this field there are several approaches. The oldest of these uses the canonical approach to relativity, a topic to which a large portion of Volume 1 is devoted. In addition to the article on quantization, other problems that are discussed using this formalism are the Cauchy problem, the positive mass conjecture, and the Hamiltonian structure of space-time. The necessary background is covered in an extensive review article.

Volume 1 also contains several articles expounding the gauge theory and the supergravity outlook. The more exotic approach through the use of twistors is described in Volume 2.

In recent years much work has also been devoted to the question, "What is the effect of a curved space-time on the quantum theory as we know it?" Investigations of this question have led to the discovery of such effects as black hole evaporation and to possible explanations of processes prevalent in the initial stages of the universe, thus providing further impetus for its study. We provide an introductory article to the subject in Volume 1.

Since the early 1960s, when conformal structures were introduced into the study of asymptotic properties, a number of important related themes

have been studied. These studies form the basis of the first part of Volume 2, beginning with analyses of null and spatial infinities (as explained in three different articles). These investigations lead naturally to the use of complexified space-times as a mathematical device. How this is done is explained in an article on complex variables in relativity. The application of complex variables to  $\mathfrak{S}$  and  $\mathfrak{S}\mathfrak{S}$  spaces is given in a further article.

Also dating from the sixties is the interest in singularities, whether and how they arise, together with their classification. Volume 2 contains a major review of this aspect of relativity theory.

Among the more important recent developments have been the astounding improvements in modern computers. It is no longer a question of computers merely being used for numerical integrations. They can now be applied constructively to the more elegant aspects of relativity. Details appear in an article which relates what has been done, explains different computing systems, and gives an intriguing hint of the future.

No survey of the fields would be complete without some review of the less spectacular but nonetheless necessary background mathematical work which still goes on and forms a necessary part of the repertoire of every relativist. To this end Volume 1 contains articles on fiber bundles, the problem of embedding, the separability of the Hamilton-Jacobi equation, conserved quantities, and the method of two-timing.

In this maze of beautiful mathematics and theories, one must not lose sight of the fact that Einstein, like every good physicist, was concerned primarily with the real world. Our work regains contact with the cosmos in Volume 2 with articles on cosmology and on gravitational collapse. Finally it comes down to earth in the form of discussions on the detection of gravitational waves, experimental tests of relativity, and statistical mechanics.

Included in Volume 1 are two articles with an historical flavor. Insight into the development of the general theory is given by the first, which traces the interaction between Einstein and the rotating disk. The second reports on Einstein's contribution to the theory of the laser. Who knows, perhaps if the technology had been available . . .

We hope that this two-volume work will serve as a quasi-textbook with something in it of benefit to the student launching his studies in this perhaps most fascinating field of modern physics as well as to the experienced researcher, who might just obtain from it a spark of inspiration to lead him in a new direction.

The Editors

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