MILDRED D. JOHNSON

PROBLEM SOLVING AND CHEMICAL CALCULATIONS

MILDRED D. JOHNSON
City College of San Francisco

PROBLEM SOLVING AND CHEMICAL CALCULATIONS

HARCOURT, BRACE & WORLD, INC.
New York / Chicago / San Francisco / Atlanta

Library of Congress Catalog Card Number: 75-87232

Printed in the United States of America

PROBLEM SOLVING AND CHEMICAL CALCULATIONS

PREFACE

This book is really two books in one—a book on problem solving and a book on chemical calculations. It was written for the student who knows arithmetic operations but has difficulty setting up a problem for solution. The first half of the book deals with problem solving in this sense, the second half with chemistry problems.

Very simple problems are taken first—problems that "anyone" can do—to focus attention on the various methods and techniques. Also, a number of unusual and even absurd problems appear in the beginning so that the student is forced to examine and use the methods presented. Problems occur in increasing order of difficulty throughout the book and are set up in logical, step-by-step fashion, usually with just one new idea presented at a time. The text constantly emphasizes analysis of the problem, stresses the use of units and dimensional analysis, and encourages problem solving by analogy wherever possible. It also encourages final synthesis of the problem into a single setup incorporating the various steps in logical order. Questions, often rhetorical in nature, appear frequently to encourage the student to develop the habit of asking himself questions, a vital factor in successful problem solving. Once a student knows how to solve problems in general he usually needs just facts and patience to solve specific problems.

In the second half of the book the major categories of elementary chemistry problems are set up according to the logical methods of attack presented earlier. The mole method is used wherever appropriate throughout the chemistry section. There is also a consistent emphasis on significant figures beginning with the chapter on significant figures.

The book has a few topics that might not be expected: a chapter on conversion of word problems into symbols and equation form, a chapter on graphing that should be useful for students in experimental classes, and a chapter on physics problems that offers many opportunities to practice problem solving and contains information particularly helpful to chemistry students who have not had physics courses. Finally, because the book is intended not only for use in specific courses but also for independent study, it includes, in addition to numerous solved examples of graded difficulty, an extensive section at the end of the book giving setups and/or answers for all problems.

The author would like to thank the students who used this material in preliminary form. She would also like to express her appreciation to her colleagues who graciously used the material in their classes: Dr. Frances Connick, Miss Mayme Fung, Mr. Stanley Furuta, Mr. William Hoskins, Mrs. Isabel Hurd, Mrs. Shirley Kelly, Dr. Alfred Lee, Mr. Wayne Matthews, Dr. William McInerny, Mr. Eugene Roberts, Mrs. Anne Thacher, Mr. Kenneth Thunem, and Dr. William Tsatsos. She also thanks Dr. Clement Skrabak for permission to use this material in classes, and Drs. John Booher and Mannfred Mueller for advice and information generously given over the years.

MILDRED D. JOHNSON

CONTENTS

1

INTRODUCTION

Well-trained, intelligent students often have difficulty solving problems. Too many students are unable to understand how or why they have "solved" problems. They know—or can find out—how to do a problem once it is set up, but they have difficulty setting up the problem. *This setting up of the problem is what we mean by problem solving.*

There is an almost infinite number of ways to approach problem solving, perhaps because the human mind is such a marvelously complex thing. No one can really teach anyone else how to solve problems. The best that anyone can do is to present a series of approaches, to arrange situations so that the mind itself discovers how to solve problems.

Problem solving could be likened to diamond cutting. There is more than one way to cut a diamond, but no matter which way is chosen the cutter must have a knowledge of crystal structure. Presumably he studies the particular diamond, decides how to cut it, and cuts it. Problems, unlike diamonds, can be cut and recut over and over, but still one must know the basic techniques of problem solving and must look for the facets of a particular problem—must *analyze* the problem, in other words—in order to solve it, regardless of method.

Naturally, not all problems are solved in the same manner, nor do all people find various methods equally congenial. We hope to help you find methods that work quickly and effectively for you, but we hope especially to help you find an *understanding* of several methods that can be used. Only with understanding can the methods be applied skillfully and surely in a variety of problems. We hope to put difficulty with problems where it belongs, in the particular set of circumstances in a particular problem. We hope to leave you with just the difficulty of the subject matter of a particular problem, not with a difficulty about how to put known and understood subject matter into workable setups.

You will be presented with a number of methods that are different from one another. You will also come in contact with what might be called the "percolation" and "hit and run" approach to problem solving. In this approach simple and apparently scattered items are presented at various times to seep into the mind, to provide the mind with a particular background, to give the mind something to work on. Please remember this when we seem to be presenting almost random items in a variety of ways.

Do not be repelled by the simplicity of many of the problems. Where possible, problems that anyone could solve have been chosen so that attention can be focused on the techniques involved. We have introduced a number of unreal and occasionally even absurd problems to encourage use of the methods being presented. Such problems usually cannot be solved by memorized methods; they yield only to methods that are understood by the problem solver.

In various books there are all manner of lists of how to solve problems. They are usually not very helpful unless you already know how to solve problems. Problem solving becomes almost intuitive after a while. However, a list of things to do in solving a problem may serve as a starting point.

1. Read the problem carefully and ask yourself exactly what you are looking for.
2. Make a list of things given in the problem.
3. Where suitable, make a drawing or diagram to help the physical reality of the problem and the phenomena involved seep below the surface of your mind.
4. Look up—in textbooks, in technical dictionaries, in dictionaries—definitions, descriptions, discussions, etc., of the factors you are dealing with. You may not have the information you need. You may have excess, irrelevant information. In fact, prior to searching suitable references you may not even know that you need certain information or that certain other information is irrelevant.
5. Look for relationships between the things you are looking for and the known factors in the problem.
6. Look for the relationships between or among the factors in the problem.

7. Look for a common factor to which both the thing you are looking for and the factors presented in the problem are related.
8. Note the nature of the relationships—direct, inverse, squares, cube roots, etc.
9. Look for limits. Something you have in the problem may be related to what you are seeking, but you may not have the information in the range or area in which you are working.
10. Be sure to attach all units to all figures. If the units do not cancel out to the proper unit for the answer, you automatically know that you have made an error. Often a glance at the units in such a case will reveal where the error was made.
11. If you still cannot solve the problem, then
    a. Look for reasonable assumptions on which to base a solution. Carried to its logical extreme this would make any problem solvable, so one has to be careful about this and state precisely what the assumption is, making sure that one has a good reason for making the assumption.
    b. Reword the problem. Put it into simple, clear English. Get rid of all unnecessary words.
    c. Dismantle the problem. Make up little problems with parts of the data. This is a valid way to help your mind find or connect factors whose connection previously eluded it.
    d. Imitate. Look at solved problems involving some or all of the data involved in the problem. This is a valid tool *if* used in an attempt to understand how and why a particular problem was solved. If one merely copies a setup into which to put numbers, then it is utterly useless as a tool for learning problem solving.

If these techniques do not help, you may have a problem that cannot be solved. It is obviously very important to know that a particular problem cannot be solved, but it is often difficult to distinguish an as yet unsolved problem from one which cannot be solved. The techniques in 4, 5, 6, and 7 are often helpful in making such a distinction.

Some minds seem to discover how to solve problems almost instantly and even intuitively, others only after full-scale, agonizing war. What matters is that one find how to solve problems. It is worth a great deal of time and effort. How often have you gone round and round with a problem until something flashed? Later you could scarcely believe that you didn't see how to do the problem immediately. We hope to decrease the round and round time almost to zero and increase the frequency of flashes to as close to infinity as possible.

2

PROBLEM SOLVING BY THE SIMPLEST METHOD POSSIBLE

Comparison and Correction

Many people find the method of comparison and correction uniquely usable and valuable in problem solving. This method can generally be used when a known factor, called a *base factor,* is changed to a new, unknown value as a result of a known change in another factor. The two values of this other factor constitute the *correcting fraction* or *correcting factor.* The trick is to find the base factor, to find what affects the base factor, and to find the nature of the relationship between the base factor and the factor whose change affects it.

Be sure not to confuse comparison and correction with proportion. Proportion is really a sophisticated and often treacherous method; other methods should be learned first.

BASIC PRINCIPLES OF COMPARISON AND CORRECTION

Problem solving by comparison and correction is best explained by illustration. The following examples are therefore presented in considerable detail.

Direct Relationship, One Variable Factor

A *direct relationship* is one in which an increase in one factor causes an increase in another factor or a decrease in one factor causes a corresponding decrease in another factor.

EXAMPLE 2–1

Five children can eat 10 quarts (qt) of ice cream in a given time. How much ice cream could 20 children eat in the same time?

1. What are you looking for? A quantity of ice cream.
2. Then what must you operate on mathematically? A quantity of ice cream.
3. What is your *base factor?* A quantity of ice cream.
4. What affects the quantity of ice cream? The *changing* number of children. (Time is a constant factor.)
5. What is the nature of the relationship between quantity of ice cream and number of children? A direct relationship. More children, all other things being equal, can eat more ice cream.
6. What is the direction of the change? An increase.
7. What is the effect of the change here? An increase in quantity of ice cream.
8. What will the *correcting fraction* be composed of? The values of the factor whose change causes a change in the base factor, in this case the different numbers of children, that is, 5 and 20.
9. What will the actual correcting fraction be? You know you need an increase; therefore the fraction must be 20/5 rather than 5/20.
10. What is the setup for this problem?

Base factor × correcting fraction: $\frac{10 \text{ qt} \times 20 \text{ children}}{5 \text{ children}} = 40 \text{ qt}$

Note that a unit is attached to each figure. The units cancel out to give quarts, a not exactly startling fact. Form the habit of attaching units to all figures in problems of this sort.

Direct Relationship, Two or More Variables

If changes in two or more different factors affect the base factor, it is absolutely necessary to consider the effect of the change in each factor *on the base factor.* The approach is the same as before. Consider the following example.

EXAMPLE 2–2

Five children can eat 10 qt of ice cream in 1 day. How much ice cream could 17 children eat in 9 days?

1. Factor sought: quantity of ice cream.
2. Base factor: quantity of ice cream.
3. Factors affecting quantity of ice cream:

Correcting factor	*Nature of relationship*	*Direction of change*	*Effect on base factor*
number of children	direct	increase	increase
number of days	direct	increase	increase

4. Setup:

$$\frac{10\text{ qt} \times 17\text{ children} \times 9\text{ days}}{5\text{ children} \times 1\text{ day}} = 306\text{ qt}$$

Remember to consider the effect of each variable on the base factor. The setup used here is a *single-line setup*. There is no point in first setting up a problem to find the amount of ice cream needed for more children and then doing another problem with the new quantity of ice cream to find out how much ice cream is needed for the longer time period.

EXAMPLE 2-3

Five children can eat 20 qt of ice cream in 2 days. At this rate, how much ice cream could 27 children eat in 1 day?

$$\frac{20\text{ qt} \times 1\text{ day} \times 27\text{ children}}{2\text{ days} \times 5\text{ children}} = 54\text{ qt}$$

Here the changing time and the changing number of children produced opposite effects on the base factor. Problems in which this happens are common and cause trouble only if one deals with the effect of the variables on one another rather than on the base factor.

Notice that all figures in the problems so far have borne units. This procedure may seem needlessly cumbersome now, but you should later find that a mere check of units in a complex setup will often reveal that you have made an error. The habit of always attaching units to all figures should pay dividends in the future.

Direct Relationship, Multiple Variables, Need to Change Unit

EXAMPLE 2–4

Five children can eat 20 qt of ice cream in 2 days. How much ice cream could 2 children eat in 3 hr if they ate half as fast as the first group of children?

Here the changing number of children, the changing period of time, and the changing rate constitute the correcting fractions. However, the time factors are in dissimilar units. Time must be in the same unit, but it is immaterial which unit is chosen.

$$\frac{20\ \text{qt} \times 2\ \text{children} \times 3\ \text{hr} \times 1}{5\ \text{children} \times 2\ \text{days} \times 24\ \frac{\text{hr}}{\text{day}} \times 2} = 0.25\ \text{qt}$$

Rate has no units attached here; it is a dimensionless figure.

Inverse Relationship, One Variable

An *inverse relationship* is one in which an increase in one factor causes a decrease in another factor or, conversely, a decrease in one factor causes an increase in another factor. For example, the more a man spends from his salary the less he can save, but the more he saves the more money he has to invest if he wishes. There is an inverse relationship between spending and saving from one's salary and a direct relationship between saving and possible investment.

EXAMPLE 2–5

At a pressure of 1 pound per square inch ($lb/in.^2$) the thickness of a sponge was 17 millimeters (mm). What would be the thickness of the sponge if the pressure were increased to 40 $lb/in.^2$? Assume that the thickness of the sponge decreases uniformly and regularly as the pressure increases.

1. Factor sought: thickness of sponge.
2. Base factor: thickness of sponge.
3. Factor affecting thickness of sponge: changing pressure.
4. Type of relationship: inverse.
5. Direction of change: increase in pressure.
6. Effect on base factor: decrease in thickness.
7. Setup:

$$\frac{17 \text{ mm} \times 1 \text{ lb}}{\text{in.}^2 \times \frac{40 \text{ lb}}{\text{in.}^2}} = 0.42 \text{ mm}$$

EXAMPLE 2–6

Forty liters of oxygen gas were under a pressure of 70 atmospheres (atm). If the pressure were changed to 35 atm, what would be the volume? Temperature remained constant.

Background information: Within limits, increasing pressure on a gas decreases its volume.

1. Base factor: volume of gas.
2. Correcting factor: changing pressure.
3. Type of relationship: inverse.
4. Direction of change: decrease in pressure.
5. Effect on base factor: increase in volume.
6. Setup:

$$\frac{40 \text{ liters} \times 70 \text{ atm}}{35 \text{ atm}} = 80 \text{ liters}$$

Multiple Variables, Direct and Inverse Relationships

EXAMPLE 2–7

At 300°K and 2 atm pressure a certain quantity of gas had a volume of 270 milliliters (ml). Find the volume at 250°K and 5 atm pressure.

Fact: Volume of a gas is directly related to absolute temperature (°K)* and inversely related to pressure.
Assumption: The gas will still be a gas under the new conditions.

Correcting factor	*Nature of relationship*	*Direction of change*	*Effect on base factor*
temperature	direct	decrease	decrease
pressure	inverse	increase	decrease

$$\frac{270 \text{ ml} \times 250°\text{K} \times 2 \text{ atm}}{300°\text{K} \times 5 \text{ atm}} = 90 \text{ ml}$$

*See Chapter 15 for discussion of absolute temperature and degrees Kelvin (°K).