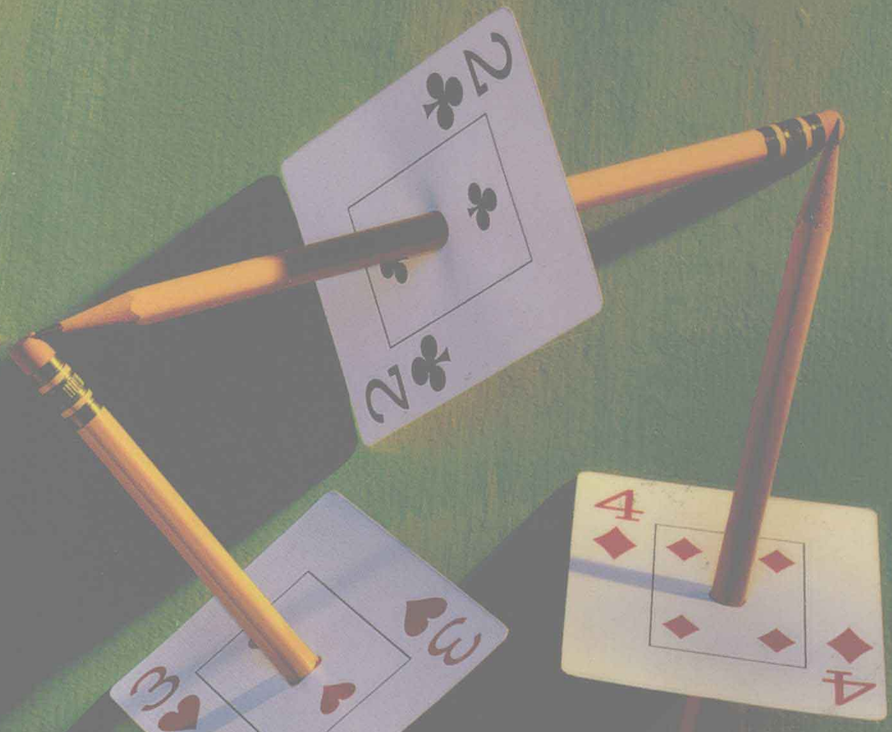
HARPERCOLLINS COLLEGE OUTLINE

FINITE MATHEMATICS WITH CALCULUS

Joan Dykes & Ronald Smith

Comprehensive Outline in Easy-To-Use Narrative Format, Supplements Major Textbooks, Graphs, Tables, Illustrations, Fully Indexed



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Finite Mathematics with Calculus

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Preface

This book is provided as a supplement to a standard college finite mathematics textbook that includes coverage of some calculus topics. Although a knowledge of intermediate algebra is assumed, as many steps as possible are provided to help the reader follow the logic involved in solving the various problems encountered in a course of this nature. Theorems are stated without proof and are often restated in words or symbols more easily understood by our own students. A set of exercises and answers appear at the end of each chapter to allow the reader to practice and receive immediate feedback. After working the exercises we have provided, use your own textbook exercises to develop the skill and speed to perform well in your course. Keep pencil and paper handy—reading and working through this book will help you succeed in finite mathematics and calculus.

Joan Dykes
Ronald Smith

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1

Lines

This chapter covers the techniques used in graphing line functions in two dimensions. Graphs of linear inequalities will be demonstrated. The concepts shown will be in processes involving systems of linear equations and business applications involving those systems of equations.

1.1 EQUATIONS OF LINES

All of the functions of the form $f(x) = mx + b$ are functions that represent a line. These functions will show a straight line when graphed. In this format, m is the slope of the line, and b is the y-intercept of the line. There are various aspects of these straight line functions that are studied in depth. These aspects will be discussed in this section.

Slope

The "steepness" of a line is called the *slope* of the line. The slope of the line is the ratio of the vertical movement along the line (*rise*) to the horizontal movement along the line (*run*).

$$\textbf{slope} = \frac{\text{rise}}{\text{run}}$$

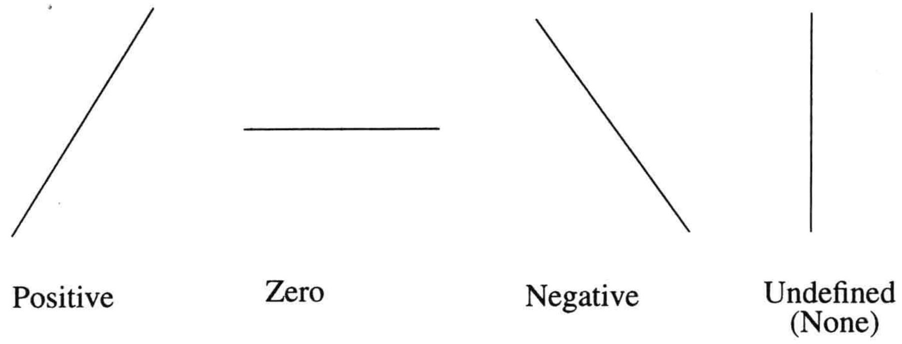

Figure 1.1 The Various Types of Slope

Figure 1.1 shows how the slope of the line relates to its steepness. The less steep the line is, the smaller the absolute value of the slope. This figure shows that a line that is **horizontal** has a **slope of 0**. As the slant of the line becomes steeper from left to right, the slope becomes a larger positive number.

Although the nuances of our language often allow the substitution of "zero" for "no," this cannot be the case when we talk about slopes. It is important to understand that a vertical line doesn't have a slope and a horizontal line has a slope equal to the number 0.

Since the rise on a graph is the difference between y-values, and the run on the graph is the difference between x-values, a slope can be determined from two points (x_1, y_1) and (x_2, y_2) on a graph by the following formula. The letter m is the mathematical designation for **slope**.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The line that connects point A, (3, 2) , and point B, (−1, 4), has a slope defined by:

$$m = \frac{4-2}{-1-3} = \frac{2}{-4} = -\frac{1}{2}$$

Note: It does not matter which point is labeled point A or which is labeled point B; the slope that is computed will be the same.

EXAMPLE 1.1

Find the slope of the line passing through the following points. It does not matter which point is labeled (x_1, y_1) and which is labeled (x_2, y_2):

(a) $(2, 1)$ and $(7, 1)$

(b) $(2, 0)$ and $(-3, -1)$

(c) $(1, -2)$ and $(1, 3)$

SOLUTION 1.1

(a) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-1}{7-2} = \dfrac{0}{5} = 0$

(b) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-(-1)}{2-(-3)} = \dfrac{1}{5}$

(c) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-(-2)}{1-1} = \dfrac{5}{0} =$ undefined

Division by 0 is undefined.

Two formulas are often used to convert information about a line into an equation for that line. These two formulas are

slope-intercept form $\quad y = mx + b$ where: $\boldsymbol{m}$ is the slope $\boldsymbol{b}$ is the y-intercept (the point where the line crosses the y-axis)	

point-slope form $\quad y - y_1 = m(x - x_1)$ where: $\boldsymbol{m}$ is the slope $(\boldsymbol{x_1}, \boldsymbol{y_1})$ are the coordinates of one point on the line Note: $\boldsymbol{x}$ and $\boldsymbol{y}$ (without subscripts) are not replaced by numbers.	

Some additional information will be helpful as we seek to find equations of lines:

(a) The slopes of two **parallel lines** with slopes of m_1 and m_2, respectively, are equal. That is, $m_1 = m_2$.

(b) The slopes of two perpendicular lines are the negative reciprocals of each other. That is, $m_1 = \dfrac{-1}{m_2}$.

(c) The coordinates of the origin are (0, 0).
(d) The point at which the line crosses the x-axis is called the x-intercept. Its coordinates are (x-value, 0).
(e) The points at which the line crosses the y-axis is called the y-intercept. The coordinates for the y-intercept are (0, y-value).

Armed with the above information and the two formulas for developing the line equations, we can now find the equation of a line given various information.

EXAMPLE 1.2

(a) Determine the equation of the line passing through the points $(2, 1)$ and $(-1, 3)$.
(b) Find the equation of the line with slope of $\frac{3}{2}$ and a y-intercept of -2.
(c) Write the equation of a line with x-intercept of 2 and passing through $(1, 5)$.
(d) Find the equation of a line passing through the origin and crossing $(-7, 1)$.
(e) Find the equation of a line with x-intercept of -5 and y-intercept of -2.
(f) Formulate the equation of the line parallel to the line $2x - 4y = 3$ and passing through $(8, 0)$.
(g) Determine the equation of a line perpendicular to $y = -\frac{1}{5}x + 1$ and passing through $(0, -1)$.
(h) Find the equation of the horizontal line passing through $(-2, 3)$.
(i) Write the equation of the vertical line crossing the x-axis at -7.

SOLUTION 1.2

(a) With two points given, we can first find the slope and then use one of the points and the slope in the point-slope formula.

$(2, 1) \quad (-1, 3)$	Two points are given.
$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3 - 1}{-1 - 2} = \dfrac{2}{-3} = -\dfrac{2}{3}$	Find the slope.
$m = \dfrac{-2}{3} \qquad (2, 1) = (x_1, y_1)$	Select either one of the points and use the slope found earlier.
$y - y_1 = m(x - x_1)$	We now need the point-slope formula.
$y - 1 = -\dfrac{2}{3}(x - 2)$	Substitute for slope and point.

$(3)(y-1) = (3)\left(-\frac{2}{3}\right)(x-2)$	Multiply both sides of the equation by 3.
$3(y-1) = -2(x-2)$	
$3y-3 = -2x+4$	Distribute to remove parentheses.
$\begin{array}{rcl} 3y-3 &=& -2x+4 \\ \underline{\quad +3} && \underline{\quad\quad +3} \end{array}$	Add 3 to both sides.
$3y \quad = -2x+7$	Add $2x$ to both sides.
$\underline{\quad +2x} \quad \underline{+2x \quad}$	
$2x+3y = 7$	This completes the equation and isolates the constant (7).

(b) Knowing the slope $(m = 3/2)$ and the y-intercept $(b = -2)$, this problem is tailor-made for the slope-intercept formula.

$y = mx+b$	Write the slope-intercept form of the equation.
$y = \frac{3}{2}x + (-2)$	Substitute slope and intercept.
$y = \frac{3}{2}x - 2$	You could leave the equation in this form.
$(2)y = 2\left(\frac{3}{2}x-2\right)$	Or you could change it into the standard form by multiplying by 2.
$2y = 2\left(\frac{3}{2}x\right) - 2(2)$	Don't forget to distribute the 2 on the right through both terms.
$2y = 3x-4$	
$-3x+2y = -4$	Isolate the constant term.
or	
$3x-2y = 4$	

(c) In this case the equation will be derived from two points (the x-intercept and the point $(1, 5)$).

An x-intercept of 2 means the line contains the point $(2, 0)$. Thus, two points this line passes through are $(2, 0)$ and $(1, 5)$.

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{5-0}{1-2} = \frac{5}{-1} = -5$ Find the slope.

$y - y_1 = m(x - x_1)$ — We now need the point-slope formula.

$m = -5 \qquad (x_1, y_1) = (2, 0)$

$y - 0 = -5(x - 2)$ — Substitute into the formula.

$y = -5(x - 2)$

$y = -5x + 10$ — Distribute.

$5x + y = 10$ — Isolate the constant term.

(d) This equation will also be built from two points. We will use the origin $(0, 0)$ and the point $(-7, 1)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 0}{-7 - 0} = \dfrac{-1}{7}$ — Find the slope.

$y - y_1 = m(x - x_1)$ — We now need the point-slope formula.

$y - 0 = \dfrac{-1}{7}(x - 0)$ — Substitute into point-slope formula.

$y = \dfrac{-1}{7}x$ — Simplify the equation.

$7y = 7\left(\dfrac{-1}{7}x\right)$ — Multiply both sides of the equation by the denominator.

$7y = -x$

$7y + x = -x + x$ — Add x to both sides of the equation.

$x + 7y = 0$

Note: The standard form of any line passing through the origin will always be an equation of the form $ax + by = 0$.

(e) This problem has two intercepts, which yield two points. The points are $(-5, 0)$ from the x-intercept of –5 and $(0, -2)$ from the y-intercept of –2.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2 - 0}{0 - (-5)} = \dfrac{-2}{5}$ — Find the slope.

$m = \dfrac{-2}{5} \qquad b = -2$ — We could use the point-slope form, but the slope-intercept is easier.

$y = mx + b$ — Write the slope-intercept form of the equation.

$y = \frac{-2}{5}x - 2$ — Substitute into the equation.

$(5)y = (5)\left(\frac{-2}{5}x - 2\right)$ — Multiply both sides of the equation by the denominator.

$5y = 5\left(\frac{-2}{5}x\right) - 5(2)$ — Distribute.

$5y = -2x - 10$

$5y + 2x = -2x + 2x - 10$ — Add $2x$ to both sides.

$2x + 5y = -10$ — Isolate the constant term.

(f) We have a slope "hidden" in the line that is given as parallel to the line we are to find and we are given a point (8, 0).

$2x - 4y = 3$ — Solve for y in terms of x to find the slope of the given line.

$-4y = -2x + 3$

$\frac{-4y}{-4} = \frac{-2x}{-4} + \frac{3}{-4}$ — Divide both sides by −4.

$y = \frac{1}{2}x - \frac{3}{4}$ — The equation is now in slope-intercept form. We will use only the slope and disregard the rest of the linear equation.

$(x_1, y_1) = (8, 0) \qquad m = \frac{1}{2}$

$y - y_1 = m(x - x_1)$ — Use the point-slope form.

$y - 0 = \frac{1}{2}(x - 8)$ — Substitute for the slope and the point.

$2(y - 0) = 2\left(\frac{1}{2}\right)(x - 8)$ — Multiply both sides by 2.

$-x + 2y = -8$ — Subtract x from both sides.

or

$x - 2y = 8$

(g) The slope of the desired line is obtained from $y = -\frac{1}{5}x + 1$. The slope of $y = -\frac{1}{5}x + 1$ is the coefficient of x which is $-\frac{1}{5}$. Perpendicular lines have slopes that are negative reciprocals of each other. We find the new slope by taking $-\frac{1}{5}$ and "flipping" it to get $\frac{5}{-1} = -5$. Next, we change the sign to get 5. The slope of the line perpendicular to the given line is 5.

$(x_1, y_1) = (0, -1) \quad m = 5$	
$y - y_1 = m(x - x_1)$	Use the point-slope form.
$y - (-1) = 5(x - 0)$	Substitute for slope and point.
$y + 1 = 5x$	Simplify.
$y + 1 - 1 = 5x - 1$	Subtract 1 from both sides.
$y = 5x - 1$	

(h) The equation of a horizontal line through (x-value, y-value) is $y = y$-value. Since we were given (–2, 3), the equation must be $y = 3$.

(i) The equation of a vertical line through (x-value, y-value) is $x = x$-value. Since we were given (–7, 0), the equation must be $x = -7$.

1.2 SOLVING INEQUALITIES

Rules for Inequalities

While linear equations with one unknown have only one solution, linear inequalities usually have many (or a range) of solutions.

Symbol	Meaning
$x > y$	x is greater than y
$x < y$	x is less than y
$x \geq y$	x is greater than or equal to y
$x \leq y$	x is less than or equal to y

Table 1.1 Inequality Symbols

Rule	Format
addition	$a + c > b + c$
subtraction	$a - c > b - c$
multiplication	$a \cdot c > b \cdot c; \quad c > 0$
division	$a \div c > b \div c; \quad c > 0$
negation	$-a < -b$

Table 1.2 Inequality Rules For $a > b$

In Table 1.2, you might notice that a negation of the inequality reverses the direction (sometimes referred to as "the sense") of the inequality. That is, the > will reverse to < if the inequality is divided or multiplied by a negative number. Addition and subtraction do not affect the direction of the inequality.

EXAMPLE 1.3

Solve the following inequalities:

(a) $x + 5 > 7$

(b) $2x - 3 < 5$

(c) $4 - 5x > 24$

(d) $6 - \frac{x}{3} < 18$

SOLUTION 1.3

(a) $x + 5 > 7$ — Copy the given inequality.

$$\begin{array}{rcr} x + 5 & > & 7 \\ \underline{\quad -5} & & \underline{-5} \end{array}$$

Subtract 5 from each side of the inequality.

$$x > 2$$

Solution set is all real numbers greater than 2.

(b) $2x - 3 < 5$ — Copy the given inequality.

$$\begin{array}{rcr} 2x - 3 & < & 5 \\ \underline{\quad +3} & & \underline{+3} \\ 2x & < & 8 \end{array}$$

Add 3 to both sides of the inequality.

$$\frac{2x}{2} < \frac{8}{2}$$

Divide both sides by 2.

$$x < 4$$

The solution set includes all real numbers less than 4. Check by substituting numbers less than 4 into the original inequality.

(c) $4 - 5x > 24$

Copy the given inequality.

$$\begin{array}{r} 4 - 5x > 24 \\ \underline{-4 \qquad\quad -4} \\ -5x > 20 \end{array}$$

Subtract 4 from each side of the inequality.

$$\frac{-5x}{-5} < \frac{20}{-5}$$

We divide by –5. Remember that when we divide by a negative number, we reverse the direction of the inequality.

$$x < -4$$

The direction of the inequality has been reversed. The solution set includes all numbers less than –4. Check by substituting numbers less than –4 into the original inequality.

(d) $6 - \frac{x}{3} < 18$

Copy the given inequality.

$$6 - \frac{x}{3} < 18$$

Subtract 6 from each side of the inequality.

$$-\frac{x}{3} < 12$$

$$-\frac{3}{1} \cdot \frac{-x}{3} > (-3)\,12$$

Multiply both sides of the inequality by (–3).

$$x > -36$$

The direction of the inequality is reversed.

$$x > -36$$

The solution set is any number greater than –36. Check by placing any number larger than –36 in the original inequality.

1.3 GRAPHING POINTS AND LINES

Graphing a Point

Each point drawn in two dimensions has an address to specify its location on the two-dimensional plane. Each address is called an *ordered pair*. The ordered pair is specified, in general, as (x, y), where x and y are real numbers. The x-value represents the distance of the point from the origin along the x-axis and the y-value represents the distance of the point from the origin along the y-axis.

Figure 1.2 shows two number lines that are perpendicular to each other. The point at which they cross is called the *origin*. These lines and the plane they define are called the *rectangular coordinate axis system*.

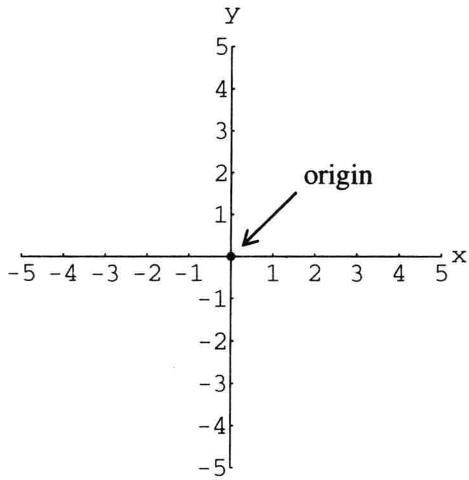

Figure 1.2 The Rectangular Coordinate System

The location of a point is defined as movement from the origin left or right and up or down. The x and y lines are called *axes*. The x-axis conveys horizontal movement and the y-axis conveys vertical movement.

The point defined by the ordered pair (3, 4) is located 3 units to the *right* of the origin and 4 units "*above*" the x-axis. The point (–1, –2) is one unit to the *left* of the origin and two units "*below*" the x-axis.

Figure 1.3 shows both of these points.

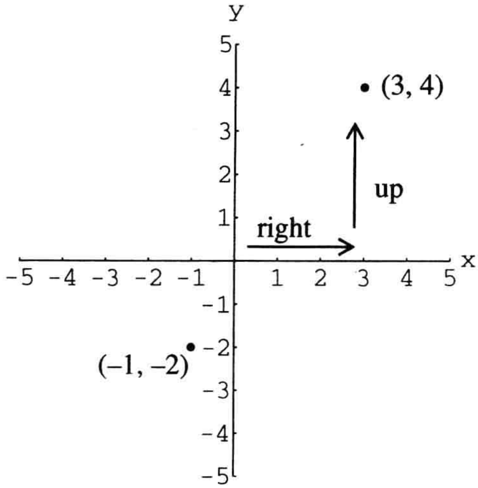

Figure 1.3 Graph of Points

The location of a point can also be found by moving first along the *y*-axis, and then parallel to the *x*-axis. However, to establish a pattern, we will always move along the *x*-axis first.

EXAMPLE 1.4

Graph the points (1, –3); (5, 2); (–4, 0) and $\left(-\frac{3}{2}, 4\right)$.

SOLUTION 1.4