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Robot Manipulators

Modeling, Performance Analysis and Control

Edited by Etienne Dombre and Wisama Khalil

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Modeling, Performance Analysis and Control of Robot Manipulators

Edited by
Etienne Dombre
Wisama Khalil

ISTE

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Chapter 1

Modeling and Identification of Serial Robots

1.1. Introduction

The design and control of robots require certain mathematical models, such as:

– transformation models between the operational space (in which the position of the end-effector is defined) and the joint space (in which the configuration of the robot is defined). The following is distinguished:

- direct and inverse geometric models giving the location of the end-effector (or the tool) in terms of the joint coordinates of the mechanism and vice versa,

- direct and inverse kinematic models giving the velocity of the end-effector in terms of the joint velocities and vice versa,

– dynamic models giving the relations between the torques or forces of the actuators, and the positions, velocities and accelerations of the joints.

This chapter presents some methods to establish these models. It will also deal with identifying the parameters appearing in these models. We will limit the discussion to simple open structures. For complex structure robots, i.e. tree or closed structures, we refer the reader to [KHA 02].

Chapter written by Wisama KHALIL and Etienne DOMBRE.

Mathematical development is based on (4 × 4) homogenous transformation matrices. The homogenous matrix ${}^{i}\mathbf{T}_j$ representing the transformation from frame R_i to frame R_j is defined as:

$$ {}^{i}\mathbf{T}_j = \begin{bmatrix} & {}^{i}\mathbf{R}_j & & {}^{i}\mathbf{P}_j \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^{i}\mathbf{s}_j & {}^{i}\mathbf{n}_j & {}^{i}\mathbf{a}_j & {}^{i}\mathbf{P}_j \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [1.1] $$

where ${}^{i}\mathbf{s}_j$, ${}^{i}\mathbf{n}_j$ and ${}^{i}\mathbf{a}_j$ of the orientation matrix ${}^{i}\mathbf{R}_j$ indicate the unit vectors along the axes $\mathbf{x}_j$, $\mathbf{y}_j$ and $\mathbf{z}_j$ of the frame R_j expressed in the frame R_i; and where ${}^{i}\mathbf{P}_j$ is the vector expressing the origin of the frame R_j in the frame R_i.

1.2. Geometric modeling

1.2.1. *Geometric description*

A systematic and automatic modeling of robots requires an appropriate method for the description of their morphology. Several methods and notations have been proposed [DEN 55], [SHE 71], [REN 75], [KHA 76], [BOR 79], [CRA 86]. The most widely used one is that of Denavit-Hartenberg [DEN 55]. However, this method, developed for simple open structures, presents ambiguities when it is applied to closed or tree-structured robots. Hence, we recommend the notation of Khalil and Kleinfinger which enables the unified description of complex and serial structures of articulated mechanical systems [KHA 86].

A simple open structure consists of n+1 links noted C_0, …, C_n and of n joints. Link C_0 indicates the robot base and link C_n, the link carrying the end-effector. Joint j connects link C_j to link C_{j-1} (Figure 1.1). The method of description is based on the following rules and conventions:

– the links are assumed to be perfectly rigid. They are connected by revolute or prismatic joints considered as being ideal (no mechanical clearance, no elasticity);

– the frame R_j is fixed to link C_j;

– axis $\mathbf{z}_j$ is along the axis of joint j;

– axis $\mathbf{x}_j$ is along the common perpendicular with axes $\mathbf{z}_j$ and $\mathbf{z}_{j+1}$. If axes $\mathbf{z}_j$ and $\mathbf{z}_{j+1}$ are parallel or collinear, the choice of $\mathbf{x}_j$ is not unique: considerations of symmetry or simplicity lead to a reasonable choice.

The transformation matrix from the frame R_{j-1} to the frame R_j is expressed in terms of the following four geometric parameters:

– α_j: angle between axes $\mathbf{z}_{j-1}$ and $\mathbf{z}_j$ corresponding to a rotation about $\mathbf{x}_{j-1}$;

– d_j: distance between $\mathbf{z}_{j-1}$ and $\mathbf{z}_j$ along $\mathbf{x}_{j-1}$;

– θ_j: angle between axes $\mathbf{x}_{j-1}$ and $\mathbf{x}_j$ corresponding to a rotation about $\mathbf{z}_j$;

– r_j: distance between $\mathbf{x}_{j-1}$ and $\mathbf{x}_j$ along $\mathbf{z}_j$.

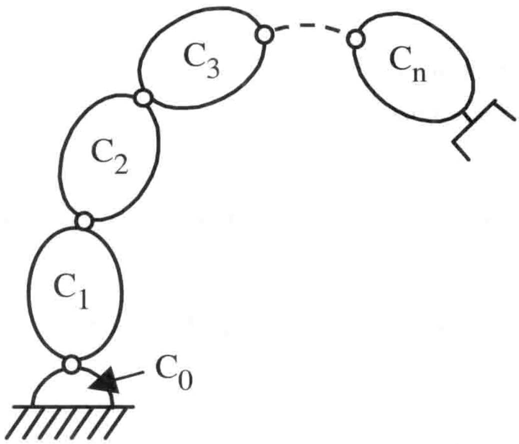

Figure 1.1. *A simple open structure robot*

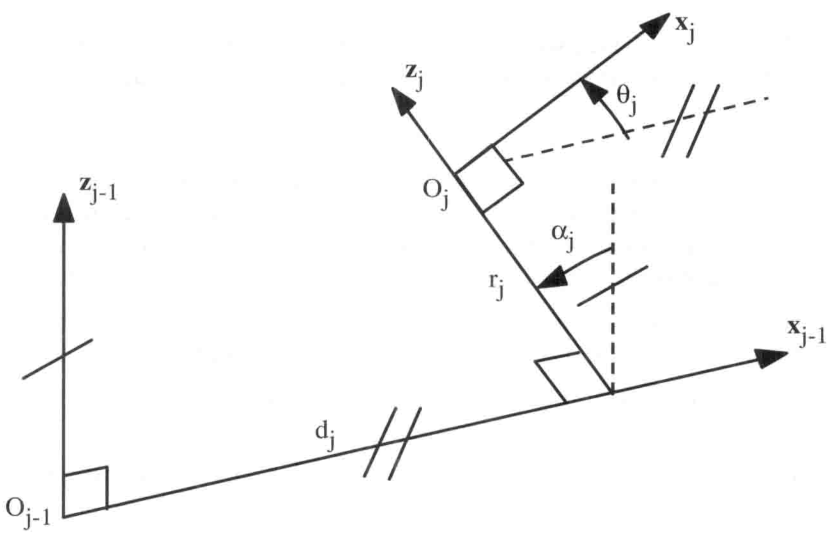

Figure 1.2. *Geometric parameters in the case of a simple open structure*

The joint coordinate q_j associated to the j^{th} joint is either θ_j or r_j, depending on whether this joint is revolute or prismatic. It can be expressed by the relation:

$$q_j = \bar{\sigma}_j \theta_j + \sigma_j r_j \qquad [1.2]$$

with:

– $\sigma_j = 0$ if the joint is revolute;

– $\sigma_j = 1$ if the joint is prismatic;

– $\bar{\sigma}_j = 1 - \sigma_j$.

The transformation matrix defining the frame R_j in the frame R_{j-1} is obtained from Figure 1.2 by:

$$^{j-1}\mathbf{T}_j = \mathbf{Rot}(\mathbf{x}, \alpha_j)\ \mathbf{Trans}(\mathbf{x}, d_j)\ \mathbf{Rot}(\mathbf{z}, \theta_j)\ \mathbf{Trans}(\mathbf{z}, r_j)$$

$$= \begin{bmatrix} C\theta_j & -S\theta_j & 0 & d_j \\ C\alpha_j S\theta_j & C\alpha_j C\theta_j & -S\alpha_j & -r_j S\alpha_j \\ S\alpha_j S\theta_j & S\alpha_j C\theta_j & C\alpha_j & r_j C\alpha_j \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [1.3]$$

where $\mathbf{Rot}(\mathbf{u}, \alpha)$ and $\mathbf{Trans}(\mathbf{u}, d)$ are (4×4) homogenous matrices representing, respectively, a rotation α about the axis $\mathbf{u}$ and a translation d along $\mathbf{u}$.

NOTES.

– for the definition of the reference frame R_0, the simplest choice consists of taking R_0 aligned with the frame R_1 when $q_1 = 0$, which indicates that $\mathbf{z}_0$ is along $\mathbf{z}_1$ and $O_0 \equiv O_1$ when joint 1 is revolute, and $\mathbf{z}_0$ is along $\mathbf{z}_1$ and $\mathbf{x}_0$ is parallel to $\mathbf{x}_1$ when joint 1 is prismatic. This choice renders the parameters α_1 and d_1 zero;

– likewise, the axis $\mathbf{x}_n$ of the frame R_n is taken collinear to $\mathbf{x}_{n-1}$ when $q_n = 0$. This choice makes r_n (or θ_n) zero when $\sigma_n = 1$ (or = 0 respectively);

– for a prismatic joint, the axis $\mathbf{z}_j$ is parallel to the axis of the joint; it can be placed in such a way that d_j or d_{j+1} is zero;

– when $\mathbf{z}_j$ is parallel to $\mathbf{z}_{j+1}$, the axis $\mathbf{x}_j$ is placed in such a way that r_j or r_{j+1} is zero;

– in practice, the vector of joint variables $\mathbf{q}$ is given by:

$$\mathbf{q} = \mathbf{K_c}\,\mathbf{q_c} + \mathbf{q_0}$$

where $\mathbf{q_0}$ represents an offset, $\mathbf{q_c}$ are encoder variables and $\mathbf{K_c}$ is a constant matrix.

EXAMPLE 1.1.– description of the structure of the Stäubli RX-90 robot (Figure 1.3). The robot shoulder is of an RRR anthropomorphic type and the wrist consists of three intersecting revolute axes, equivalent to a spherical joint. From a methodological point of view, firstly the axes $\mathbf{z}_j$ are placed on the joint axes and the axes $\mathbf{x}_j$ are placed according to the rules previously set. Next, the geometric parameters of the robot are determined. The link frames are shown in Figure 1.3 and the geometric parameters are given in Table 1.1.

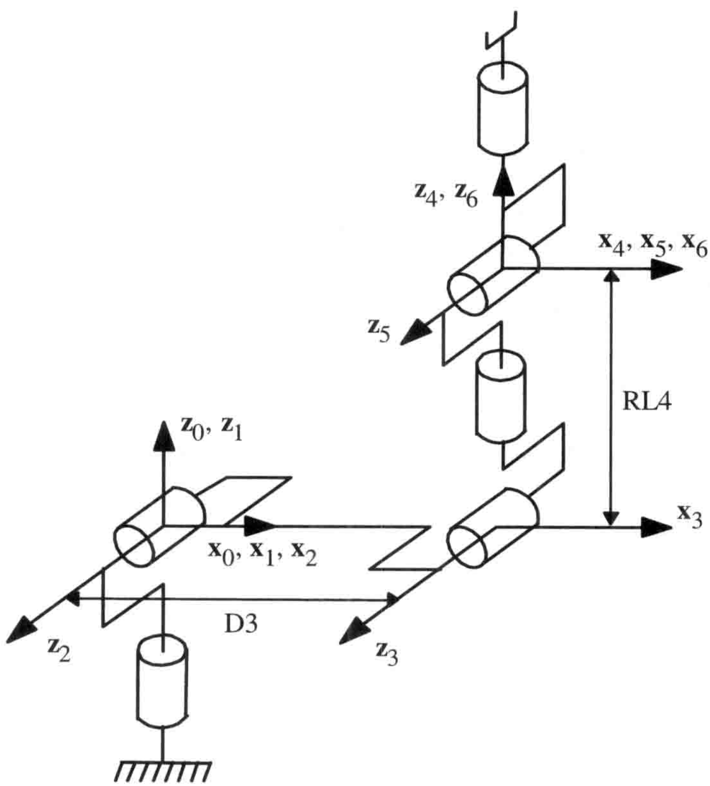

Figure 1.3. *Link frames for the Stäubli RX-90 robot*