

Quantitative Methods for Business Decisions

**SECOND
EDITION**

Lawrence L. Lapin

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New York San Diego Chicago San Francisco Atlanta
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*To my brothers,
Charles and Robert*

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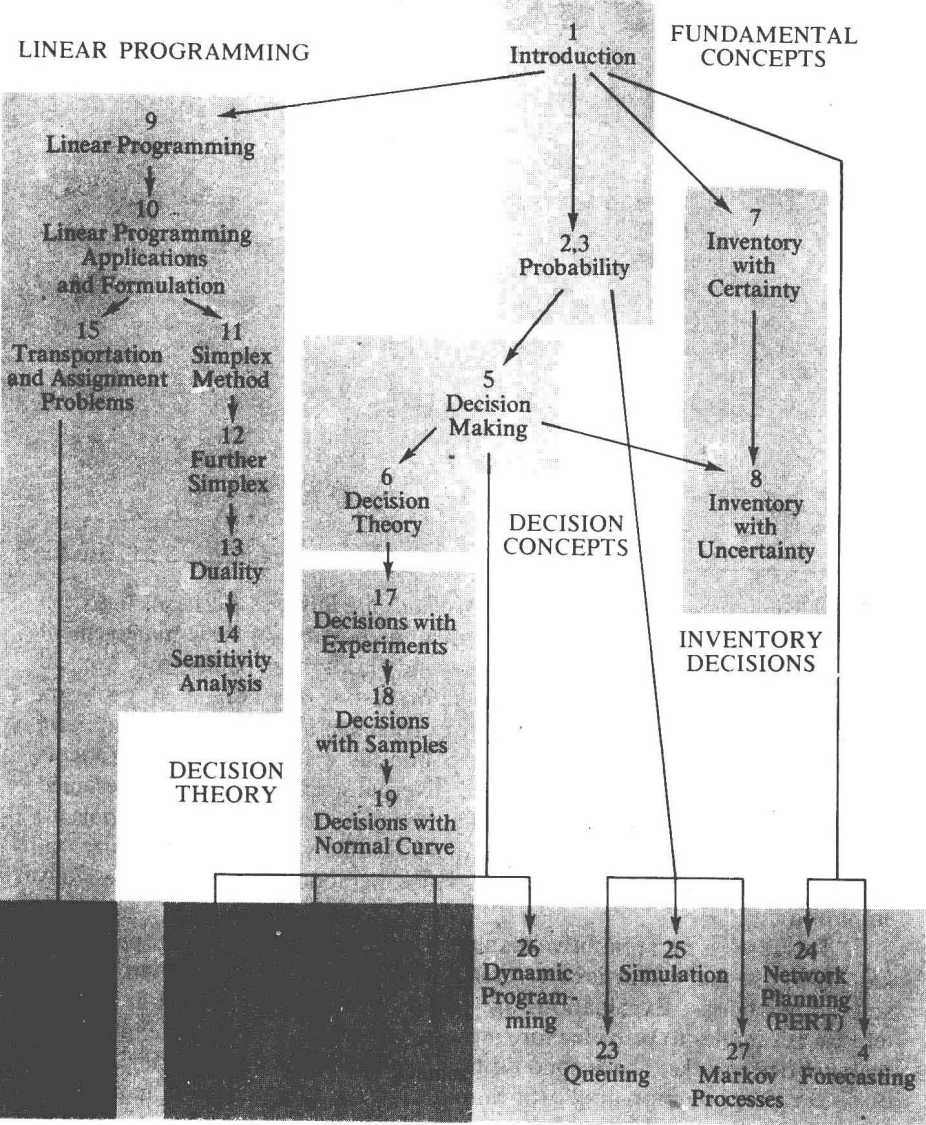
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Preface

My goal in writing *Quantitative Methods for Business Decisions* has been to provide as complete a treatment as possible of basic management science methodology. The Second Edition expands this topical coverage to include several new subjects: forecasting, the assignment problem, Vogel's approximation (for transportation problems), and integer programming and the branch-and-bound method. This book is written for college students who have only an algebra background. Even more important, it is designed to provide a feeling for the variety and power of management science tools, to alleviate apprehension of the subject, and to enable students to recognize on-the-job situations where management science methodology can be successfully employed.

This book is more intuitive than most. I have treated difficult topics "with kid gloves," so that discussions devoted to this material are longer than those in some other books. Explanations are richly illustrated with relevant and interesting examples to provide more meaningful and *easier* learning experiences than in briefer books. The Second Edition now includes an entirely new chapter on linear programming applications and problem formulation (Chapter 10). Chapter 11 thoroughly describes in nonmathematical terms the underlying rationale of the simplex method, so that the student can learn why—as well as how—this method works. Because many instructors may wish to omit the more advanced simplex concepts, such as artificial variables, these more difficult



topics are now grouped in a second optional simplex chapter (Chapter 12). Chapter 24 discusses network planning (PERT or CPM) in a broad context, including management implications, milestone and activity scheduling, time-cost trade-off, and (in an appendix to the chapter) probabilistic aspects. Chapter 25 introduces Monte Carlo simulation as a simple substitute for the stopwatch observation of an actual system operation. Highly intuitive decision trees are used extensively throughout to explain a variety of concepts.

This book also highlights the limitations and pitfalls associated with various mathematical models and algorithms. For example, some basic models, such as the EOQ model used in inventory decisions and the simple queuing formulas, are based on assumptions that rarely apply in real life. Wherever practical, alternative approaches such as Monte Carlo simulation are indicated and fully described. Traditional probabilistic PERT assumptions are accompanied by a critical analysis of their applicability. The severe limitations on the use of Markovian decision models are also noted.

Because hand calculations are required (a difficulty encountered in any quantitative methods course), I have designed the problem material to minimize computational chores and to emphasize concepts and formulation. Coverage of the computer is provided wherever appropriate, although specific programs are not described in detail—a subject properly covered elsewhere.

As the tinted areas in the preface figure indicate, the overall design of the book is modular to provide maximum flexibility for adaptation to the requirements of a particular course. All or portions of any part of these subject groupings may be used in constructing a one- or two-quarter or a one- or two-semester quantitative methods course. For example, Chapters 2 and 3 may be bypassed by students who have had a prior course in statistics or by instructors who teach a purely deterministic course (a viable possibility with this book). The specific sequencing constraints to be followed are also shown in the figure.

The chapters on probability (or some prior knowledge of this subject) serve as the prerequisite to all the stochastic material. Much of the book follows directly from Chapter 5, which examines the basic concepts of decision making and how to cope with uncertainty, and applies expected value and decision tree analysis to general problem solving. After reading Chapter 5, students should be able to handle any of the special topics and to pursue decision theory in detail.

The book has been thoroughly class-tested several times in a variety of different courses, which has resulted in the culling, revising, and grading of the problem material. In general, the problems are broken into several distinct parts to make the student's job easier and to permit the instructor added flexibility in making assignments. As an added bonus, brief answers to selected problems are provided in the back of the book, so that students can check their own work.

The *Instructor's Manual* contains specific recommendations for various course designs and teaching suggestions and provides detailed solutions to the nearly 400 problems in the text. As an additional aid to the instructor, a set of

more than 150 solved problems of slight to moderate difficulty for supplementary homework or examinations is available to adopters (from the publisher). A comprehensive bibliography is included in the back of the book for students who wish to pursue a particular topic in greater detail.

I wish to thank my colleagues who were instrumental in helping me shape the manuscript: C. Randall Byers of the University of Idaho, Ross Lanser of San José State University, Don McBrien of Boston University, Zeb Vancura of the University of Santa Clara, and William D. Whisler of the California State University at Hayward. I also wish to acknowledge the valuable assistance of my students, and to extend special mention to Erika Heider, who helped find errors and assisted in preparing the *Instructor's Manual*.

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