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Elements of Psychophysical Theory

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Preface

This book is intended as a self-contained graduate text for a two-semester course discussing the basic concepts of measurement and psychophysics. The level of the material is geared to students in experimental psychology with a relatively modest background in mathematics (e.g., two semesters of undergraduate calculus), but well motivated and prepared to work hard. In a two-semester course, all the material in the book can be covered, with the exception of Chapter 7 and the starred sections, which contain technical material and are generally more difficult. In a one-semester course on psychophysical theory, a possible sequence might include the first four sections in Chapter 2, Chapters 4–6, 8–10, 13, and whatever else is deemed important by the instructor in the remaining chapters (again, omitting the starred sections and also the proofs of a number of results).

I have used drafts of the book in several courses, and the students' reactions have been most helpful. I am especially grateful to Charlie Chubb, Ching-Fan Sheu, Martin Gizzi, and John Van Praag for their remarks.

Jean-Paul Doignon, Geoffrey Iverson, and Misha Pavel, my longtime friends and co-workers, also had a positive influence on my writing. Geoffrey's careful reading of drafts of many chapters and his detailed comments have been particularly valuable.

The viewpoint on psychophysics given here owes much to the work of several researchers in the field. Among others, the names of David Krantz, Michael Levine, and especially Duncan Luce, whose contribution was seminal, should be mentioned. It is not that this viewpoint is startlingly new. To the contrary. The novelty of the presentation only resides in an attempt to discuss the concepts of classical psychophysics in the framework of measurement theory, and more generally, in the language of contemporary mathematics. Much more will be said on this matter in the initial chapter, entitled "Preliminaries."

I am indebted to my colleagues at New York University for the intellectually stimulating atmosphere that characterizes our program. I am especially grateful to Murray Glanzer, Lloyd Kaufman, Michael Landy, Tony Movshon, and George Sperling for the encouragement they gave me regarding this project.

At some time or other, Mary Peters, Amy Kritz, Kathleen Williams, Odella Schattin, and Saritha Clements typed drafts of chapters of this book. I am happy to thank them here for their expert, conscientious work, and for their friendly acceptance of my idiosyncrasies. I am also grateful to my daughter Catherine Landergan-Falmagne, who kindly offered to do the artwork. The final appearance of this book owes much to Joan Knizeski-Bossert, from Oxford, whose painstaking vigilance, coupled with patience and understanding, was very helpful in the final stage.

The work reported here was partly supported by grants from the National Science Foundation to New York University. Part of the writing was done during my tenure as a Guggenheim fellow and as a von Humboldt fellow. The support of these institutions is gratefully acknowledged.

I am also thankful to my wife, Cecilia, for her unwavering, good-humored support of what may perhaps have seemed, at times, a rather whimsical activity.

This book is dedicated to Lloyd Kaufman, whose friendship and guidance over the years have been invaluable.

New York City J.-C. F.
January 1985

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Elements of Psychophysical Theory

Preliminaries

CLASSICAL VERSUS MODERN PSYCHOPHYSICS

Traditionally, research in psychophysics has attempted to answer the following questions:

Q1. What are the basic "sensation" scales underlying the subject's responses in psychophysical experiments?
Q2. How are these scales related to the physical scales?

These questions prompted a program of research that originated with G. T. Fechner and was remarkably successful in generating a considerable array of useful data as well as a theoretical framework for these data. Over the years, however, Fechner's methods and concepts have been criticized. Today, many consider classical (or Fechnerian) psychophysics outmoded, and of historical interest only. In standard textbooks, it is dispatched rather than expounded, the typical presentation reading like a summary of a treatise that could have been written at the end of the nineteenth century. Little effort is made to analyze the filiation between classical and modern concepts, and the progress realized.

This situation deserves to be corrected. In fact, the influence of Fechner's program on contemporary psychophysics is overwhelming, even though the favored terminology may have evolved somewhat. Rather than Q1 or Q2, a modern psychophysicist would ask:

Q3. How is physical intensity coded by a particular sensory system?

This reformulation of the fundamental questions reflects the tendency of contemporary psychophysics to seek explanations of the data in terms of models or mechanisms. This involves a generalization of the classical viewpoint, embodied in Q1 and Q2, that a stimulus intensity induces an event in the organism, which may be represented by a single number. A less restrictive position is consistent with Q3. The effect of a stimulus intensity on the organism may have a representation as an intricate mathematical object, such as a random variable, or a stochastic process of neural events.

Let us be more concrete. In a discrimination experiment, let $P_{a,b}$ be the probability that a stimulus of intensity a is judged as exceeding a stimulus of intensity b in some sensory attribute. The fundamental equation of classical psychophysics is

$$P_{a,b} = F[u(a) - u(b)] \qquad (1)$$

in which u and F are real valued, strictly increasing, continuous (but otherwise unspecified) functions. Thus, the probability of choosing a over b increases with the difference between these stimuli on the sensory scale u. This scale is taken to be a measure of the magnitude of sensations evoked by the stimuli. (The foundation of this position will be discussed in Chapter 13.) Equation 1 must have a solution, that is, the functions u and F must exist, for Fechner's procedures to be valid. This equation is never discussed, does not even appear, in typical contemporary texts (for an exception, see Kaufman, 1974).[1] However, almost all models for discrimination data considered seriously by working psychophysicists involve generalizations or special cases of this equation (or equivalent ones).[2] For instance, Equation 1 may appear as a consequence of a complicated model. In such a case, the function F may take the form of some distribution function, and $u(a)$, $u(b)$ may be parameters of some stochastic process.

Our presentation will clarify the relations between past and current concepts and methods.[3] An apt title for a large part of this book (Chapters 1–9) would have been "Classical Psychophysics from a Modern Viewpoint.'

ON THE UNIQUENESS OF MODELS AND REPRESENTATIONS

Questions Q1 and Q2 implicitly presuppose that the sensation scales somehow exist independently of both the experimental paradigm and the mathematical model used to analyze the data, and that they have to be "uncovered." Similarly, but more generally, Q3 may suggest that the stimuli, or more accurately, the effects they produce in the organism, have a mathematical representation the form of which does not depend on paradigm or model. This requires some elaboration. Consider the following typical example of psychophysical lingo, describing a model for the discrimination probabilities $P_{a,b}$ appearing in Equation 1:

> We assume that the presentation of a stimulus of intensity a induces, in some neural location, some excitation of intensity $\mathbf{U}_a$, a random variable. If a pair of stimuli (a,b) is presented in a discrimination paradigm, the subject will choose a over b as the more intense stimulus whenever the excitation induced by stimulus a exceeds that

1. In all fairness, it must be admitted that this equation was only implicit in the "Elements of Psychophysics" (Fechner, 1860/1966).

2. Even Stevens's magnitude estimation or cross-modality matching methods (and data) may be formalized by a theory closely related to the Fechnerian Equation 1. (See the Krantz-Shepard relation theory in Chapter 13.)

3. The opposition between Stevens's and Fechner's school of psychophysics has been grossly exaggerated, at the cost of much confusion. It can be argued convincingly that, apart from semantical issues, the two positions are quite consistent.

induced by stimulus b; that is, whenever $\mathbf{U}_a > \mathbf{U}_b$. This means that the discrimination probabilities $P_{a,b}$ satisfy the equation

$$P_{a,b} = \mathbb{P}\{\mathbf{U}_a > \mathbf{U}_b\}, \tag{2}$$

where $\mathbb{P}$ is the probability measure.

This illustrates a convenient and suggestive descriptive style, which we shall use on occasion. It may be misleading, however, if employed carelessly. The quoted text may suggest, for example, that the random variables $\mathbf{U}_a$ and $\mathbf{U}_b$ appearing in (2) are a natural, invariant mathematical representation of the events taking place in the organism as a result of the stimulation. Such a presumption, even though it may very well be logically unassailable (there may be no practical way to disprove it) is nevertheless controversial. It is indeed at odds with the considerable arbitrariness presiding at various important stages of a research enterprise. The most critical choice to be made concerns the model used to explain the data. A particular model, say M, found to provide an acceptable fit to some data, is rarely unique in this regard. There usually are many models fitting the data equally well, and thus equivalent to M in this respect. However, equivalent models may lead to very different representations.

The model of Equation 2 (which, incidentally, generalizes that of Equation 1) will provide an example. Let us suppose that a further stage in the sensory coding of the stimuli is taking place, and that the stimuli are represented by the random variable $g(\mathbf{U}_a)$, $g(\mathbf{U}_b)$, etc., in which g is some real valued, strictly increasing function defined on the real numbers. Thus, the subject's decisions are based on a comparison of the random variables $g(\mathbf{U}_a)$ and $g(\mathbf{U}_b)$, and (1) is replaced by

$$P_{a,b} = \mathbb{P}\{g(\mathbf{U}_a) > g(\mathbf{U}_b)\}. \tag{3}$$

But obviously, (2) and (3) are equivalent. Nothing in the discrimination data considered would justify choosing $\mathbf{U}_a$ over $g(\mathbf{U}_a)$ as being a more genuine representation of the neural effects of stimulus a. Nevertheless, the form of the distributions of the random variables $\mathbf{U}_a$ and $g(\mathbf{U}_a)$ may differ drastically.[4] A researcher taking such representations too literally may be led astray and venture into unwarranted speculations. One might argue that the choice of a representation may be influenced by taking into account additional data, collected with a different paradigm, or even of a different nature (e.g., physiological). Actually, some reflection indicates that, even in light of such data, the situation would not be altered significantly. The arbitrariness of the choice of a representation may be reduced, but does not disappear.

Moreover, models that are not formally equivalent may have predictions that are not distinguishable experimentally. This lack of "uniqueness" of the models is by no means peculiar to psychophysics, but is especially dramatic in this field. The fact is that the data basis is scarce, as compared with the ambitious explanations often entertained by researchers.

4. Some important properties will remain. For example, if the random variables $\mathbf{U}_a$ are independent, then the random variables $g(\mathbf{U}_a)$ are also independent.

Such considerations will lead us to adopt, in our presentation of psychophysical theory, a rather sober attitude toward models and scales. But what, one might ask, will then be at the center of the discussion?

LAWS VERSUS MODELS

Two central questions confront the psychophysical researcher:

1. What mechanisms, or models, may serve to explain the observed sensory responses?
2. What regularities, or laws, are suggested by the data?

The relative importance attached to these questions provides an interesting way of classifying workers in the field. One might argue that the opposition that I am hinting at here is artificial, and claim, virtuously, that everyone wants to know how the brain and the sensory systems work, and proceeds to find out by studying the regularities in the data.[5] Nevertheless, I believe that the classification is useful, and that there really are two schools of thought, or at least two research styles.

Classical psychophysics was certainly focused on the second question. Probably influenced by the successful history of physics, the early psychophysicists were searching for regularities or laws, rather than for models or mechanisms, hoping to build up a solid experimental foundation for ulterior, more ambitious, theoretical constructions. Stevens and his followers are direct descendants of Fechner's, while many other contemporary psychophysicists definitely belong in the first school, and see their research as an attempt at discovering mechanisms.

Some would certainly say that too much emphasis on the second question leads to rather dull enterprises, which at best prepare the way for more creative accomplishments. This may very well be the dominant opinion in psychophysics today. Despite this pessimistic appraisal, this book is organized around empirical laws, regularities, or invariants (these may be taken as synonyms). Models will play a role, however, and many will be discussed in detail; but they will appear as illustrations or special cases of more general structures. The main reason for this choice is that laws tend to have a better life expectancy than models. A law may be useful scientific device even when it is known to be falsified systematically in some conditions. Weber's law is a case in point. It is known to hold well at medium intensities, but fails dramatically at the lower end of the stimulus scale (and for some sensory continua, also at the upper end).[6] Nevertheless, Weber's law

5. One might also object that the distinction between models and laws is not clear, and could not easily be formalized, since it is a matter of degree. Both a model and a law define constraints on the data. In the statement of a law, economy of though presides, and a minimum of hypothetical concepts is used. Such preoccupation with economy is less prominent in the statement of a model, the domain of application of which can often be stretched to include the prediction of empirical relations beyond its intended scope, through the evocative power of its concepts.

6. A comparable example in physics is Boyle's law, which fails at low temperature. A more comprehensive description of the data is offered by van der Waal's law.

remains an important criterion for models. A discrimination model is judged acceptable only if, to a good approximation, it is consistent with Weber's law at medium intensities.

The search for regularities tends to be neglected in contemporary psychophysics. The organization of this book is a step toward a change of focus in psychophysical research.[7]

ON THE CONTENT OF THIS BOOK

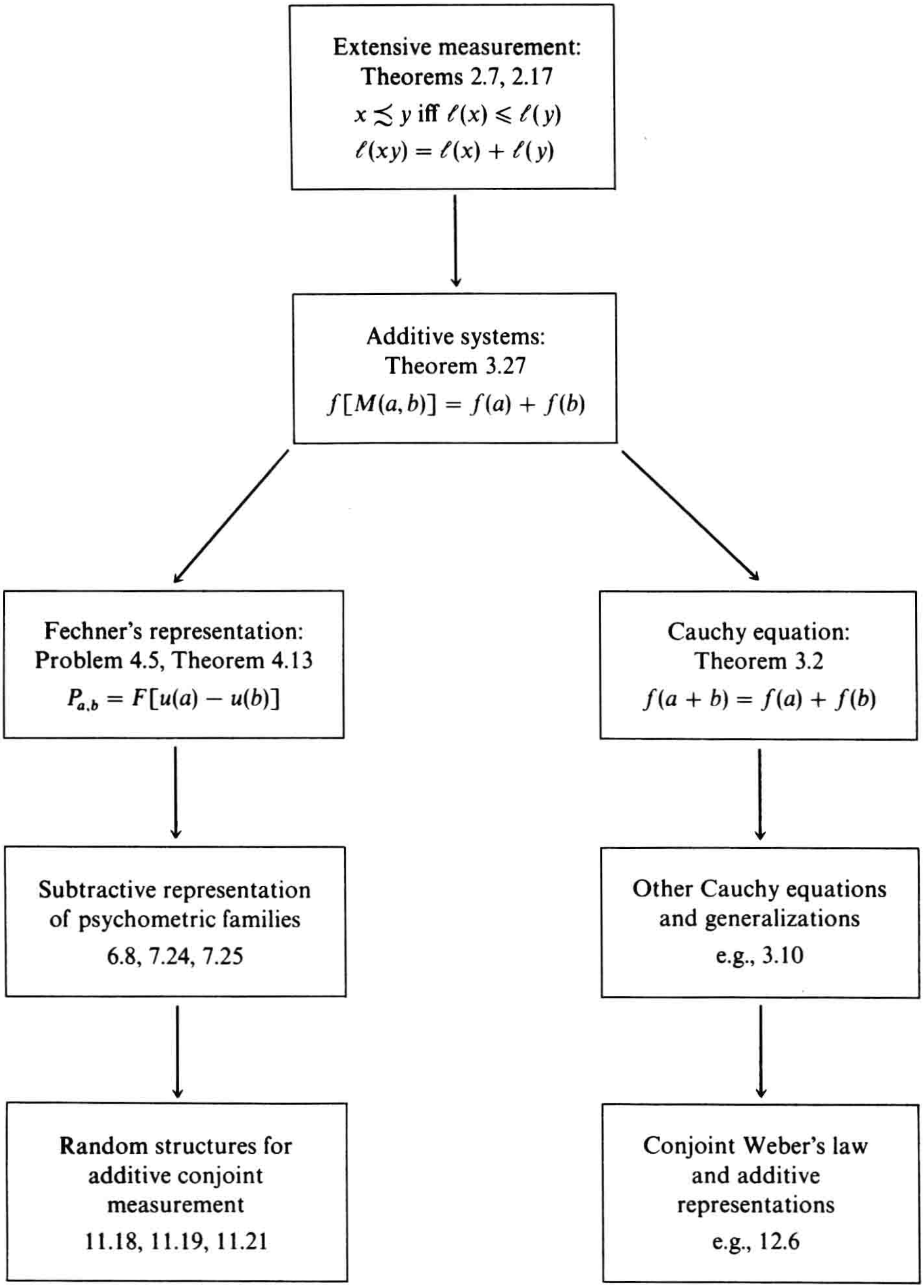
The Contents of this book may puzzle some readers, who will wonder about the role, in psychophysics, of the esoteric topics of Part I, such as "functional equations" or "extensive measurement." This material is included since we wanted our discussion of psychophysical theory to be accessible to any student with a bit of mathematical training (say, a couple of courses in calculus, maybe one college-level course in algebra), but equipped with a great deal of perseverance. Many fundamental results in psychophysical theory can be obtained through relatively straightforward applications of functional equations or extensive measurement techniques. Unfortunately, these techniques are only presented in specialized texts, and then in a manner which is often not exactly suitable for our purpose. Omitting them in this book would have rendered the task of the thorough student of psychophysical theory much more difficult. The interdependence of some of the most important mathematical results discussed here is illustrated in the following figure, which displays the "mathematical skeleton" of the book.

It is my hope that the efforts made to present the material of Part I in a style congenial to a wide audience will be perceptible and appreciated.

The reader seeking only a survey of psychophysical theory can start on Chapter 4, with the understanding that the proofs of a number of important results may have to be skipped.

Finally, it must be said that not all the topics that could be evoked by the title of this book will be dicussed. Among the regrettable omissions are color theory and multidimensional scaling. A chapter on the historically important question of the sensory threshold had been planned, but was dropped in the final draft and replaced by a few paragraphs in Chapter 10. Green and Luce's counting and timing models are only mentioned in passing (Green and Luce, 1974; Luce and Green, 1972, 1974). Levine's projective geometry representations of magnitude estimation and related data are not discussed (Levine, 1974). Sequential dependencies in psychophysical data, even though they certainly are theoretically significant, and empirically pervasive, are only alluded to.

7. This view of the role of models or theories in the scientific enterprise did not originate with the writer. It has been propounded by Ernst Mach, and critized by others, such as Max Planck. In the celebrated words of Mach (1871, English translation published as *History and Root of the Principle of the Conservation of Energy*, 1911): "The aim of natural science is to obtain connections among phenomena. Theories, however, are like withered leaves, which drop off after having enabled the organism of science to breathe for a time." (An account of the debate can be found in Frank, 1941.)



Mathematical backbone of the book. Only the main lines are indicated. An arrow linking two boxes indicates that the upper results can be used to infer the lower ones.