ENGINEERING MECHANICS

STATICS

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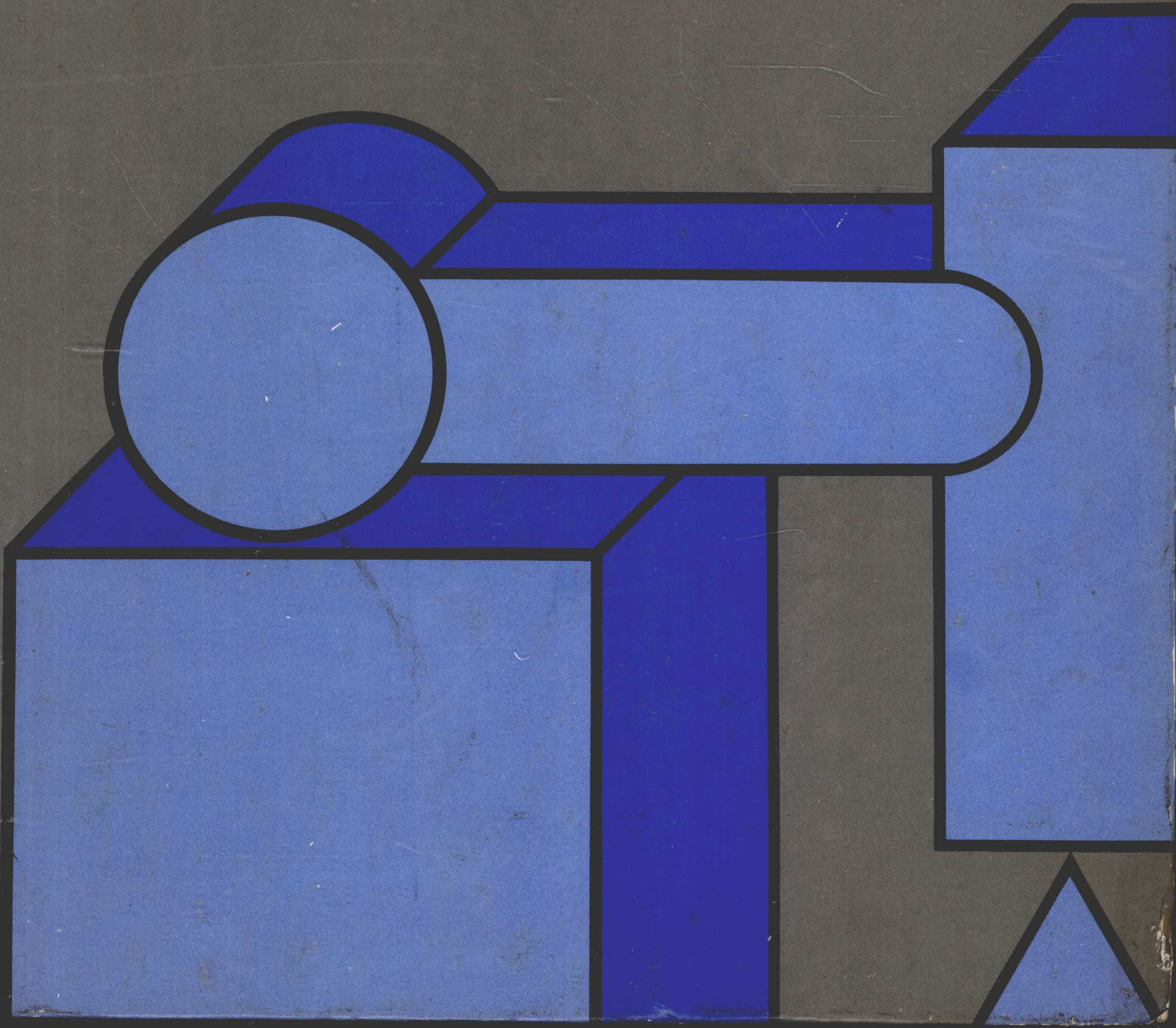

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ENGINEERING MECHANICS

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To Our Parents

SHANTI DEVI ANAND	DELLA JOSEPHINE CUNNIFF
ANANT RAM ANAND	MARTIN JOSEPH CUNNIFF

Preface

This book is the first part of a two-volume *Statics* and *Dynamics* set. The material covered in *Statics* has been extensively used in our classes; however, the basic format of each chapter also facilitates self-study. The theoretical material is carefully developed, and supported by a generous selection of worked-out examples, to assist the reader in acquiring problem-solving capabilities. As an aid to the student, each chapter concludes with a summary of the important ideas and equations covered.

After each major topic there are numerous exercise problems arranged approximately in order of increasing complexity. The instructor may choose to assign even-numbered problems for which the answers can be found at the back of the book. Also, we note that both the International System of units (SI) and the standard English gravitational system of units are used throughout the book. Consequently, the examples and assigned problems incorporate both sets of units.

Chapter 1 reviews the subject of vectors and the fundamental laws useful for the study of statics. The concepts of force and couple are introduced in Chapter 2, where we also consider the equilibrium of a particle. The equilibrium of rigid bodies is then covered in Chapter 3. Although two- as well as three-dimensional equilibrium is considered, stress is placed on the former since the salient points can be grasped without introducing algebraic complexity. The equilibrium of structures and frames is presented in Chapter 4, where the graphical approach has been adopted in light of current developments in computer graphics.

Centroids and distributed loads are considered in Chapter 5, which also includes the analysis of the cable. Following up on the concept of the first moment, we introduce the area moment of inertia and the mass moment of inertia in Chapter 6. Here we have deliberately avoided a lengthy treatment of three-dimensional integration procedures. Area and mass moments of inertia are placed early in the book to ensure that the student is exposed to the concept of inertia before studying dynamics.

All of Chapter 7 is devoted to beam analysis. Friction is then treated in Chapter 8. We have not included fluid friction, which from our experience, can be more adequately covered in subsequent coursework. The concept of virtual work is introduced in Chapter 9. We believe that it is important to include this chapter for two reasons. First, it provides an alternate method for deriving equations of static equilibrium, and second, it serves as a logical introduction to more advanced problems in mechanics.

To work effectively with the concepts presented here, students should be at least concurrently registered in the second-semester calculus course of a four-year engineering program. This assumes that they have had some exposure to limits, continuity, derivatives, definite and indefinite integrals, and are currently learning techniques of integration, improper integrals, and applications of integration.

All of the material in this book is suitable for a four-credit semester course. In the case of a three-credit semester course, the instructor may wish to delete all of Chapter 7 along with special topics such as the equilibrium of space trusses in Chapter 4 (Section 4.5), cables in Chapter 5 (Section 5.8), principal axes in Chapter 6 (Section 6.4), and potential energy and stability of equilibrium in Chapter 9 (Sections 9.4 and 9.5).

The material in this book has been used in many schools, in addition to the University of Maryland. This revised edition was reviewed by Professors T. J. Zilka and John T. Tielking, to whom we are most grateful. We also wish to acknowledge the contributions of those students, teachers, and colleagues who have used the earlier edition and have made many worthwhile suggestions.

D. K. A.
P. F. C.

List of Symbols

A	Area
$\mathbf{A}$	Vector with magnitude A
A_x, A_y, A_z	Components of $\mathbf{A}$ in the direction of X, Y, and Z
a	Constant or length
$\mathbf{B}$	Vector with magnitude B
B_x, B_y, B_z	Components of $\mathbf{B}$ in the direction of X, Y, and Z
b	Constant or length
b	Distributed load
C	Signifies a compression force
$\mathbf{C}$	Vector of magnitude C
c	Constant or length
C_x, C_y, C_z	Components of $\mathbf{C}$ in the direction of X, Y, and Z
$\mathbf{D}$	Vector of magnitude D
D	Distance or length
D_x, D_y, D_z	Components of $\mathbf{D}$ in the direction of X, Y, and Z
d	Distance or length
$\bar{d}$	Centroidal distance
$\mathbf{e}$	Unit vector
$\mathbf{e}_A$	Unit vector in the direction of vector $\mathbf{A}$
$\mathbf{E}$	Vector of magnitude E
E_x, E_y, E_z	Components of $\mathbf{E}$ in the direction of X, Y, and Z
$\mathbf{F}$	Force vector of magnitude F
F_x, F_y, F_z	Components of force in the direction of X, Y, Z

List of Symbols

F_{AB}	Force magnitude along truss member *AB*
f_k	Kinetic friction force
f_m	Static friction force
G	Universal gravitational constant
g	Gravitational acceleration
h	Distance or constant
I_x	Area or mass moment of inertia about *X* axis
I_{xy}	Area product or mass product moment of inertia about *X*-*Y* axes
$\bar{I}_x$	Centroidal area or mass moment of inertia about *X* axis
$\bar{I}_{xy}$	Centroidal area or mass moment of inertia about *X*-*Y* axes
$\mathbf{i}$	Unit vector along *X* axis
J	Polar moment of inertia
$\bar{J}$	Centroidal polar moment of inertia
$\mathbf{j}$	Unit vector along *Y* axis
k	Radius of gyration
k	Constant
$\mathbf{k}$	Unit vector along *Z* axis
k	Spring modulus
L	Length
ℓ, l	Length
M	Bending moment
$\mathbf{M}$	Moment vector of magnitude *M*
$\mathbf{M}_o$	Moment vector about point *o*
m	Mass
N	Normal reaction
p	Pressure
P	Load or force magnitude
p_a	Absolute pressure
p_o	Atmospheric pressure
Q	Shear force or magnitude of force

R	Resultant vector of magnitude R
R_x, R_y, R_z	Components of vector **R** in X, Y, and Z directions
r	Radius or distance
r	Position vector
s	Distance or length
T_o	Tension in cable evaluated at origin
T	Tension vector of magnitude T
t	Plate thickness
u	Derivative of y with respect to x
V	Volume
V	Potential energy
W	Work
W	Watts
w	Magnitude of weight
w	Distributed weight or load
w	Weight vector
X, Y, Z	Cartesian coordinate axes
$\bar{x}$, $\bar{y}$, $\bar{z}$	Centroidal distance
α	Angle related to pitch and radius of thread
δ	Signifies virtual displacement of virtual work
ω	Angular velocity
θ	Angle
ϕ	Angle
ϕ_0	Angle of repose
μ	Coefficient of kinetic friction
μ_0	Coefficient of static friction
π	Constant

List of Abbreviations

cos	cosine
cosh	hyperbolic cosine
da	derivative of variable *a*
ft	foot
J	joule or N·m
kg	kilogram
kip	kilopound
kN	kilonewton
lb	pound
ln	natural logarithm
m	meter
N	newton
Pa	pascal or N/m^2
PW	pitch of wrench
rad	radian
s, sec	second
sin	sine
sinh	hyperbolic sine
t	1000 kg
tan	tangent
tanh	hyperbolic tangent

Contents

Contents

Beam Analysis 239 7

Friction 267 8

Virtual Work and Energy 293 9

Basic Concepts 1

Mechanics is the science that deals with stationary and moving bodies under the action of forces. Theoretical mechanics is generally the concern of physicists and applied mathematicians; engineering mechanics is primarily of interest to engineers. The study of mechanics may be categorized into a study of the mechanics of fluids, the mechanics of bodies that deform, and the mechanics of rigid bodies. Here, we are concerned primarily with the mechanics of rigid bodies.

The mechanics of rigid bodies can be considered to consist of two parts, statics and dynamics. In general, statics treats the equilibrium of stationary bodies, as well as bodies moving at constant velocity, under the influence of various kinds of forces. In this volume we restrict our treatment of statics to include only the equilibrium of stationary bodies. The study of the motion of bodies and the forces that cause the motion is the subject of a separate volume entitled *Dynamics*.

Mechanics is among the oldest of the physical sciences. Interest in mechanical problems goes back to the time of Aristotle (384–322 B.C.). The lever fulcrum, as well as the theory of buoyancy, was first clearly explained by Archimedes (287–212 B.C.). After that, there were very few developments until, in the fifteenth century A.D., Leonardo da Vinci (1452–1519) continued Archimedes' work on levers and deduced the concept of moments as they apply to the equilibrium of bodies. Next, the laws of equilibrium and the parallelogram law were developed by Stevinus (1548–1620), and the relationship between the moment of a force and its components was established by Varignon (1654–1722). The idea of virtual work, which is an important concept in the formulation of advanced techniques used in mechanics, was established by Descartes

(1596–1650), whereas the concept of virtual displacements was used by Pascal (1623–1662) to establish analytically the direction of the propagation of stresses.

Despite the many important contributions made to mechanics before him, Newton's (1642–1727) three famous laws and his law of universal gravitation were perhaps the most important steps in the progress of mechanics. Newton's work on particles, based on geometry, was extended to rigid-body systems by Euler (1707–1783), whose work was based on calculus. At about this time, much of the work in mechanics was reformulated by Lagrange (1736–1813), whose analytical approach used the concepts of energy. The contributions of D'Alembert, Hamilton, Routh, and others, coupled with the fundamental work described earlier, has helped establish what we know as engineering mechanics, statics and dynamics.

We begin our study of statics by first reviewing some basic concepts. These include a few fundamental ideas pertaining to newtonian mechanics, dimensions and units, and finally a review of vectors.

1.1 Fundamental Concepts

The concepts fundamental to a study of mechanics are those of space, time, inertia, and force. The following is a brief description of these and other important concepts:

Space. By space we mean a geometric region in which physical events occur. It can have one, two, or three dimensions. Indeed, more than three dimensions can be conceptualized. Here we shall be concerned with, at most, three-dimensional space. Position in space is established relative to some reference system. The basic reference system necessary for newtonian mechanics is one that is considered fixed in space.* Measurements relative to this system are called absolute.

Time. Essentially a measure of the orderly succession of events occurring in space, time is considered as an absolute quantity. The unit of time is a second, which was originally related directly to the earth's rate of spin. Today, the standard of time is established by the frequency of oscillation of a cesium atom.

* In dynamics we show that a nonaccelerating reference frame is also an acceptable reference frame for newtonian mechanics.

Inertia. The ability of a body to resist a change in motion is called inertia.

Mass. The mass of a body is a quantitative measure of its inertia.

Force. A force is the action of one body on another. This action may exist because of contact between the bodies, which is called the push-pull effect; or it may exist with the bodies apart, which is called the field-of-force effect.

Particle. If the dimensions of a body are treated as negligible, the body is said to be a particle. The mass of a particle is assumed to be concentrated at a point, and a particle is sometimes called a mass point.

Rigid Body. A rigid body is characterized by the condition that any two points of the body remain at a fixed distance relative to each other for all time.

The study of mechanics derives from several fundamental laws, the most important of which were formulated by Sir Isaac Newton in 1687.

Newton's Laws

I. Every particle remains at rest or continues to move in a straight line with a uniform motion if there is no unbalanced force acting upon it.

II. The time rate of change of the linear momentum of a particle is proportional to the unbalanced force acting upon it and occurs in the direction in which the force acts. It can be shown that when the mass is constant the time rate of change of the linear momentum is equivalent to the product of the mass and its acceleration. Hence, we often use the more familiar relationship that says force equals mass times acceleration.

III. To every action there is an equal and opposite reaction. The mutual forces of two bodies acting upon each other are equal in magnitude, collinear, and opposite in direction.

In addition to these three basic laws, Newton also formulated the **law of gravitation**, which governs the mutual attraction between two isolated bodies. This law is expressed mathematically as

$$F = G\frac{m_1 m_2}{r^2}$$