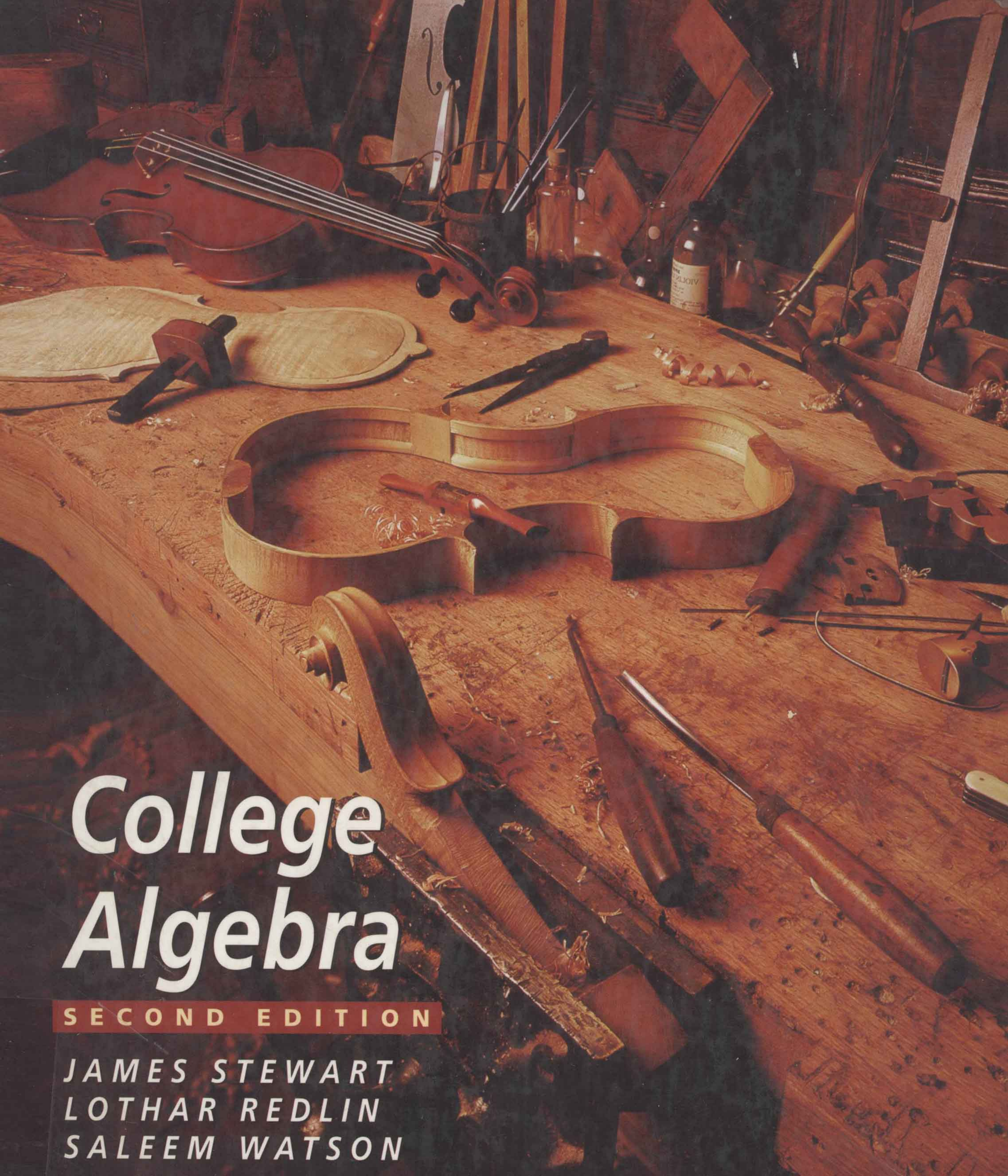

College Algebra

SECOND EDITION

JAMES STEWART
LOTHAR REDLIN
SALEEM WATSON

COLLEGE ALGEBRA

Second Edition

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BROOKS/COLE PUBLISHING COMPANY
I(T)P™ An International Thomson Publishing Company
Pacific Grove ■ Albany ■ Bonn ■ Boston ■ Cincinnati ■ Detroit ■ London ■ Madrid ■ Melbourne
Mexico City ■ New York ■ Paris ■ San Francisco ■ Singapore ■ Tokyo ■ Toronto ■ Washington

A GARY W. OSTEDT BOOK

Publisher: *Gary W. Ostedt*
Assistant Editor: *Elizabeth Barelli Rammel*
Marketing Team: *Patrick Farrant, Margaret Parks, Michele LeBoeuf*
Editorial Associate: *Carol Ann Benedict*
Production Editor: *Jamie Sue Brooks*
Production Service: *TECHarts*
Manuscript Editor: *Kathi Townes, TECHarts*
Cover Design: *Vernon T. Boes*
Cover Photo: *David Ash, Tony Stone Worldwide*
Illustrations: *TECHarts*
Typesetting: *Beacon Graphics*
Cover Printing: *Phoenix Color Corp.*
Printing and Binding: *R. R. Donnelley & Sons, Crawfordsville*

For more information, contact:

BROOKS/COLE PUBLISHING COMPANY
511 Forest Lodge Road
Pacific Grove, CA 93950
USA

International Thomson Publishing Europe
Berkshire House 168-173
High Holborn
London WC1V 7AA
England

Thomas Nelson Australia
102 Dodds Street
South Melbourne, 3205
Victoria, Australia

Nelson Canada
1120 Birchmount Road
Scarborough, Ontario
Canada M1K 5G4

International Thomson Editores
Campos Eliseos 385, Piso 7
Col. Polanco
11560 México D. F. México

International Thomson Publishing GmbH
Königswinterer Strasse 418
53227 Bonn
Germany

International Thomson Publishing Asia
221 Henderson Road
#05-10 Henderson Building
Singapore 0315

International Thomson Publishing Japan
Hirakawacho Kyowa Building, 3F
2-2-1 Hirakawacho
Chiyoda-ku, Tokyo 102
Japan

Printed in the United States of America

10 9 8 7 6 5 4 3

LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA
Stewart, James
College algebra/James Stewart, Lothar Redlin, Saleem Watson. 2nd ed.
p. cm.
Includes index.
ISBN 0-534-33983-2
1. Algebra I. Redlin, L. II. Watson, Saleem. III. Title.
QA152.2.2.S74 1996 95-32697
512.9–dc20 CIP

THIS BOOK IS PRINTED ON ACID-FREE RECYCLED PAPER

EXPONENTS AND RADICALS

$x^m x^n = x^{m+n}$

$(x^m)^n = x^{mn}$

$(xy)^n = x^n y^n$

$x^{1/n} = \sqrt[n]{x}$

$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$

$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$

$\frac{x^m}{x^n} = x^{m-n}$

$x^{-n} = \frac{1}{x^n}$

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

SPECIAL PRODUCTS

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

FACTORING FORMULAS

$x^2 - y^2 = (x + y)(x - y)$

$x^2 + 2xy + y^2 = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)^2$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

INEQUALITIES AND ABSOLUTE VALUE

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$|x| = a$ means $x = a$ or $x = -a$.

$|x| < a$ means $-a < x < a$.

$|x| > a$ means $x > a$ or $x < -a$.

DISTANCE AND MIDPOINT FORMULAS

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of P_1P_2: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

LINES

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through $P_1(x_1, y_1)$ with slope m:

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope m and y-intercept b:

$$y = mx + b$$

Two-intercept equation of line with x-intercept a and y-intercept b:

$$\frac{x}{a} + \frac{y}{b} = 1$$

LOGARITHMS

$y = \log_a x$ means $a^y = x$

$\log_a a^x = x$ $\qquad$ $a^{\log_a x} = x$

$\log_a 1 = 0$ $\qquad$ $\log_a a = 1$

$\log x = \log_{10} x$ $\qquad$ $\ln x = \log_e x$

$\log_a xy = \log_a x + \log_a y$

$\log_a \left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^b = b \log_a x$ $\qquad$ $\log_b x = \dfrac{\log_a x}{\log_a b}$

SEQUENCES AND SERIES

Arithmetic:

$$a,\ a+d,\ a+2d,\ a+3d,\ a+4d,\ \ldots$$

$$a_n = a + (n-1)d$$

$$S_n = \sum_{k=1}^{n} a_k = \frac{n}{2}[2a + (n-1)d] = n\left(\frac{a + a_n}{2}\right)$$

Geometric:

$$a,\ ar,\ ar^2,\ ar^3,\ ar^4,\ \ldots$$

$$a_n = ar^{n-1}$$

$$S_n = \sum_{k=1}^{n} a_k = a\,\frac{1-r^n}{1-r}$$

If $|r| < 1$, then the sum of an infinite geometric series is

$$S = \frac{a}{1-r}$$

THE BINOMIAL THEOREM

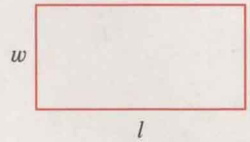
$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

GEOMETRIC FORMULAS

Formulas for area A, circumference C, and volume V:

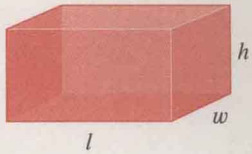
Rectangle

$A = lw$



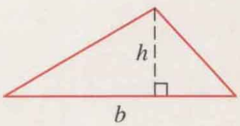
Box

$V = lwh$



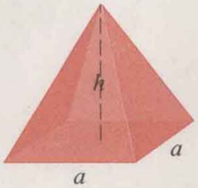
Triangle

$A = \frac{1}{2}bh$



Pyramid

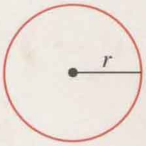
$V = \frac{1}{3}ha^2$



Circle

$A = \pi r^2$

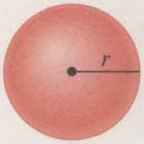
$C = 2\pi r$



Sphere

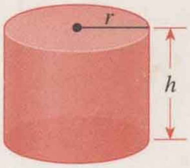
$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$



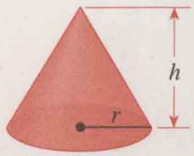
Cylinder

$V = \pi r^2 h$



Cone

$V = \frac{1}{3}\pi r^2 h$



COUNTING

1. **Fundamental Counting Principle:** Suppose that two events occur in order. If the first can occur in m ways and the second can occur in n ways (after the first has occurred), then the two events can occur in order in $m \times n$ ways.
2. The number of **permutations** of n objects taken r at a time is
$$P(n, r) = \frac{n!}{(n-r)!}$$
3. The number of **combinations** of n objects taken r at a time is
$$C(n, r) = \frac{n!}{r!\,(n-r)!}$$
4. The number of **subsets** of a set with n elements is 2^n.

PROBABILITY

Complement of an event:
$$P(E') = 1 - P(E)$$

Union of two events:
$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Intersection of two events:
$$P(E \cap F) = P(E)\,P(F \mid E)$$

If a game gives payoffs of $a_1, a_2, \ldots, a_n$ with probabilities $p_1, p_2, \ldots, p_n$, then the **expected value** is
$$E = a_1 p_1 + a_2 p_2 + \cdots + a_n p_n$$

FINANCE

Compound interest:
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where P is the principal, A is the amount after t years, r is the interest rate, and the interest is compounded n times per year.

Amount of an annuity:
$$A_f = R\,\frac{(1+i)^n - 1}{i}$$

where A_f is the final amount, R is the size of each payment, n is the number of payments, and i is the interest rate per time period.

Present value of an annuity:
$$A_p = R\,\frac{1 - (1+i)^{-n}}{i}$$

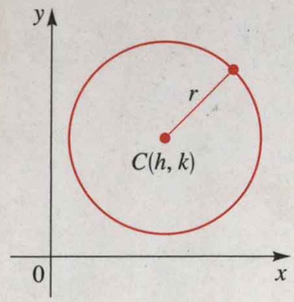
where A_p is the present value, R is the size of each payment, n is the number of payments, and i is the interest rate per time period.

Installment buying:
$$R = \frac{iA_p}{1 - (1+i)^{-n}}$$

where R is the size of the payment, A_p is the amount of the loan, n is the number of payments, and i is the interest rate per time period.

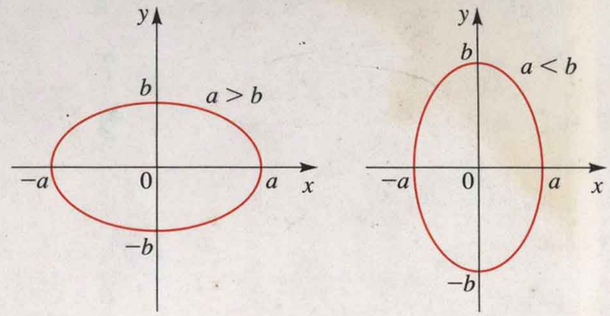
CONIC SECTIONS

CIRCLES



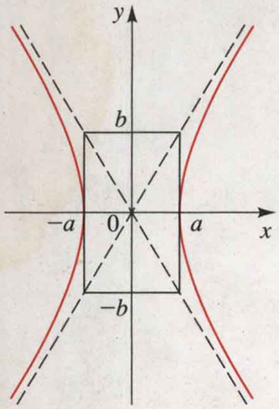
$$(x - h)^2 + (y - k)^2 = r^2$$

ELLIPSES

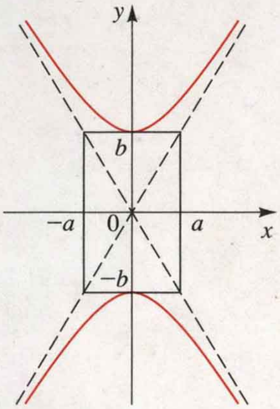


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

HYPERBOLAS

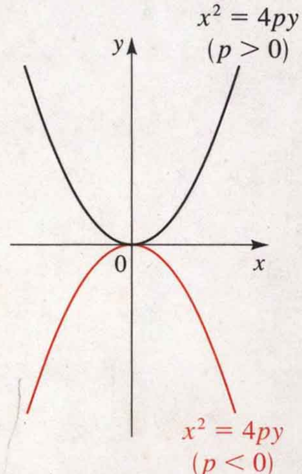


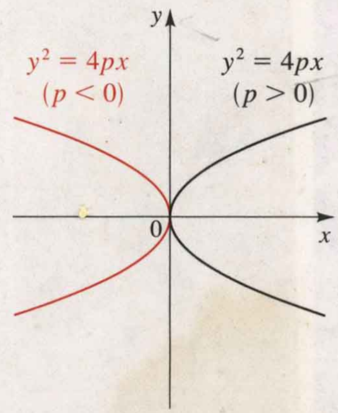
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

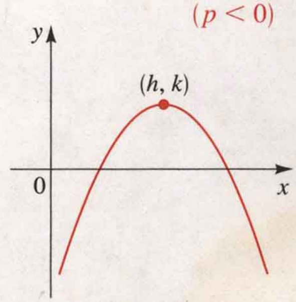


$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

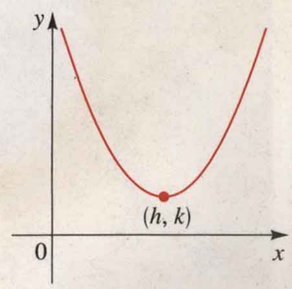
PARABOLAS







$y = a(x - h)^2 + k,$
$a < 0, h > 0, k > 0$



$y = a(x - h)^2 + k,$
$a > 0, h > 0, k > 0$

COLLEGE ALGEBRA

ABOUT THE COVER

The College Algebra course is an opportunity to learn about the beauty and practical power of mathematics. The course may also be a stepping stone to the study of calculus.

The violin, with its sound hole in the shape of an integral sign, has become a symbol of lead author James Stewart's calculus textbook series, which includes *Calculus, Third Edition,* and *Calculus: Early Transcendentals, Third Edition.*

The cover of this edition, which shows violin-making as a "work in progress," reflects both the Stewart authorship and the foundational importance of the College Algebra course.

ABOUT THE AUTHORS

James Stewart was educated at the University of Toronto and Stanford University, did research at the University of London, and now teaches at McMaster University. His research field is harmonic analysis.

He is the author of a best-selling calculus textbook series published by Brooks/Cole, including *Calculus, 3rd Ed.,* and *Calculus: Early Transcendentals, 3rd Ed.,* as well as a series of high school mathematics textbooks.

A talented violinist, Stewart was concertmaster of the McMaster Symphony Orchestra for eight years and played professionally in the Hamilton Philharmonic Orchestra. One of his greatest pleasures is playing string quartets.

Lothar Redlin grew up on Vancouver Island, received a Bachelor of Science degree from the University of Victoria, and a Ph.D. from McMaster University in 1978. After completing his education, he did research and taught at the University of Washington, the University of Waterloo, and California State University, Long Beach.

He is currently Associate Professor of Mathematics at The Pennsylvania State University, Abington-Ogontz Campus. His research field is topology.

Saleem Watson received his Bachelor of Science degree from Andrews University in Michigan. He did his graduate studies at Dalhousie University and McMaster University, where he received his Ph.D. in 1978. After completing his education, he did research at the Mathematics Institute of the University of Warsaw in Poland. He subsequently taught and did research at McMaster University and The Pennsylvania State University.

He is currently Professor of Mathematics at California State University, Long Beach. His research field is functional analysis.

The authors have also published *Mathematics for Calculus, Second Edition* (Brooks/Cole, 1993).

TO PHYLLIS

PREFACE

The art of teaching is the art of assisting discovery.
MARK VAN DOREN

For many students a College Algebra course represents the first opportunity to learn about the beauty and practical power of mathematics. Thus, teachers and textbook authors are faced with the challenge of teaching the techniques of the subject while at the same time imparting a concept of the true nature of mathematics. This text represents our view of how the subject can best be taught.

Our main goal in writing this edition was to sharpen the clarity of the exposition, while retaining the main features which have contributed to the success of this book. To assist students in understanding the examples we have included step-by-step comments on solutions, given in the margins so as not to interrupt the flow of the solutions. We have added many graphs and figures to remind students of the geometric meaning behind a calculation and to promote visual insight into formulas and theorems. We continue to present College Algebra as a *problem-solving* activity because we think this is what mathematics is all about. We have included many examples and exercises that make substantial use of college algebra to solve real-life problems. Our historical "vignettes" add interest to the subject and also serve to show the universality of mathematics. We feel that all these features make for a user-friendly book.

Graphing calculators are powerful tools for developing the students' understanding of equations and functions. We have devoted entire sections to the use of this tool so as to make substantial use of it where it is appropriate. Those who choose to teach these sections can follow up by using the subsections and exercises in the rest of the book which are clearly identified with a special icon: .

Many of the changes in the structure of the present book are a result of our own experience in teaching College Algebra. We have experimented with different ways of incorporating graphing calculators into the course and with different orders of presenting the material. We have benefited greatly from the insightful comments of our colleagues who have used the text and critiqued it section by section as they taught it. We have also benefited from the sharp insight of our reviewers.

SPECIAL FEATURES

Focus on Problem Solving

We are committed to helping students develop mathematical thinking rather than "memorizing all the rules," an endless and pointless task that many students are not taught to avoid. We believe that mathematical thinking is best taught by careful exposition and examples and well-graded exercises, but also by giving guidelines for problem solving. Thus, an emphasis on problem solving is integrated throughout the text. In addition, we have concluded each chapter with a section entitled *Focus on Problem Solving,* which highlights a particular problem-solving technique. The first of these sections, on pages 61–67, gives a general introduction to the principles of problem solving. Each *Focus* section includes problems of a varied nature that encourage students to apply their problem-solving skills. In selecting these problems we have kept in mind the following advice from David Hilbert: "A mathematical problem should be difficult in order to entice us, yet not inaccessible lest it mock our efforts."

Graphing Calculators and Computers

Calculator and computer technology provides completely new ways of visualizing mathematics and is affecting not only how a topic is taught but also what is emphasized. We have integrated the technology carefully in four sections devoted to the fundamentals of graphing devices—Sections 3.3, 4.2, 4.6, and 4.9. In other sections we have included subsections, examples, and exercises to show how the graphing calculator can be used (see, for instance, pages 223 and 351). One example of how technology enhances understanding is that we can use it to draw accurate graphs of *families* of functions, so that students can see how varying a parameter can affect the graph of a function (see pages 191–92 and 271). These graphing calculator sections, subsections, examples, and exercises, all marked with the special logo, are optional and may be skipped without loss of continuity, but we encourage you to try them. You may in fact wish to integrate graphing devices more fully into your course by using them for the "regular" sections of the text as well. It has been our experience that students enjoy working with the graphing calculator, and they learn the material better and with more enthusiasm when we require or recommend its use in our classes.

The availability of graphing calculators makes it not less important but far more important to clearly understand the concepts that underlie the image the calculator produces on its screen. Accordingly, all our calculator-oriented sections and subsections are preceded by sections in which students must sketch graphs by hand and analyze them, so they can understand precisely what the graphing device is doing when they later use it to simplify the routine, mechanical part of their work. We must never lose sight of the fact that we are teaching mathematical ideas, not the use of any particular tool. Thus, we treat the calculator as an aid to understanding, an extension of pencil and paper, not as the central feature of the course.

"Real-World" Applications

We have included substantial applied problems that we believe will capture the attention of students. These are integrated throughout the text in both examples and exercises. Applications from engineering, physics, chemistry, business, biology, environmental studies, and other fields show how mathematics is used to model real-life situations. We have carefully chosen applications that show the relevance of algebra to our daily lives and its remarkable power as a problem-solving tool. (See, for instance, the examples and exercises on energy expended in bird flight, page 110; determining the optimal shape for a can, pages 225–26; terminal velocity of a skydiver, page 350, Exercise 28; establishing time of death, page 387, Exercise 30.)

Mathematical "Vignettes"

Throughout this book we make use of the margins to provide short biographies of interesting mathematicians as well as applications of algebra to the "real world." The biographies often include a key insight that was discovered by the mathematician and is relevant to College Algebra. (See, for instance, the vignettes on Viète, page 91; Salt Lake City, page 140; and radiocarbon dating, page 371.) They serve to enliven the material and show that mathematics is an important, vital activity, and that even at this elementary level, it is fundamental in everyday life.

Review Sections and Chapter Tests

Each chapter ends with an extensive review section, including a Chapter Test designed to help the students gauge their progress. Brief answers to the odd-numbered exercises in each section (including the review exercises), and to all questions in the Chapter Tests, are given at the back of the book.

MAJOR CHANGES FOR THE SECOND EDITION

- Chapter 1, which is a review chapter, has been restructured so that the material on integer and rational exponents appears together in one section, and the material on algebraic expressions and factoring also appears in one section.
- Chapter 2 introduces the *Check Your Answer* feature, which is used everywhere in the book that equations occur, to emphasize the importance of looking back to check whether an answer is reasonable. Section 2.7 now contains the method of sign diagrams for solving nonlinear inequalities.
- Chapter 3 now combines the material on the coordinate plane and the material on functions. This permits earlier introduction of the graphing calculator. The elementary discussion on conics has been deleted—a full treatment of this topic now appears in Chapter 7.

(Major changes continue on page xiii)

CHECK YOUR ANSWER

$x = 4$:

$$\text{LHS} = 2 + \frac{5}{4-4} = 2 + \frac{5}{0}$$

$$\text{RHS} = \frac{4+1}{4-4} = \frac{5}{0}$$

Impossible—can't divide by 0. LHS and RHS are undefined, so $x = 4$ is not a solution. ✕

$$2x - 8 + 5 = x + 1 \quad \text{Distributive Property}$$

$$2x - 3 = x + 1 \quad \text{Simplify}$$

$$2x = x + 4 \quad \text{Add 3}$$

$$x = 4 \quad \text{Subtract } x$$

But now if we try to substitute $x = 4$ back into the original equation, we would be dividing by 0, which is impossible. So, this equation has no solution. ■

The first step in the preceding solution, multiplying by $x - 4$, had the effect of multiplying by 0. (Do you see why?) Multiplying each side of an equation by an expression that contains the variable may introduce extraneous solutions. That is why it is important to check every answer.

EXAMPLE 5 ■ Solving for one Variable in Terms of Others

The surface area A of the closed rectangular box shown in Figure 1 can be calculated from the length l, the width w, and the height h according to the formula

$$A = 2lw + 2wh + 2lh$$

Solve for w in terms of the other variables in this equation.

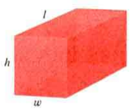


FIGURE 1
A closed rectangular box

SOLUTION

Although this equation involves more than one variable, we solve it as usual by isolating w on one side, treating the other variables as we would numbers.

$$A = (2lw + 2wh) + 2lh \quad \text{Collect terms involving } w$$

$$A - 2lh = 2lw + 2wh \quad \text{Subtract } 2lh$$

$$A - 2lh = (2l + 2h)w \quad \text{Factor } w \text{ from RHS}$$

$$\frac{A - 2lh}{2l + 2h} = w \quad \text{Divide by } 2l + 2h$$

The solution is $w = \dfrac{A - 2lh}{2l + 2h}$. ■

It is, of course, irrelevant which letters we use for the variables in an algebra problem—the letters have no effect on the solution. Nevertheless, it is helpful in applied problems to use letters that are somehow related to the quantities they represent. In other problems, it is customary to use letters from the end of the alphabet $(\ldots, x, y, z)$ for unknown quantities that we are solving for, and to use

◀ Author notes provide step-by-step comments on solutions.

◀ *Check Your Answer* emphasizes the importance of looking back at your work to check whether an anwer is reasonable.

SECTION 1.5 FRACTIONAL EXPRESSIONS 53

It is sometimes useful to use the formula

$$\frac{A}{C} + \frac{B}{C} = \frac{A+B}{C}$$

backward to write a quotient as a sum of fractions; that is, we write

$$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$$

For example, in certain calculus problems it is advantageous to write

$$\frac{x+3}{x} = \frac{x}{x} + \frac{3}{x} = 1 + \frac{3}{x}$$

But remember to avoid the following common error:

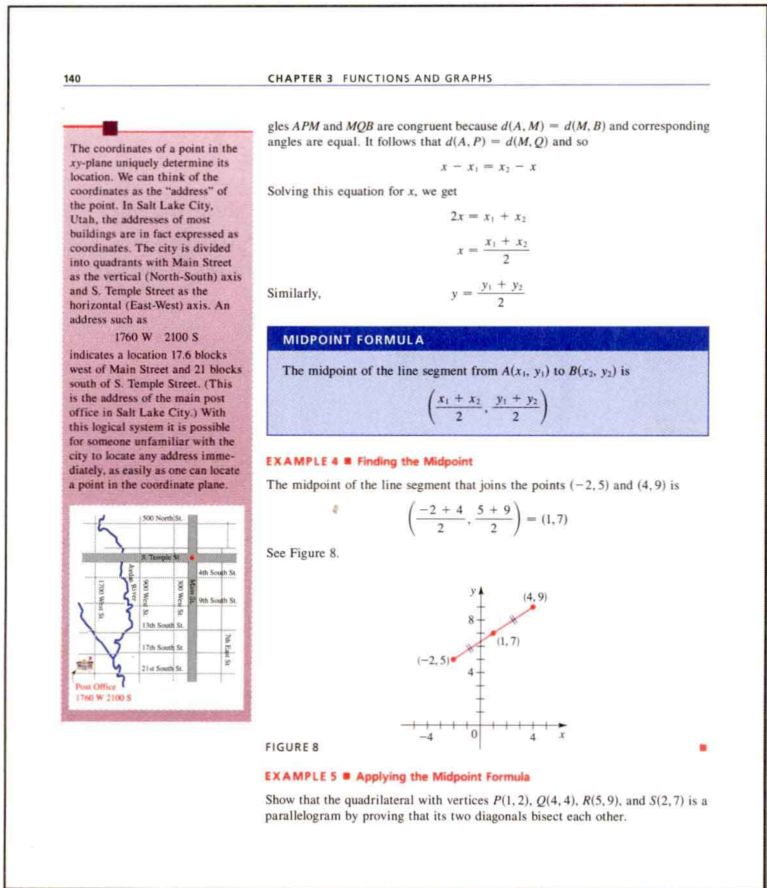
$$\frac{A}{B+C} \neq \frac{A}{B} + \frac{A}{C}$$

Do not make the mistake of applying properties of multiplication to the operation of addition. Many of the common errors in algebra involve doing just that. The following table states several properties of multiplication and illustrates the error in applying them to addition.

Correct multiplication property	Common error with addition
$(a \cdot b)^2 = a^2 \cdot b^2$	$(a + b)^2 \neq a^2 + b^2$
$\sqrt{a \cdot b} = \sqrt{a}\sqrt{b} \quad (a, b \geq 0)$	$\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$
$\sqrt{a^2 \cdot b^2} = a \cdot b \quad (a, b \geq 0)$	$\sqrt{a^2 + b^2} \neq a + b$
$\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{a \cdot b}$	$\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a + b}$
$\frac{ab}{a} = b$	$\frac{a + b}{a} \neq b$
$a^{-1} \cdot b^{-1} = \frac{1}{a \cdot b}$	$a^{-1} + b^{-1} \neq \frac{1}{a + b}$

To verify that the formulas in the right-hand column are wrong, simply substitute numbers for a and b and calculate each side. For example, if we take $a = 2$

This *Warning* symbol points out situations where many students make the same mistake. ▶


140 CHAPTER 3 FUNCTIONS AND GRAPHS

The coordinates of a point in the xy-plane uniquely determine its location. We can think of the coordinates as the "address" of the point. In Salt Lake City, Utah, the addresses of most buildings are in fact expressed as coordinates. The city is divided into quadrants with Main Street as the vertical (North-South) axis and S. Temple Street as the horizontal (East-West) axis. An address such as

1760 W 2100 S

indicates a location 17.6 blocks west of Main Street and 21 blocks south of S. Temple Street. (This is the address of the main post office in Salt Lake City.) With this logical system it is possible for someone unfamiliar with the city to locate any address immediately, as easily as one can locate a point in the coordinate plane.

gles APM and MQB are congruent because $d(A, M) = d(M, B)$ and corresponding angles are equal. It follows that $d(A, P) = d(M, Q)$ and so

$$x - x_1 = x_2 - x$$

Solving this equation for x, we get

$$2x = x_1 + x_2$$

$$x = \frac{x_1 + x_2}{2}$$

Similarly, $$y = \frac{y_1 + y_2}{2}$$

MIDPOINT FORMULA

The midpoint of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

EXAMPLE 4 ■ Finding the Midpoint

The midpoint of the line segment that joins the points $(-2, 5)$ and $(4, 9)$ is

$$\left(\frac{-2 + 4}{2}, \frac{5 + 9}{2}\right) = (1, 7)$$

See Figure 8.

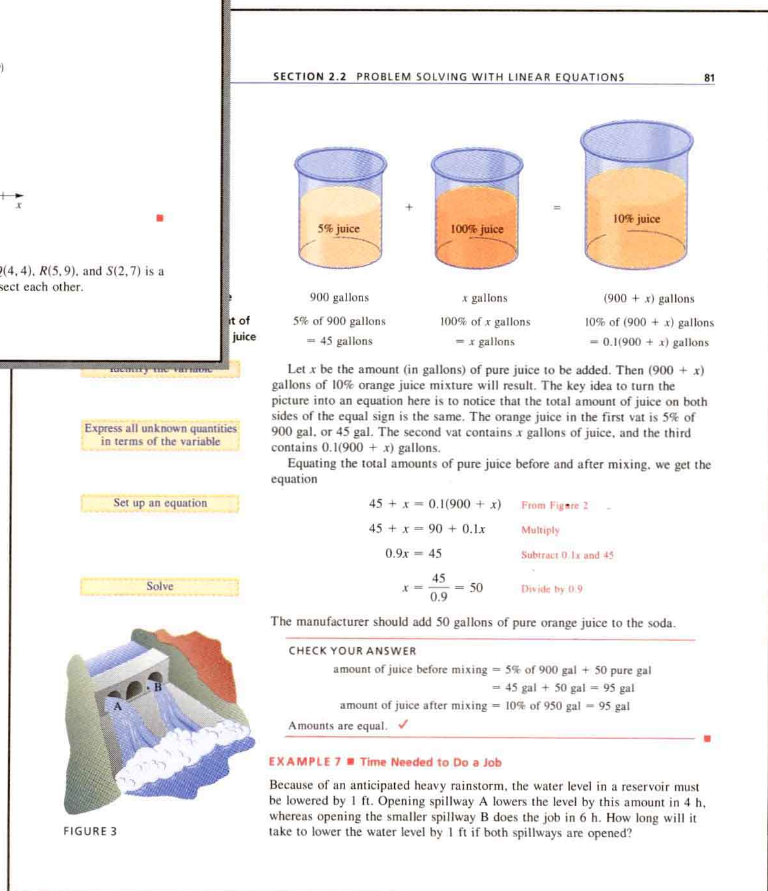
FIGURE 8 ■

EXAMPLE 5 ■ Applying the Midpoint Formula

Show that the quadrilateral with vertices $P(1, 2)$, $Q(4, 4)$, $R(5, 9)$, and $S(2, 7)$ is a parallelogram by proving that its two diagonals bisect each other.

◀ Mathematical vignettes provide applications of algebra to the "real world" or give short biographies of mathematicians, contemporary as well as historical.

◀ Summary boxes organize and clarify topics and highlight key ideas.


SECTION 2.2 PROBLEM SOLVING WITH LINEAR EQUATIONS 81

900 gallons	x gallons	$(900 + x)$ gallons
5% of 900 gallons	100% of x gallons	10% of $(900 + x)$ gallons
= 45 gallons	= x gallons	= $0.1(900 + x)$ gallons

t of juice

Let x be the amount (in gallons) of pure juice to be added. Then $(900 + x)$ gallons of 10% orange juice mixture will result. The key idea to turn the picture into an equation here is to notice that the total amount of juice on both sides of the equal sign is the same. The orange juice in the first vat is 5% of 900 gal, or 45 gal. The second vat contains x gallons of juice, and the third contains $0.1(900 + x)$ gallons.

Express all unknown quantities in terms of the variable

Equating the total amounts of pure juice before and after mixing, we get the equation

Set up an equation

$$45 + x = 0.1(900 + x) \quad \text{From Figure 2}$$

$$45 + x = 90 + 0.1x \quad \text{Multiply}$$

$$0.9x = 45 \quad \text{Subtract 0.1x and 45}$$

Solve

$$x = \frac{45}{0.9} = 50 \quad \text{Divide by 0.9}$$

The manufacturer should add 50 gallons of pure orange juice to the soda.

CHECK YOUR ANSWER

amount of juice before mixing = 5% of 900 gal + 50 pure gal
= 45 gal + 50 gal = 95 gal
amount of juice after mixing = 10% of 950 gal = 95 gal

Amounts are equal. ✓ ■

FIGURE 3

EXAMPLE 7 ■ Time Needed to Do a Job

Because of an anticipated heavy rainstorm, the water level in a reservoir must be lowered by 1 ft. Opening spillway A lowers the level by this amount in 4 h, whereas opening the smaller spillway B does the job in 6 h. How long will it take to lower the water level by 1 ft if both spillways are opened?

Problem-solving notes in margins apply problem-solving steps to examples in the text. ▶

Check Your Answer ▶

Example titles clarify the purpose of examples. ▶

◀ Graphing calculator exercises are indicated with the graphing calculator logo.

◀ Real-world applications show the relevance of algebra to everyday life and indicate its remarkable problem-solving power.

350 CHAPTER 5 EXPONENTIAL AND LOGARITHMIC FUNCTIONS

25. Under ideal conditions, a certain type of bacteria has a relative growth rate of 220% per hour. A number of these bacteria are introduced accidentally into a food product. Two hours after contamination, a bacteria count shows that there are about 40,000 bacteria in the food.
(a) Find the initial number of bacteria introduced into the food.
(b) Find the estimated number of bacteria in the food three hours after contamination.

26. A radioactive substance decays in such a way that the amount of mass remaining after t days is given by the function

$$m(t) = 13e^{-0.015t}$$

where $m(t)$ is measured in kilograms.
(a) Find the mass at time $t = 0$.
(b) How much of the mass remains after 45 days?

27. Radioactive iodine is used by doctors as a tracer in diagnosing certain thyroid gland disorders. This type of iodine decays in such a way that the mass remaining after t days is given by the function

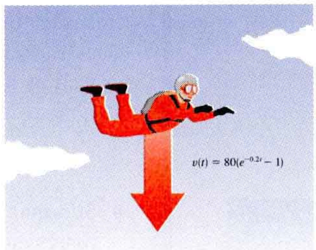
$$m(t) = 6e^{-0.087t}$$

where $m(t)$ is measured in grams.
(a) Find the mass at time $t = 0$.
(b) How much of the mass remains after 20 days?

28. A sky diver is dropped from a reasonable height above the ground. The air resistance she experiences is proportional to her velocity, and the constant of proportionality is 0.2. It can be shown that the velocity of the sky diver at time t is given by

$$v(t) = 80(e^{-0.2t} - 1)$$

where t is measured in seconds and $v(t)$ is measured in feet per second (ft/s).
(a) Find the initial velocity of the sky diver.
(b) Find the velocity after 5 s and after 10 s.
(c) Draw a graph of the velocity function $v(t)$.
(d) The maximum velocity of a falling object with wind resistance is called its *terminal velocity*. From the graph in part (c), find the terminal velocity of this sky diver.



29. A 50-gallon barrel is filled completely with pure water. Salt water with a concentration of 0.3 lb/gal is then pumped into the barrel, and the resulting mixture overflows at the same rate. The amount of salt in the barrel at time t is given by

$$Q(t) = 15(1 - e^{-0.04t})$$

where t is measured in minutes and $Q(t)$ is measured in pounds.
(a) How much salt is in the barrel after five minutes?
(b) How much salt is in the barrel after ten minutes?
(c) Draw a graph of the function $Q(t)$.
(d) From the graph in part (c), what value does the amount of salt in the barrel approach as t becomes large? Is this what you would expect?

Families of functions can be studied using the graphing calculator, so that students can see how varying a parameter affects the graph of a function. ▶

SECTION 4.2 USING GRAPHING DEVICES TO GRAPH POLYNOMIALS 271

SOLVING INEQUALITIES

In the next example, we solve an inequality using the graph of a polynomial.

EXAMPLE 6 ■ Solving an Inequality Using a Graphing Calculator

Solve the inequality

$$2x^4 - 6x^3 - 5x^2 - 3x - 3 \leq 0$$

SOLUTION

First we graph the polynomial $P(x) = 2x^4 - 6x^3 - 5x^2 - 3x - 3$, as in Figure 8. To solve the inequality $P(x) \leq 0$ we must find all values of x for which the graph of the function lies on or below the x-axis. From the graph we see that the solution is the interval that lies between the two x-intercepts. By zooming in we find that the x-intercepts are -0.79 and 3.79 (correct to two decimal places), so the approximate solution of the inequality is the interval $[-0.79, 3.79]$. ■

4
−100

FIGURE 8
$P(x) = 2x^4 - 6x^3 - 5x^2 - 3x - 3$

FAMILIES OF POLYNOMIALS

A graphing calculator enables us to quickly draw the graphs of many functions at once, on the same viewing screen. This enables us to see how changing a value in the definition of the functions affects the shape of its graph. In the next example we apply this principle to a family of third-degree polynomials.

EXAMPLE 7 ■ A Family of Polynomials

Sketch the family of polynomials $P(x) = x^3 - cx^2$ for $c = 0, 1, 2,$ and 3. How does changing the value of c affect the graph?

SOLUTION

The polynomials

$$P_0(x) = x^3 \qquad P_1(x) = x^3 - x^2$$
$$P_2(x) = x^3 - 2x^2 \qquad P_3(x) = x^3 - 3x^2$$

are graphed in Figure 9. We see that increasing the value of c causes the graph to develop an increasingly deep "valley" to the right of the y-axis, creating a local maximum at the origin and a local minimum at a point in quadrant IV. This local minimum moves lower and further to the right as c increases. ■

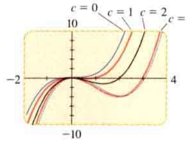
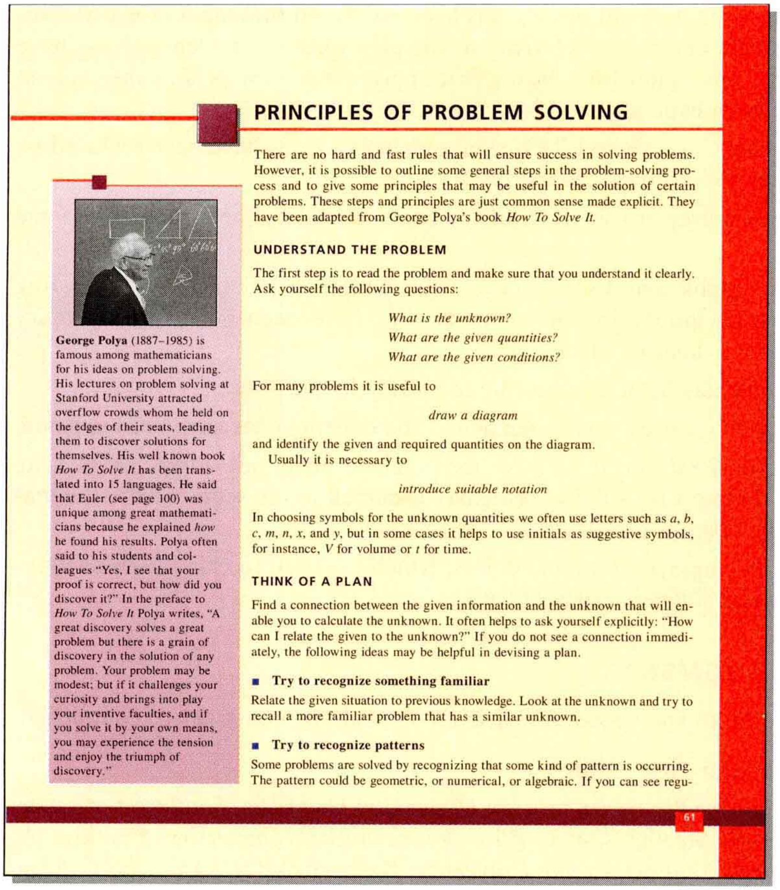


FIGURE 9
$P(x) = x^3 - cx^2$


PRINCIPLES OF PROBLEM SOLVING

There are no hard and fast rules that will ensure success in solving problems. However, it is possible to outline some general steps in the problem-solving process and to give some principles that may be useful in the solution of certain problems. These steps and principles are just common sense made explicit. They have been adapted from George Polya's book *How To Solve It.*

UNDERSTAND THE PROBLEM

The first step is to read the problem and make sure that you understand it clearly. Ask yourself the following questions:

What is the unknown?
What are the given quantities?
What are the given conditions?

For many problems it is useful to

draw a diagram

and identify the given and required quantities on the diagram.
Usually it is necessary to

introduce suitable notation

In choosing symbols for the unknown quantities we often use letters such as a, b, c, m, n, x, and y, but in some cases it helps to use initials as suggestive symbols, for instance, V for volume or t for time.

THINK OF A PLAN

Find a connection between the given information and the unknown that will enable you to calculate the unknown. It often helps to ask yourself explicitly: "How can I relate the given to the unknown?" If you do not see a connection immediately, the following ideas may be helpful in devising a plan.

- **Try to recognize something familiar**

Relate the given situation to previous knowledge. Look at the unknown and try to recall a more familiar problem that has a similar unknown.

- **Try to recognize patterns**

Some problems are solved by recognizing that some kind of pattern is occurring. The pattern could be geometric, or numerical, or algebraic. If you can see regu-

George Polya (1887–1985) is famous among mathematicians for his ideas on problem solving. His lectures on problem solving at Stanford University attracted overflow crowds whom he held on the edges of their seats, leading them to discover solutions for themselves. His well known book *How To Solve It* has been translated into 15 languages. He said that Euler (see page 100) was unique among great mathematicians because he explained *how* he found his results. Polya often said to his students and colleagues "Yes, I see that your proof is correct, but how did you discover it?" In the preface to *How To Solve It* Polya writes, "A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery."

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◀ *Focus on Problem Solving* sections include many new problems that are related to the problem-solving principle discussed in the section or to the material of the chapter itself.

◀ Mathematical vignettes provide short biographies of interesting mathematicians, contemporary as well as historical, or give applications of algebra to the real world.

(Major changes continued from page ix)

- Chapter 4 on polynomial and rational functions has been restructured to begin with the idea of the graph of a polynomial function; the material on the theory of equations now comes later in the chapter. Many of the exercise sets in this chapter have been rewritten to include problems that involve more straightforward factorizations.
- Chapter 5 has been completely rewritten to emphasize the importance of the natural exponential function and its role in applications. Exponential and logarithmic equations are now treated in a separate section, Section 5.5.
- In Chapter 6, the concept of echelon form has been introduced to clarify the Gaussian elimination process, and many of the Gaussian elimination exercises have been simplified from a computational point of view.
- Chapter 7 is a new chapter—it contains a complete treatment of conic sections, material that in the first edition was divided between Chapter 3 and an appendix. The summary boxes for each conic have been restructured and clarified, and we now summarize guidelines for sketching hyperbolas.
- Chapters 8 and 9 retain the structure of the first edition, with all the new features and enhancements.