An Outline for the Study of Calculus

Volume I

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AN OUTLINE FOR THE STUDY OF CALCULUS Volume I

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AN OUTLINE FOR THE STUDY OF CALCULUS Volume I

Preface

Each section of the outline includes all of the most important definitions and theorems that are usually found in a course in calculus and analytic geometry. Often these are followed by a discussion that elaborates the concepts and presents a summary of problem solving techniques. A selection of exercises with complete and detailed solutions, including all graphs, is given for each section. At the end of each chapter there is a set of review exercises, also with complete solutions. In the Appendix there is a test for each chapter with a time limit indicated, followed by solutions for the test.

For those exercises that are more easily solved by using a computer, general flow charts that show how to apply the computer are given. Each flow chart is followed by a sample program, written in BASIC, that illustrates the solution of a particular exercise. The computer solutions are found in Chapters 7, 16, and 21.

The outline may be used for self study or to supplement any standard three semester course in calculus. Volume I contains Chapters 1-8, Volume II contains Chapters 9-16, and Volume III contains Chapters 17-21. The definitions, theorems, and exercises are taken from *The Calculus with Analytic Geometry, third edition,* by Louis Leithold. The chapter and section numbers and the exercise numbers agree with those used in Leithold. However, the chapter tests found in the Appendix are compiled from test questions that I have used with my own students at De Anza College.

J.H.M

Contents

1

Real numbers, introduction to analytic geometry, and functions

1.1 SETS, REAL NUMBERS, AND INEQUALITIES

Inequalities are used to define the limit of a function—a fundamental concept of calculus. The technique for finding the solution set of a first degree inequality is similar to that for solving a first degree equation, except that you must remember to reverse the sense of the inequality when you multiply or divide each member of the inequality by a negative number. However, to solve an inequality that is not first degree requires special techniques that are quite different from those used to solve equations. Frequently, the steps involve finding the union or the intersection of sets. Following is a list of those definitions and theorems that are most frequently used when solving inequalities.

1.1.4 Definition Let A and B be two sets. The *union* of A and B, denoted by $A \cup B$ and read "A union B," is the set of all elements that are in A or in B or in both A and B.

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

1.1.5 Definition Let A and B be two sets. The *intersection* of A and B, denoted by $A \cap B$ and read "A intersection B," is the set of all elements that are in both A and B.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

1.1.22 Theorem If $a < b$, then $a + c < b + c$, and $a - c < b - c$ if c is any real number.

1.1.24 Theorem If $a < b$ and c is any positive number, then $ac < bc$.

1.1.25 Theorem If $a < b$ and c is any negative number, then $ac > bc$.

1.1.33 Definition The *open interval* from a to b denoted by (a, b), is defined by

$$(a, b) = \{x \mid a < x < b\}$$

1.1.34 Definition The *closed interval* from a to b, denoted by $[a, b]$, is defined by

$$[a, b] = \{x \mid a \leqslant x \leqslant b\}$$

1.1.35 Definition The *interval half-open on the left,* denoted by $(a, b]$, is defined by

$$(a, b] = \{x \mid a < x \leqslant b\}$$

1.1.36 Definition The *interval half-open on the right,* denoted by $[a, b)$, is defined by

$$[a, b) = \{x \mid a \leqslant x < b\}$$

1.1.37 Definition

(i) $(a, +\infty) = \{x \mid x > a\}$
(ii) $(-\infty, b) = \{x \mid x < b\}$
(iii) $[a, +\infty) = \{x \mid x \geqslant a\}$
(iv) $(-\infty, b] = \{x \mid x \leqslant b\}$
(v) $(-\infty, +\infty) = R^1$

Exercises 1.1

10. If $A = \{0, 2, 4, 6, 8\}, B = \{1, 2, 4, 8\}, C = \{1, 3, 5, 7, 9,\}$, and $D = \{0, 3, 6, 9\}$, then represent $(A \cup B) \cap (C \cup D)$ by listing its members within braces.

SOLUTION

$$A \cup B = \{0, 1, 2, 4, 6, 8\}$$
$$C \cup D = \{0, 1, 3, 5, 6, 7, 9\}$$
$$(A \cup B) \cap (C \cup D) = \{0, 1, 6\}$$

In Exercises 14-30, find the solution set of the given inequality and illustrate the set on the real number line.

14. $3x - 5 < \frac{3}{4}x + \frac{1 - x}{3}$

SOLUTION: First, we "eliminate" the fractions.

$$12(3x - 5) < 12\left(\frac{3}{4}x + \frac{1 - x}{3}\right)$$

$$36x - 60 < 3 \cdot 3x + 4(1 - x)$$
$$36x - 60 < 9x + 4 - 4x$$
$$31x < 64$$
$$x < \tfrac{64}{31}$$

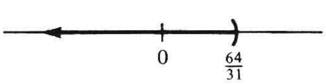

Figure 1.1.14

Hence, the solution set is $(-\infty, \frac{64}{31})$, as illustrated in Fig. 1.1.14.

16. $2 \leqslant 5 - 3x < 11$

SOLUTION: We reduce the middle expression to "x" by first adding -5 to each of the three expressions.

$$-3 \leqslant -3x < 6$$

Next, we divide each expression by -3 and reverse the sense of the inequality.

$$1 \geqslant x > -2$$
$$-2 < x \leqslant 1$$

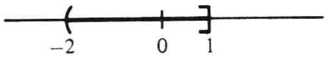

Figure 1.1.16

Thus, the solution set is $(-2, 1]$, as illustrated in Fig. 1.1.16.

20. $\dfrac{2}{1-x} \leqslant 1$

SOLUTION: Since the multiplier needed to eliminate the fraction is $1 - x$, which may be either positive or negative, we must consider two cases.

Case 1: $1 - x > 0$, or, equivalently, $x < 1$

Multiplying both sides of the given inequality by $1 - x$, we get

$$2 \leqslant 1 - x$$
$$x \leqslant -1$$

Thus, the solution set for Case 1 is $\{x | x < 1 \text{ and } x \leqslant -1\}$, or, equivalently, $\{x | x < 1\} \cap \{x | x \leqslant -1\} = \{x | x \leqslant -1\} = (-\infty, -1]$.

Case 2: $1 - x < 0$, or, equivalently, $x > 1$.

Multiplying both sides of the given inequality by $1 - x$ and reversing the sense of the inequality, we have

$$2 \geqslant 1 - x$$
$$x \geqslant -1$$

Thus, the solution set for Case 2 is $\{x | x > 1\} \cap \{x | x \geqslant -1\} = \{x | x > 1\} = (1, +\infty)$.

Finally, the solution set for the given inequality is the union of the solution sets for Case 1 and Case 2, namely $(-\infty, -1] \cup (1, +\infty)$, as shown in Fig. 1.1.20.

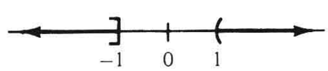

Figure 1.1.20

ALTERNATE SOLUTION: We do not eliminate the fraction, but rather we write the given inequality in zero form; that is, with zero on one side, and then simplify.

$$\frac{2}{1-x} - 1 \leqslant 0$$

$$\frac{2-(1-x)}{1-x} \leqslant 0$$

$$\frac{x+1}{1-x} \leqslant 0$$

$$\frac{x+1}{x-1} \geqslant 0 \qquad (1)$$

Note that we reverse the sense of the inequality on the last step, because the multiplier is -1. Next, we consider the factors $x + 1$ and $x - 1$ separately. The factor $x + 1$ has a positive value if $x + 1 > 0$, or, equivalently, if $x > -1$, and $x + 1$ has a negative value if $x < -1$. Similarly, $x - 1$ is positive if $x > 1$ and negative if $x < 1$. Table 20 summarizes this discussion about the factors $x + 1$ and $x - 1$ and also indicates for each interval whether the fraction $(x + 1)/(x - 1)$ is positive or negative. The signs for this fraction are found by using the fact that the quotient of two positive or of two negative numbers is positive, whereas the quotient of one positive and one negative number is negative. Since strict inequality (1) is satisfied whenever the fraction has a positive value, and equation (1) is satisfied if $x = -1$, the solution set for (1) is $\{x | x \leqslant -1 \text{ or } x > 1\} = (-\infty, -1] \cup (1, +\infty)$.

Table 20

	$x<-1$	$x=-1$	$-1<x<1$	$x=1$	$x>1$
$x+1$	−	0	+	+	+
$x-1$	−	−	−	0	+
$\dfrac{x+1}{x-1}$	+	0	−	does not exist	+

24. $x^2 - 3x + 2 > 0$

SOLUTION: First we factor.

$$(x-2)(x-1) > 0 \tag{2}$$

As in the alternate solution for Exercise 20, we consider the factors separately. Table 24 indicates that $x - 2$ has a positive value if $x > 2$ and a negative value if $x < 2$. Similarly, $x - 1$ is positive if $x > 1$ and negative if $x < 1$. The product $(x-2)(x-1)$ is positive whenever both factors are positive or both factors are negative. Thus, the solution set for (2) is $\{x \mid x > 2 \text{ or } x < 1\} = (-\infty, 1) \cup (2, +\infty)$, as illustrated in Fig. 1.1.24.

Figure 1.1.24

Table 24

	$x<1$	$x=1$	$1<x<2$	$x=2$	$x>2$
$x-2$	−	−	−	0	+
$x-1$	−	0	+	+	+
$(x-2)(x-1)$	+	0	−	0	+

28. $2x^2 - 6x + 3 < 0$

SOLUTION: First, we divide by 2.

$$x^2 - 3x + \frac{3}{2} < 0$$

To "complete the square" we add and subtract $(-3/2)^2$.

$$x^2 - 3x + \left(\frac{-3}{2}\right)^2 - \left(\frac{-3}{2}\right)^2 + \frac{3}{2} < 0$$

$$\left(x - \frac{3}{2}\right)^2 - \frac{3}{4} < 0$$

Now we factor this "difference of squares."

$$\left[\left(x - \frac{3}{2}\right) + \frac{\sqrt{3}}{2}\right]\left[\left(x - \frac{3}{2}\right) - \frac{\sqrt{3}}{2}\right] < 0$$

$$\left(x - \frac{3-\sqrt{3}}{2}\right)\left(x - \frac{3+\sqrt{3}}{2}\right) < 0 \tag{3}$$

We consider the factors separately, as in the alternate solution for Exercise 20. As Table 28 indicates, the factor $x - (3-\sqrt{3})/2$ "changes sign" when $x = (3-\sqrt{3})/2$, whereas the factor $x - (3+\sqrt{3})/2$ changes sign when $x = (3+\sqrt{3})/2$. Since inequality (3) is satisfied whenever the product of the two factors is negative, the solution set is

$$\left\{x \mid \frac{3-\sqrt{3}}{2} < x < \frac{3+\sqrt{3}}{2}\right\} = \left(\frac{3-\sqrt{3}}{2}, \frac{3+\sqrt{3}}{2}\right)$$

as illustrated in Fig. 1.1.28.

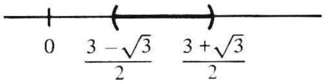

Figure 1.1.28

Table 28

	$-\infty$ to $\frac{3-\sqrt{3}}{2}$	$\frac{3-\sqrt{3}}{2}$ to $\frac{3+\sqrt{3}}{2}$	$\frac{3+\sqrt{3}}{2}$ to $+\infty$
$x - \frac{3-\sqrt{3}}{2}$	−	+	+
$x - \frac{3+\sqrt{3}}{2}$	−	−	+
$\left(x - \frac{3-\sqrt{3}}{2}\right)\left(x - \frac{3+\sqrt{3}}{2}\right)$	+	−	+

We note that the end points in the solution set are the solutions of the equation $2x^2 - 6x + 3 = 0$, which is obtained from the given inequality. Thus, we may find these end points by solving the equation by the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{3 \pm \sqrt{3}}{2}$$

30. $\frac{x+1}{2-x} < \frac{x}{3+x}$

SOLUTION: First, we write the inequality in zero form and then simplify.

$$\frac{x+1}{2-x} - \frac{x}{3+x} < 0$$

$$\frac{2x^2 + 2x + 3}{(2-x)(3+x)} < 0$$

Because the coefficient of x in the factor $2 - x$ is negative, we multiply by -1 and reverse the sense of the inequality.

$$\frac{2x^2 + 2x + 3}{(x-2)(x+3)} > 0 \qquad \textbf{(4)}$$

We consider each factor separately. First, we complete the square on the expression that appears in the numerator.

$$2x^2 + 2x + 3 = 2\left(x^2 + x + \frac{1}{4}\right) - 2 \cdot \frac{1}{4} + 3$$

$$= 2\left(x + \frac{1}{2}\right)^2 + \frac{5}{2}$$

Because the square of any real number is nonnegative, we see that $2x^2 + 2x + 3$ is always positive. However, $x - 2$ changes sign at $x = 2$, and $x + 3$ changes sign at $x = -3$. Table 30 shows that the solution set for inequality (4) is $(-\infty, -3) \cup (2, +\infty)$, as illustrated in Fig. 1.1.30.

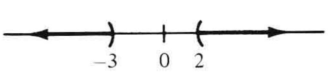

Figure 1.1.30

Table 30

	$x < -3$	$-3 < x < 2$	$x > 2$
$2x^2 + 2x + 3$	+	+	+
$x - 2$	−	−	+
$x + 3$	−	+	+
$\dfrac{2x^2 + 2x + 3}{(x - 2)(x + 3)}$	+	−	+

40. Prove: If $x < y$, then $x < \frac{1}{2}(x + y) < y$

SOLUTION: We prove the inequalities in the conclusion one at a time.

STATEMENT	REASON
$x < y$	Hypothesis
$2x < x + y$	Theorem 1.1.22
$x < \frac{1}{2}(x + y)$	Theorem 1.1.24

Similarly, if $x < y$, then $x + y < 2y$, and $\frac{1}{2}(x + y) < y$. Thus, if $x < y$, then $x < \frac{1}{2}(x + y) < y$.

1.2 ABSOLUTE VALUE

The most important definitions and theorems in this section are the following ones.

1.2.1 Definition

$|x| = x$ if $x \geqslant 0$
$|x| = -x$ if $x < 0$

1.2.2 Theorem If $a > 0$, then $|x| < a$ if and only if $-a < x < a$.

1.2.4 Theorem If $a > 0$, then $|x| > a$ if and only if $x > a$ or $x < -a$.

1.2.6 Theorem $|ab| = |a| \cdot |b|$

1.2.7 Theorem If $b \neq 0$, then $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$

1.2.8 Theorem (Triangle Inequality) $|a + b| \leqslant |a| + |b|$

In addition to the above, we often use the following theorem, which is a corollary to exercises 36 and 37 in Exercises 1.1.

Theorem If $a \geqslant 0$, then

(i) $a < b$ if and only if $a^2 < b^2$
(ii) $a < b$ if and only if $\sqrt{a} < \sqrt{b}$

Exercises 1.2

8. Find the solution set.

$$2x + 3 = |4x + 5| \tag{1}$$

SOLUTION: We consider two cases.

Case 1: $4x + 5 \geqslant 0$

By Definition 1.2.1, $|4x + 5| = 4x + 5$. Thus, Eq. (1) is equivalent to

$$2x + 3 = 4x + 5$$
$$x = -1$$

Since -1 satisfies the Case 1 assumption, $4x + 5 \geqslant 0$, we conclude that -1 also satisfies Eq. (1).

Case 2: $4x + 5 < 0$

By Definition 1.2.1, $|4x + 5| = -(4x + 5)$. Thus, Eq. (1) is equivalent to

$$2x + 3 = -(4x + 5)$$
$$x = -\frac{4}{3}$$

Since $-4/3$ satisfies the Case 2 assumption, $4x + 5 < 0$, we conclude that $-4/3$ also satisfies Eq. (1).

Hence, the solution set for Eq. (1) is

$$\{-1\} \cup \left\{-\frac{4}{3}\right\} = \left\{-1, -\frac{4}{3}\right\}$$

14. Find the set of all replacements of x for which $\sqrt{x^2 + 2x - 1}$ is real.

SOLUTION: Since the square root of a negative number is not real, x must satisfy the inequality

$$x^2 + 2x - 1 \geqslant 0$$

We complete the square and take the principal square root of each member.

$$x^2 + 2x + 1 \geqslant 2$$
$$(x + 1)^2 \geqslant 2$$
$$|x + 1| \geqslant \sqrt{2} \qquad (2)$$

Note that we must use absolute value bars to represent the principal square root of $(x + 1)^2$. Now, by Theorem 1.2.4, inequality (2) is satisfied if

$$x + 1 \geqslant \sqrt{2} \quad \text{or} \quad x + 1 \leqslant -\sqrt{2}$$

That is, if

$$x \geqslant -1 + \sqrt{2} \quad \text{or} \quad x \leqslant -1 - \sqrt{2}$$

Hence, the set we seek is $[-1 + \sqrt{2}, +\infty) \cup (-\infty, -1 - \sqrt{2}]$.

In Exercises 15-28 find the solution set of the given inequality and illustrate the solution set on the real number line.

18. $|6 - 2x| \geqslant 7$

SOLUTION: By Theorem 1.2.4, the given inequality is satisfied if either $6 - 2x \geqslant 7$ or $6 - 2x \leqslant -7$. We solve each inequality.

$$6 - 2x \geqslant 7 \qquad\qquad 6 - 2x \leqslant -7$$
$$-2x \geqslant 1 \qquad\qquad -2x \leqslant -13$$
$$x \leqslant -\frac{1}{2} \qquad\qquad x \geqslant \frac{13}{2}$$

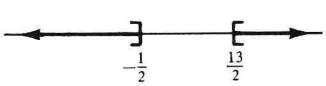

Figure 1.2.18

The solution set is $(-\infty, -\frac{1}{2}] \cup [\frac{13}{2}, +\infty)$ and is illustrated in Fig. 1.2.18.

20. $|3 + 2x| < |4 - x|$

SOLUTION: First, we square each member.

$$|3 + 2x|^2 < |4 - x|^2$$
$$9 + 12x + 4x^2 < 16 - 8x + x^2$$
$$3x^2 + 20x - 7 < 0$$
$$(x + 7)(3x - 1) < 0 \qquad (3)$$

Figure 1.2.20

As Table 20 indicates, the factor $x + 7$ changes sign at $x = -7$, whereas $3x - 1$ changes sign at $x = \frac{1}{3}$. Since inequality (3) is satisfied whenever $(x + 7)(3x - 1)$ is negative, the table indicates that the solution set is $(-7, \frac{1}{3})$, and is illustrated in Fig. 1.2.20.

Table 20

	$-\infty$ to -7	-7 to $\frac{1}{3}$	$\frac{1}{3}$ to $+\infty$
$x + 7$	−	+	+
$3x - 1$	−	−	+
$(x + 7)(3x - 1)$	+	−	+

24. $\left|\dfrac{6 - 5x}{3 + x}\right| \leqslant \dfrac{1}{2}$

SOLUTION: If $3 + x \neq 0$, we may multiply on both sides of the inequality by the positive number $2|3 + x|$.

$$2|3 + x| \cdot \left|\frac{6 - 5x}{3 + x}\right| \leqslant 2|3 + x| \cdot \frac{1}{2}$$
$$2|6 - 5x| \leqslant |3 + x|$$
$$[2|6 - 5x|]^2 \leqslant |3 + x|^2$$
$$4(36 - 60x + 25x^2) \leqslant 9 + 6x + x^2$$
$$100x^2 - 240x + 144 \leqslant x^2 + 6x + 9$$
$$99x^2 - 246x + 135 \leqslant 0$$
$$33x^2 - 82x + 45 \leqslant 0$$
$$(11x - 9)(3x - 5) \leqslant 0 \qquad (4)$$

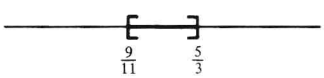

Figure 1.2.24

As Table 24 indicates, the factor $11x - 9$ changes sign at $x = \frac{9}{11}$, and the factor $3x - 5$ changes sign at $x = \frac{5}{3}$. Since the strict inequality (4) is satisfied whenever $(11x - 9)(3x - 5)$ is negative, and the equation (4) is satisfied either when $x = \frac{9}{11}$ or when $x = \frac{5}{3}$, by Table 24 we see that the solution set for (4) is $[\frac{9}{11}, \frac{5}{3}]$, and this set is illustrated in Fig. 1.2.24. Note that our assumption $3 + x \neq 0$ is satisfied by every element in $[\frac{9}{11}, \frac{5}{3}]$.

Table 24

	$x < \frac{9}{11}$	$x = \frac{9}{11}$	$\frac{9}{11} < x < \frac{5}{3}$	$x = \frac{5}{3}$	$x > \frac{5}{3}$
$11x - 9$	−	0	+	+	+
$3x - 5$	−	−	−	0	+
$(11x - 9)(3x - 5)$	+	0	−	0	+

32. Solve for x and use absolute value bars to write the answer.

$$\frac{x + 5}{x + 3} < \frac{x + 1}{x - 1}$$

SOLUTION:

$$\frac{x+5}{x+3} - \frac{x+1}{x-1} < 0$$

$$\frac{(x+5)(x-1) - (x+1)(x+3)}{(x+3)(x-1)} < 0$$

$$\frac{-8}{(x+3)(x-1)} < 0$$

$$\frac{8}{(x+3)(x-1)} > 0 \qquad (5)$$

Since inequality (5) is satisfied if and only if $(x+3)(x-1)$ is positive, we see that (5) is equivalent to

$$\begin{aligned}(x+3)(x-1) &> 0\\ x^2+2x-3 &> 0\\ x^2+2x+1 &> 4\\ (x+1)^2 &> 4\\ \sqrt{(x+1)^2} &> \sqrt{4}\\ |x+1| &> 2\end{aligned}$$

34. Prove: $|a| - |b| \leq |a-b|$

SOLUTION: We have the following equation

$$|a| = |(a-b)+b| \qquad (6)$$

From Theorem 1.2.8 we have

$$|(a-b)+b| \leq |a-b| + |b| \qquad (7)$$

Substituting from Eq. (6) into inequality (7), we have

$$\begin{aligned}|a| &\leq |a-b| + |b|\\ |a| - |b| &\leq |a-b|\end{aligned}$$

1.3 THE NUMBER PLANE AND GRAPHS OF EQUATIONS

To draw a sketch of the graph of an equation by plotting points is slow and inaccurate unless a computer is used to make the calculations and many points are found. As we proceed through the course, we will discover theorems that will enable us to draw a sketch of a graph by plotting only a few points. The first of these theorems is the test for symmetry.

1.3.6 Theorem The graph of an equation in x and y is

(i) symmetric with respect to the x-axis if and only if an equivalent equation is obtained when y is replaced by $-y$ in the equation.
(ii) symmetric with respect to the y-axis if and only if an equivalent equation is obtained when x is replaced by $-x$ in the equation.
(iii) symmetric with respect to the origin if and only if an equivalent equation is obtained when x is replaced by $-x$ and y is replaced by $-y$ in the equation.

It is important to remember that if $x > 0$, then $\sqrt{x}$ represents only the *positive* square root of x. Thus, $\sqrt{25} \neq \pm 5$. Rather, $\sqrt{25} = 5$ and $-\sqrt{25} = -5$.

Exercises 1.3

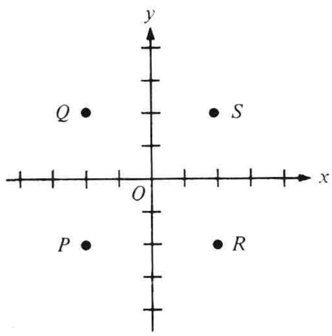

Figure 1.3.4

4. Let $P = (-2, -2)$. Plot P and points Q, R, and S so that P and Q are symmetric with respect to the x-axis, P and R are symmetric with respect to the y-axis, and P and S are symmetric with respect to the origin.

SOLUTION: By Theorem 1.3.6, to find Q we replace y by $-y$ in P. Hence, $Q = (-2, 2)$. Similarly, to find R we replace x by $-x$ and obtain $R = (2, -2)$, and to find S we replace x by $-x$ and y by $-y$ to obtain $S = (2, 2)$. The points are plotted in Fig. 1.3.4.

In Exercises 7-28 draw a sketch of the graph of the equation.

10. $y = -\sqrt{x - 3}$

SOLUTION: Since the square root of a negative number is not real, we must choose as replacements for x only those numbers that satisfy $x - 3 \geqslant 0$, or, equivalently, $x \geqslant 3$. For each such replacement we use the given equation to calculate y. Table 10 gives the results of such calculations. We use a decimal approximation for y whenever y is irrational. A sketch of the graph is obtained by plotting the points from Table 10 and drawing a smooth curve that contains these points, as shown in Fig. 1.3.10.

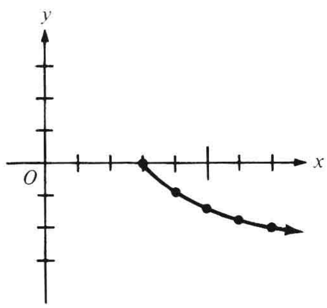

Figure 1.3.10

Table 10

x	3	4	5	6	7
y	0	−1	−1.4	−1.7	−2

14. $x = y^2 + 1$

SOLUTION: Replacing y by $-y$ in the equation gives

$$x = (-y)^2 + 1$$

which is equivalent to

$$x = y^2 + 1$$

By Theorem 1.3.6, the graph is symmetric with respect to the x-axis. We plot several points for which $y \geqslant 0$, draw a smooth curve that contains them, and use symmetry to complete the sketch. For Table 14 we choose y first and then calculate x from the given equation. The sketch of the graph is shown in Figure 1.3.14.

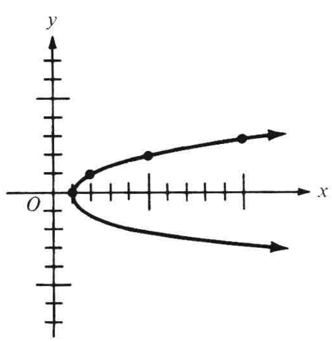

Figure 1.3.14

Table 14

x	1	2	5	10
y	0	1	2	3

18. $y = -|x| + 2$

SOLUTION: Replacing x by $-x$ in the equation gives

$$y = -|-x| + 2 \tag{1}$$

and since $|-x| = |x|$, Eq. (1) is equivalent to

$$y = -|x| + 2$$

By Theorem 1.3.6, the graph is symmetric with respect to the y-axis. We plot several points for which $x \geqslant 0$, draw a smooth curve that contains them, and use symmetry