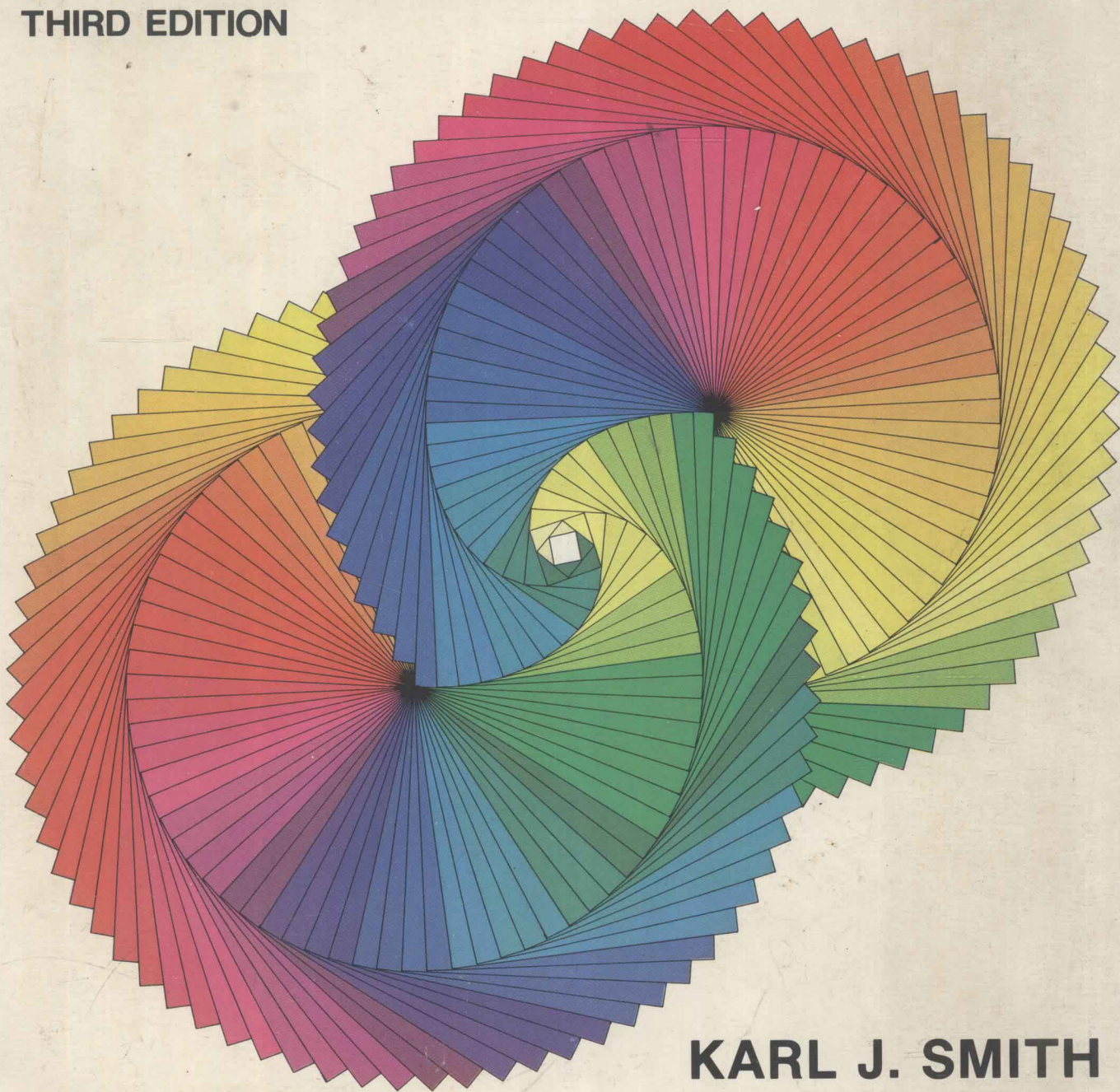


THE NATURE OF MODERN MATHEMATICS

THIRD EDITION



KARL J. SMITH

**THE NATURE OF
MODERN MATHEMATICS**
THIRD EDITION

KARL J. SMITH

Santa Rosa Junior College

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Karl J. Smith

To my wife, Linda,
and our children, Melissa and Shannon

PREFACE

This book was written to create a positive attitude toward mathematics—a realization that mathematics is not an endless procession of dull manipulations, theorems, proofs, and irrelevant topics. Rather than simply presenting the technical details needed to proceed to the next course, the book attempts to give insight into what mathematics is, what it accomplishes, and how it is pursued as a human enterprise.

Intended primarily for a one- or two-semester course addressed to the liberal arts student, this book presupposes almost no knowledge from previous courses. The student is asked not to “do math problems” but rather to “experience math.” For this reason many of the topics may seem unorthodox when compared with previous experiences the student has had in mathematics courses.

Historical notes and history charts are provided to give the student some insight into the humanness of mathematics. Rather than being strictly biographical reports, these notes focus on the people. Nearly every major mathematician will have some part of his or her life to tell on the pages of this book. I hope that a glimpse of the people of mathematics will give you a glimpse of the nature of mathematics.

I frequently encounter people who tell me about their unpleasant experiences with mathematics. I have a true sympathy for these people, and I recall one of my elementary teachers who assigned additional arithmetic problems as punishment. This can only create negative attitudes toward mathematics, which is indeed unfortunate. If elementary teachers and parents have a positive attitude toward mathematics, their children will see some of the beauty of the subject. I want the student to come away from this course with the feeling that mathematics can be pleasant, useful, and practical—and enjoyed for its own sake.

This text is designed so that professor and students have many options. Although some chapters are dependent on others, the development of the course allows for great flexibility in changing the order of the chapters and in deemphasizing or eliminating certain sections without disrupting the continuity or requiring supplementary material. For example, if you don't like to consider logic as the second chapter, you can insert it wherever you wish, if at all. A typical three-unit course could cover from five to six chapters.

One of the most frequently occurring suggestions that I received from reviewers and users of the book was to rearrange the chapters. Nearly everyone had their "own arrangement." This simply points out one of the strongest features of this book—that the order in which the material is presented depends on the preference of the instructor and the make-up of the students taking the course. The material can be easily adapted to almost any order or arrangement. Several alternative course outlines are described in the Instructor's Manual accompanying this text. The dependence of the chapters is as follows:

<i>Chapter</i>	<i>Prerequisite</i>
1	none
2	none
3	none
4	none
5	none
6	Chapter 4
7	none
8	none
9	Chapter 8
10	none

Chapter 1 introduces the student to the spirit and style of the course by showing how patterns are used in mathematics. Patterns are applied to such diverse ideas as the golden section and the metric system. However, any of the sections in this chapter can be omitted without any loss of continuity in the development of the remaining material in this book.

Chapter 2 combines two fundamental concepts of mathematics—sets and logic—by showing how logical reasoning applies to everyday arguments as well as to relationships among sets and to the material that will follow in the book.

Calculator and computer applications from Chapter 3 are integrated throughout the rest of the book, but they are always found in the problem sets and are indicated by the words *Calculator Problem* or *Computer Problem*. The instructor has several options for handling the computer material.

1. Omit Chapter 3.
2. Treat Chapter 3 merely as an introduction to calculators and computers. In this case, only Sections 3.1 through 3.3 need be covered.
3. Treat calculators (Section 3.2) early in the course and computers as the last section in the course (time permitting).
4. Emphasize Chapter 3, including programming, and use the computer throughout the course. I have experienced success using this material with students of varied abilities. The computer problems can be discussed in class or assigned, and computer solutions can be used as part of the course's development. This is one of the features of the book: *computers can be integrated into the entire course*. The computer language used is BASIC, but the material could easily be adapted to FORTRAN or other computer languages.

Chapter 4 concerns itself with the development of our numeration system as well as with the different sets of numbers that are of interest to mathematicians and laypersons.

Chapter 5 treats arithmetic from a layperson's point of view by discussing interest as it is applied to installment buying, automobile buying, home buying, and insurance. Although important to the consumer, Sections 5.3, 5.4, and 5.5 are not typical of a usual mathematics course. Depending on the class, the instructor might wish to skip these sections or to cover them only as a reading assignment. Chapter 5 is designed to help the reader make more informed decisions regarding personal finances.

Chapter 6, on the nature of algebra, returns to the thread begun in Chapter 4 and builds the idea of a mathematical system related to the structure of algebra. Chapter 7, on the nature of geometry, takes a layperson's look at non-Euclidean geometry, topology, and the famous Königsberg Bridge Problem.

Chapter 8 develops ways of counting without actually enumerating all possibilities. Permutations and combinations lead quite naturally to the notion of probability in Chapter 9, although Chapter 9 may be studied without Chapter 8 if the instructor is selective in the problems assigned. The final Chapter, Chapter 10, deals with statistics, with emphasis on how statistics can be manipulated to support almost any argument.

A wide range of problems is offered, since students who enroll in a survey course come from a wide range of backgrounds. There are quite a number of problems of the routine or drill type, as well as a large selection of thought-provoking problems. Much interesting material appears in the problems, and the reader can learn quite a bit just by reading them. The answers to the odd-numbered problems are provided in the back of the book, and complete solutions to all of the problems, as well as sample tests, are available in the Instructor's Manual. The "Mind Bogglers" and "Problems for Individual Study" sections should be utilized as interest demands. They are not to be forced on the student but should be investi-

gated for their own sake. Hopefully, these sections will stimulate class discussion and still other avenues of investigation.

I also thank the many people who wrote me offering suggestions for revising the book: Jan Boal, T. W. Buquoi, Chris C. Braunschweiger, T. A. Bronikowski, Eugene Callahan, Michael W. Carroll, Joseph Cavanaugh, Ralph De Marr, Samuel L. Dunn, Ernest Fandreyer, Ralph Gellar, Charles E. Johnson, Linda H. Kodama, Daniel Koral, Helen Kriegsman, C. Deborah Laughton, John Mullen, John Palumbo, Gary Peterson, O. Sassian, Clifford H. Wagner, and Barbara Williams, as well as all those who responded to my questionnaire.

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Karl J. Smith

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