RECENT DEVELOPMENTS IN STATISTICAL INFERENCE AND DATA ANALYSIS

K. Matusita Editor

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PREFACE

Recently, statistics has made remarkable progress in application as well as in theory, and its applications have come to range over fields such as environments, natural resources, medical treatments, etc. To encourage further researches and disseminate new ideas in this science, the International Conference in Statistics in Tokyo was held on November 28-30, 1979 at the Institute of Statistical Mathematics. It had about 160 participants from Belgium, Canada, China, France, Hong Kong, Korea, the U.S.A. and Japan, and 65 papers were read.

The papers in this volume were presented at the Conference and have passed referees' examination. In these papers recent developments in statistical inference and data analysis are discussed by prominent workers in these fields. The book will be of interest to statisticians and to research workers who apply statistics.

The Conference was supported by various industrial and commercial organizations in Japan. We wish to express deep gratitude to them on behalf of the Organizing Committee. The Conference was sponsored, as well, by the Institute of Statistical Mathematics, Tokyo, the Japan Statistical Society, the Bernoulli Society, and the Institute of Mathematical Statistics, U.S.A., each in its own fashion. We wish to express our thanks to them also.

In editing the book, we had help and cooperation from many people in refereeing papers. We wish to express our thanks to them. In addition, thanks are due to Mr. R. Price and to Miss F. Rizzardi, of Berkeley, California, U.S.A. for their work in improving the English language usage in some of the Japanese authored papers, and also, to Miss Y. Okada and Mrs. U. Mizuno for their assistance in editorial affairs.

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Recent Developments in Statistical Inference and Data Analysis; K. Matusita, editor

ENLARGED MATHEMATICAL REPRESENTATIONS FOR STOCHASTIC PHENOMENA

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This paper is to stress the commonality of some of our recent publications wherein we have begun to adduce more general mathematical structures than appear in today's literature on stochastic processes to describe certain stochastic phenomena. We give particular attention to one of these structures, and we supply explicit proofs here for certain results that were previously stated without proof.

1. INTRODUCTION

We have recently put out two pieces of work in each of which the burden was to set forth a mathematical description of a non-deterministic process in such a manner that the phenomenon in question would be reasonably well accounted for and that the proposed mathematical description would be recognizably generalizing of the current mathematical form that is applied to describing stochastic processes. The first of these pieces of work is the article [3], already in print. The second, [4], was presented at the 2nd International Conference on Mathematical Modeling, in St. Louis, and the Proceedings of that Conference should have appeared by the time this present paper is published. Our motivation and guidance in efforts of this kind come from, and serve, our fundamental research aim, which is the establishment of a general theory of behavior asserting that all behavior is the cumulative acts in stochastic processes. This aim is formulated with due regard for the possible necessity of enlarging appreciably the mathematical form that we take as descriptive of stochastic processes. The limits of enlargement--the extent of this necessity--will be determined by the eventual successful formulation of the general dynamical law in the theory. (For more extensive discussion of these general theoretical ideas, see [1] and [2]--to start with.)

The article [4] --together with a companion article, [5], still to be published--undertakes the relatively limited objective of finding the proper rearrangement of the standard mathematical givens in an extended finite game so that a playing of the game takes on a description that is recognizable as a stochastic process form. Starting out to approach this problem, one might expect that the necessary rearrangement would be routine and that the stochastic process form that resulted would be the usual standard form that is found in today's probability literature. But the answer is not quite so simple. The information partition of the game-tree vertices in general precludes the possibility of such a classical description. What did emerge as the general answer to the question is this: the playing of any finite game may be seen as an evolving stochastic process provided the admitted mathematical stochastic process form is a (finite) collection of component spaces with, however, no assumed prior temporal ordering among them, and the successive conditional probability measures are on the successively smaller maximal direct sums of the still unactualized spaces, thus admitting actualization of the component spaces in any order. (see [4] and [5] for a more detailed description and elaboration of this result.)

We shall not here pursue any further the specific subject of the article [4]. The behavioral question that was confronted and dealt with in the article [3] appears to have led us much further into the generality that will be needed for the full formulation of the dynamical law. It is to the stochastic process form set forth in that article [3] that we shall devote the rest of the discussion in this paper. In the next section we shall present this stochastic process description, including the proofs of various facts regarding it--proofs that we did not give in [3]. In the course of this development we shall correct a small error made in [3]. Some of the statements made there are in fact true only if the strain in question is a complete pride, not merely a pride. We have therefore modified our definition of a stochastic process in this present article, calling for its strain to be a complete pride of eventualities. (Definitions of pride, complete pride, strain, etc. are given in Section 2.) In Section 3 we give some elaborative comments on our present description of a stochastic process. For the particular substantive psychological and sociological phenomena that were treated with this stochastic process description in the article [3], we respectfully refer the reader to that article.

2. REAL STRUCTURE

In the theory of behavior that we expect to emerge from our work it is conceived that all reality is structured of eventualities and acts. The collection of all eventualities has a partial order relation, <u>$\ll$</u>, and a meet operation, $\wedge$, under this order. Thus, the collection is a semi-lattice which we denote by the symbol 生 . We postulate that this semi-lattice has no least element.

A sub-semi-lattice of 生 that has no least element will be said to be <u>unbounded below</u>, abbreviated <u>u.b.</u> (生 itself is u.b.). A subset ש of 生 will be called <u>conditionally meet-closed</u>, or <u>c.m.c.</u>, if it has the following property: If Ξ_1 and Ξ_2 are elements of ש , and if $\Xi_1 \wedge \Xi_2 \gg \Xi_3$ for some element Ξ_3 of ש , then $\Xi_1 \wedge \Xi_2 \in$ ש. Generalizing this, we define a subset ש of 生 to be <u>completely conditionally meet-closed</u> (<u>c.c.m.c.</u>) if for any subcollection $\{\Xi_k, k \in K\}$ of elements of ש , such that there exists $\bigwedge_{k\in K} \Xi_k \gg \Xi'$ for some $\Xi' \in$ ש , we have $\bigwedge_{k\in K} \Xi_k \in$ ש . (Clearly there are intermediate notions as well, for any specific maximum cardinality of the subcollections of ש .) A c.m.c. subset will also be called a <u>pride</u> or a <u>simple pride</u>, and a c.c.m.c. subset will be called a <u>complete pride</u>. We shall contract these last two expressions to <u>simpride</u> and <u>compride</u>, resp.

We can now state our central definition:

<u>Definition 1</u>. <u>A stochastic process is a compride of eventualities together with the acts resulting from actualization of eventualities in this compride</u>.

A stochastic process S can then be presented notationally as (ש$,B$), where ש denotes the compride of eventualities that characterizes S, and B denotes the <u>behavior in</u> S, that is, the acts resulting from actualization of eventualities in ש . We shall call ש the <u>strain</u> of the stochastic process S.

If ש is the strain of a stochastic process S, we define

(2.1) ש$^U \overset{\text{def.}}{=} \{\Gamma \in$ 生 $| \Gamma \ll T$ for some $T \in$ ש $\}$.

With the understanding that a complete sub-semi-lattice of $\mathfrak{E}$ is one that contains existing meets of arbitrary subcollections of its elements, we have the following:

Proposition 1. $\mathfrak{W}^U$ is a u.b. complete sub-semi-lattice of $\mathfrak{E}$, and $\mathfrak{W} \subseteq \mathfrak{W}^U$.

Proof: It is immediately clear that $\mathfrak{W}^U$ is a semi-lattice and that $\mathfrak{W} \subseteq \mathfrak{W}^U$. To see that $\mathfrak{W}^U$ is u.b., suppose, to the contrary, that Γ_0 is the least element of $\mathfrak{W}^U$. Then $\mathfrak{W}^U$ cannot be all of $\mathfrak{E}$, and so there exists an eventuality $\Gamma' \notin \mathfrak{W}^U$. If for every $\Gamma' \notin \mathfrak{W}^U$ we had $\Gamma_0 \wedge \Gamma' = \Gamma_0$, then $\mathfrak{E}$ itself would have a lower bound. Hence, there must be some $\Gamma' \notin \mathfrak{W}^U$ such that $\Gamma_0 \wedge \Gamma' \ll \Gamma_0$. But then $\Gamma_0 \wedge \Gamma' \in \mathfrak{W}^U$ and we have a contradiction to the assumption that Γ_0 is a lower bound for $\mathfrak{W}^U$. Therefore, as asserted, $\mathfrak{W}^U$ is u.b.

Finally, to see that $\mathfrak{W}^U$ is a complete sub-semi-lattice, let Γ_k, $k \in K$, be elements of $\mathfrak{W}^U$ whose combined meet exists. For each k, $\Gamma_k \underline{\ll} T_k$ for some $T_k \in \mathfrak{W}$. For any particular chosen k_0, we have $\bigwedge_k \Gamma_k \underline{\ll} T_{k_0}$, and therefore this meet is, by definition, a member of $\mathfrak{W}^U$. This completes the proof of Proposition 1.

Being a complete sub-semi-lattice, $\mathfrak{W}^U$ is in particular a compride and so it is the strain of a stochastic process. We denote this process by $\mathcal{S}^U$ and we call it the universal process of $\mathcal{S}$, or the universe of $\mathcal{S}$. We call $\mathfrak{W}^U$ the universal strain of the strain $\mathfrak{W}$.

Letting K denote any index set, we next define

$$(2.2)\qquad \mathfrak{W}^R \overset{\text{def.}}{=} \{\Gamma \in \mathfrak{W}^U \mid [\Gamma \underline{\ll} T_k \in \mathfrak{W}, \forall k \in K, \text{ and } \exists \bigwedge_{k\in K} T_k] \Rightarrow \bigwedge_{k\in K} T_k \in \mathfrak{W}\}.$$

We now prove

Proposition 2. $\mathfrak{W}^R$ is a compride, and $\mathfrak{W} \subseteq \mathfrak{W}^R$.

Proof. Let Ξ be any element of $\mathfrak{W}$, and suppose that T_k, $k \in K$, are a collection of elements of $\mathfrak{W}$ such that $\Xi \underline{\ll} T_k$ for every k, and $\bigwedge_{k\in K} T_k$ exists. Then these T_k are elements of $\mathfrak{W}$ whose meet exists and has an element of $\mathfrak{W}$, namely Ξ, as a sub-eventuality. Therefore, by the characterizing property of $\mathfrak{W}$ as a compride, it follows that $\bigwedge_{k \in K} T_k \in \mathfrak{W}$. And thus it is established that $\mathfrak{W} \subseteq \mathfrak{W}^R$.

To prove that $\mathfrak{W}^R$ is a compride, suppose that Γ_k, $k \in K$, are elements of $\mathfrak{W}^R$ whose combined meet exists and is the eventuality $\hat{\Gamma}$, say, and suppose that $\hat{\Gamma}$ satisfies $\hat{\Gamma} \gtrapprox \Gamma_0 \in \mathfrak{W}^R$. If $T_{k'}$, $k' \in K'$, are elements of $\mathfrak{W}$, whose meet exists, and are such that $\hat{\Gamma} \lessapprox T_{k'}$ for all k', then we will have $\Gamma_0 \lessapprox T_{k'}$ for every k', and, therefore, since $\Gamma_0 \in \mathfrak{W}^R$, we have $\bigwedge_{k' \in K'} T_{k'} \in \mathfrak{W}$. This shows that $\hat{\Gamma} \in \mathfrak{W}^R$, and it is therefore demonstrated that $\mathfrak{W}^R$ is a compride. Proposition 2 is therefore proved.

Since $\mathfrak{W}^R$ is a compride, it is the strain of a stochastic process. We denote this process by $\mathfrak{S}^R$, and we call it the <u>reach of the process</u> $\mathfrak{S}$. We call $\mathfrak{W}^R$ the <u>reach of the strain</u> $\mathfrak{W}$.

Now we set

$$\mathfrak{W}^E \overset{\text{def.}}{=\!=} \mathfrak{W}^U - \mathfrak{W}^R. \tag{2.3}$$

Explicitly, we have

$$\mathfrak{W}^E = \{\Gamma \in \mathfrak{W}^U \mid \underline{\text{for some}}\ K\text{: } \Gamma \lessapprox T_k \in \mathfrak{W}, \forall k \in K, \ \underline{\text{and}}\ \exists \bigwedge_k T_k \notin \mathfrak{W}\}. \tag{2.4}$$

We prove

<u>Proposition 3</u>. $\mathfrak{W}^E$ <u>is a u.b. complete sub-semi-lattice of</u> $\mathfrak{E}$.

<u>Proof</u>. Let $\Gamma_{k'}$, $k' \in K'$, be a collection of elements of $\mathfrak{W}^E$, whose meet exists. For a particular index value, say k_0', we have $\Gamma_{k_0'} \lessapprox T_k$, $\forall k \in K$, for some collection $\{T_k, k \in K\}$ of elements of $\mathfrak{W}$, and $\exists \bigwedge_k T_k \notin \mathfrak{W}$. But then it is also true that $\bigwedge_{k' \in K'} \Gamma_{k'} \lessapprox T_k$, $\forall k \in K$. Thus, the collection $\{T_k, k \in K\}$ verifies also the membership of $\bigwedge_{k'} \Gamma_{k'}$ in $\mathfrak{W}^E$, and it is proved that $\mathfrak{W}^E$ is a complete sub-semi-lattice.

To prove that $\mathfrak{W}^E$ is u.b., suppose, to the contrary, that it is not, and that Γ_0 is the least element of $\mathfrak{W}^E$. Then, as in the proof of Proposition 1, we can demonstrate that there is an eventuality $\Gamma_{00} \ll \Gamma_0$. But from the argument in the preceding paragraph it is clear that any sub-eventuality of an eventuality in $\mathfrak{W}^E$ is again in $\mathfrak{W}^E$. Hence, we have $\Gamma_{00} \in \mathfrak{W}$, thus contradicting the assumption that Γ_0 is the least element. Hereby the proof of Proposition 3 is completed.

Again as in the case of $\mathfrak{W}^U$, since $\mathfrak{W}^E$ is a complete sub-semi-lattice, it is a compride and is, therefore, the strain of a stochastic process, S^E. This process we call the eternal process of S or the eternity of S. And $\mathfrak{W}^E$ we call the eternal strain of the strain $\mathfrak{W}$.

The terminology we have introduced above for the S-related stochastic processes S^U, S^R and S^E emerges as meaningful when we give two more definitions and then state our present conception of an at least partial detailing of the dynamical law.

Definition 2. A stochastic process $S = (\mathfrak{W}, \mathcal{B})$ is said to be evolving, or to be in evolution, if there is ongoing actualization of eventualities in $\mathfrak{W}$, but no actualization of an eventuality in $\mathfrak{W}^E$.

Definition 3. A stochastic process $S = (\mathfrak{W}, \mathcal{B})$ is said to lapse, or to be lapsed, or to have lapsed, if there is actualization of an eventuality in $\mathfrak{W}^E$.

Dynamical law (partial statement):

(i) If $\Xi' \ll \Xi''$ and Ξ' actualizes, then Ξ'' actualizes;

(ii) if every Ξ_k of a collection $\{\Xi_k\}$ actualizes, then necessarily $\bigwedge_k \Xi_k$ exists and it actualizes;

(iii) in any u.b. semi-lattice of eventualities, at least one of which has actualized, some eventuality which has not actualized will actualize.

Notice that, according to (ii), if the collection of eventualities $\{\Xi_k\}$ is such that $\bigwedge_k \Xi_k$ does not exist, then it does not happen that all the eventualities Ξ_k actualize. Thus, regarding (iii), we see that it does not happen that all the eventualities in a u.b. semi-lattice actualize; that is, whatever the eventualities in a u.b. semi-lattice that have actualized, there are other eventualities in that semi-lattice that have not actualized.

It should be mentioned here that the strain of any stochastic process--in fact, any pride of eventualities--can be presented as a semi-lattice with a least element. This is accomplished by the introduction of a fictitious element, an element that does not represent an eventuality. For a fuller explanation of this fact we refer the reader to the article [3]. In the discussion there we have furthermore pointed out that this construction may be seen as accounting for the null set in the classical representation of a collection of eventualities as a σ-field of sets.

3. SOME COMMENTS

We now wish to make a few remarks on various features of the theorization that has been laid out above--to point up its promise in the direction of more complete description of real phenomena.

Our theorization--it is seen--ventures a completely explicit characterization of the distinction between the thriving of a process and the demise of that process. The former is the on-going actualization of only eventualities in $\mathfrak{W}^R$; the latter comes to be if an eventuality in $\mathfrak{W}^E$ --and, by Proposition 3 and the Dynamical Law, endlessly thereafter another and another and another eventuality in $\mathfrak{W}^E$ -- actualizes. Traditional theorizations dealing with moderately and more complex processes say nothing specific about the demise, or the lapsing--or the death--of a process, but imply that this state is, or entails, the cessation of actualization of eventualities in the process. In the case of very complex processes, like human beings, there is frequently a tacit or explicit postulation of another realm of being into which the process passes at its lapsing. But there is no scientifically tenable theorization offered relating to that other realm or to that passage. Our proposed formulation maintains, to the contrary, that there is only one realm of being--only one mode of reality. And the lapsing of a process is accordingly more complicated than a mere "cessation".

In the article [3] we cited the example of a tossed coin splitting in two at its median plane and the two halves coming to rest with, respectively, the coin-head up and the coin-tail up--this as an illustration of the set of concepts being set forth; namely, the semi-lattice property of the collection of all eventualities, the characterization of the strain of a stochastic process as in general only conditionally (completely) meet-closed, and the defined lapsing of a process. Other examples can be adduced to support this conceptual framework. For instance, consider a block of dry ice. (This example was suggested by F. Mogerman.) The process that is this particular block of dry ice might contain, for each n=1,2,..., the eventuality, Γ_n, that the volume of the block is measured and is found not to exceed 1/n cubic centimeters. Empirically we do not consider that this entity continues as a block of dry ice if it has evaporated completely, that is, if it yields a zero volume measurement. Thus, while it is reasonably supposed that there exists the eventuality $\Gamma' = \bigwedge_{n=1,2,\ldots} \Gamma_n$, this eventuality does not belong to the process that is the block of ice. The actualization of Γ' signals the lapse of the block of ice. We see in this example a case that argues strongly for the conditionality of meet closure in the definition of a process.

There is much further examination to be made of this proposed concept structure. Many more examples can usefully be looked at. And in the case of empirically familiar but fairly complex processes it can, it seems, be a challenging problem to account for empirically standard manners of demise in direct terms of the above definition of lapsing.

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Recent Developments in Statistical Inference and Data Analysis; K. Matusita, editor

SPECIFICATION OF STATISTICAL MODELS BY SUFFICIENCY

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A reformulation of the specification of statistical models is given, with several examples discussed. The basis for this new mode of specification is a generalized concept of sufficiency inspired by Dawid. It is brought out also that the degree of universality of assertions regarding certain variables will depend on the stochastic properties which will be allowed to attach to these and other variables. The sufficiency of statistics and parameters in a model is redefined using the notation of conditional independence, following along the line of Dawid [4].

§0 *Introduction.*

Looking at the mathematical structure of statistical theory, we find several important concepts which turn out to have a common mathematical expression; such concepts are sufficiency and ancillarity of a statistic, and identifiability and estimability of a parameter. When the Bayesian concept of prior distribution is brought into statistical theory, the number of such concepts grows still larger, as seen in the definition of sufficiency by Kolmogoroff [7] and the concept of parametric sufficiency due to Barankin [2]. A duality between parameters and statistics was brought out in the paper [2] of Barankin (and also by Kolmogoroff), and in more recent times several authors— Petit [11], Kudō [9], Picci [12] and Dawid [4] — have worked on such a duality. Dawid [4], in particular, established a unification of several concepts in diverse fields by using the concept of conditional independence.

All of these works, however, presented their discussions within the traditional framework which conceives a strict distinction between parameters and statistics. In this paper we shall break down the towering wall standing between parameters and statistics and try to establish a new approach to statistical problems by taking the whole range of the variables appearing in a statistical problem as the basic space, regarding these variables as random variables associated with the family of their possible distributions; the distinction between parameters and statistics will emerge in the opposed concepts of a variable accessible through observation and a variable inaccessible through observation. In this paper we shall call a pair of an accessible variable x and an inaccessible variable θ a statistical model if the conditional distribution of x given θ is definite whichever member of the family of possible joint distributions of x and θ may be true. Since the variables act only as partitions of the basic space in this paper, we may and shall carry on our discussion using σ-field-language instead of variable-language.

In Section 1, we shall discuss the traditional distinction existing between parameters and statistics and declare our intention to drop this distinction from our discussions. In Section 2, we characterize the basic space of a statistical problem as a certain measurable space, and we illustrate this with several examples from the classical theory of statistics. Looking to the identification of parameters and statistics, we distinguish the class of accessible variables from the class of all other variables (Section 3), and then in Section 4 the statistical model is introduced as a pair of an accessible σ-field (the sampling σ-field) and an inaccessible σ-field (the parametric σ-field). We regard in this paper all variables appearing in a statistical problem as random variables with a distribution whose conditional and marginal parts are not specified if they are unknown. These unspecified parts of the distribution may affect the degree of universality of assertions on the parametric σ-field. Such effects are discussed in Sections 5 and 6. The informational properties, such as sufficiency, of sub-σ-fields of the sampling and parametric σ-fields are defined in the last section. These are expressed in terms of conditional independence, following Dawid [4].

§1 *Background of the discussion.*

In the traditional first approach to mathematical statistics, it is customary to make a clear distinction between parameters and samples (or functions of samples, which we call statistics). They are usually distinguished by the feature that the value of the sample comes to be known through observation while the value of the parameter remains unknown (although inferable) after observation.

The distinction between statistics and parameters is traditionally also reflected in the stochastic assumptions. The samples and statistics were to be random variables whereas the parameters were not. Some statistical problems, however, contain even third kinds of variables, other than parameter and statistic: such third variables are, for example, a future variable in a prediction problem and a missing variable in an estimation problem. Once we try to place these variables within the general context of variables in a statistical problem, we see that the distinction between parameters and statistics becomes vague indeed. According to either of the above two criteria for the distinguishing of parameters and statistics, the future variable in a prediction problem and the missing variable in an estimation problem cannot be classified as either one or the other. As another example, the marginal frequency of the contingency table in the testing for independence cannot be regarded as a parameter.

The affirmations for variables that arise in a discussion as here above may be dependent on the meaning one takes for probability. Early in the development of mathematical statistics, it very likely was the case that the relationship between parameter and sample was confused with the relationship between cause and effect. It is our guess that this is the reason such a firm distinction was originally made between parameter and sample. One might raise an objection to this conjecture of ours. Are not missing variables to be regarded as the parameters in their respective problems of statistical inference? Yes, but they have also the aspect of being themselves outcomes of previous stochastic situations, so they have the quality of statistics as well.

§2 *The framework of statistical problems.*

In this paper we regard all variables in statistical problems as random variables, irrespective of whether their probability distribution is (partly or completely) known or not. Let Ω denote the set of all values taken by those variables, and S a σ-field of subsets of Ω which is considered to be that which makes variables measurable. Let M be a set of possible distributions of the full set of variables, which the problem concerned provides us; mathematically speaking, M is a set of probability measures on S.

Let us consider some simple, typical examples to illustrate our approach.