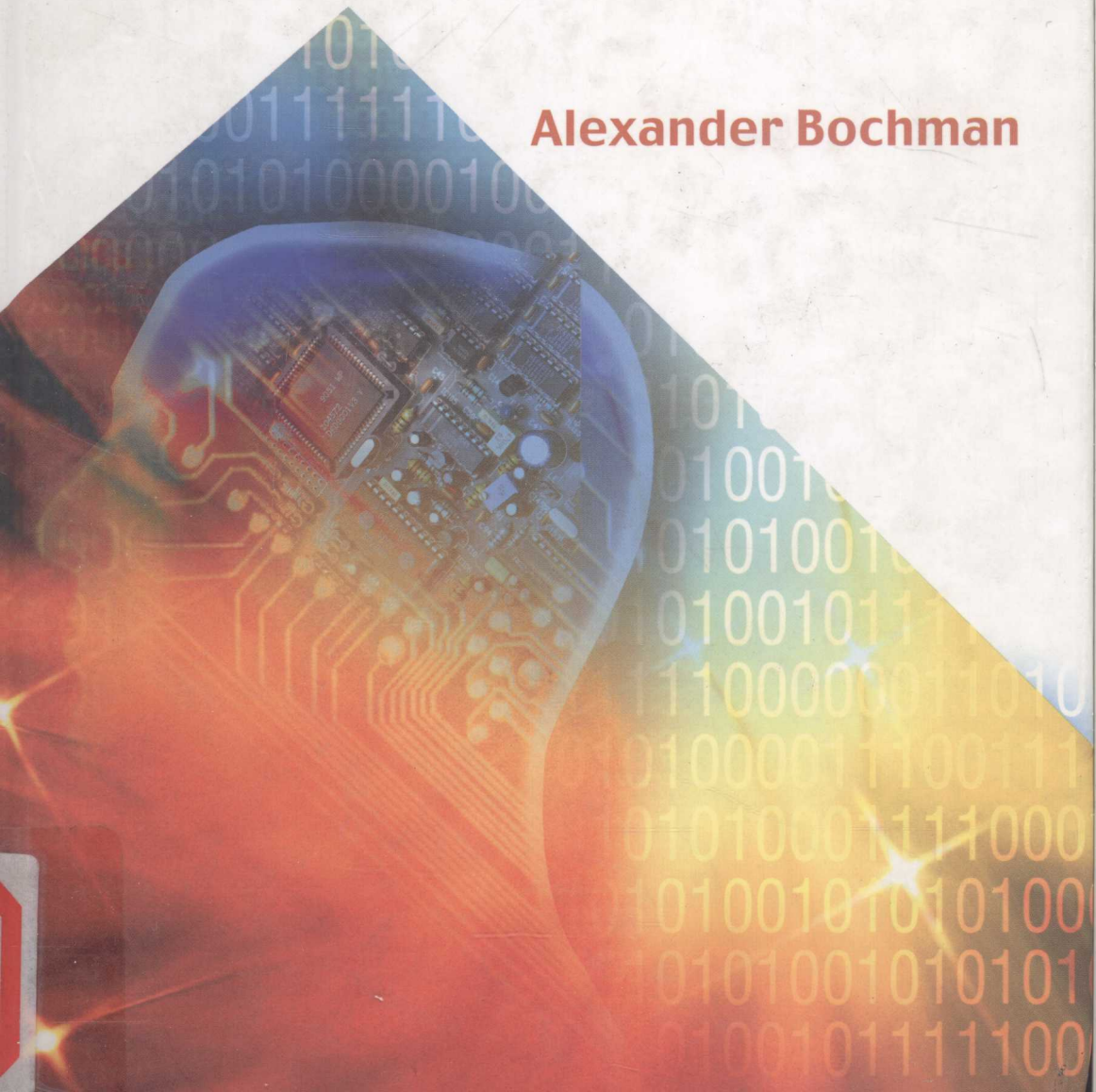


Explanatory Nonmonotonic Reasoning

Alexander Bochman





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Advances in Logic – Vol. 4

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Alexander Bochman

Holon Academic Institute of Technology, Israel



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**Explanatory
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To Jonathan Stavi and my family.

Preface

Like my preceding book, [Bochman, 2001], the present study is also a systematic attempt to answer the question ‘what is nonmonotonic reasoning?’. It complements the previous book by giving a logical formalization to the original approach to nonmonotonic reasoning that includes default logic, autoepistemic and modal nonmonotonic logics, and logic programming. We call this approach *explanatory* nonmonotonic reasoning, since the notion of explanation can be seen as the ultimate and unifying basis behind these nonmonotonic formalisms.

Three aspects distinguish this book from previous studies in this area. First, the book provides a uniform generalized theory of explanatory nonmonotonic reasoning rather than a description of existing nonmonotonic logics. Though the latter are shown to be covered by this theory, the formalism of biconsequence relations, taken as a logical basis of this study, suggests a powerful generalization going in most cases far beyond existing nonmonotonic formalisms. Second, the book shifts attention to some relatively recent, non-epistemic approaches to nonmonotonic reasoning, such as four-valued biconsequence relations, causal reasoning and argumentation theory. These formalisms actually fill the gap between logic programming, on the one hand, and traditional nonmonotonic logics, on the other. Accordingly, default and modal nonmonotonic logics are covered only in the last two chapters, and only as parts of the more general formalism of epistemic biconsequence relations. Last but not least, the book focuses on the logical, monotonic basis of explanatory nonmonotonic reasoning. In this sense, it is as much about logic as it is about nonmonotonic reasoning. As the main benefit of this approach, it will be shown that different formalisms of explanatory nonmonotonic reasoning are based on essentially the same principles and models, the main distinction being the underlying logical

formalisms that host such a reasoning.

The intended audience of this book consists of graduate students and researchers in AI, on the one hand, and general logicians, on the other. In fact, one of the aims of the book consists in persuading the former about the need of logic, and the latter that nonmonotonic reasoning should be an integral part of general logical research.

A. Bochman

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Chapter 1

Introduction

In this introductory chapter I will delineate the subject of this study, and the questions it is going to answer.

1.1 Two Theories of Nonmonotonic Reasoning

Studies in nonmonotonic reasoning have given rise to two basically different approaches that will be called, respectively, *preferential* and *explanatory* nonmonotonic reasoning, with little interaction between them¹. The first approach encompasses nonmonotonic inference relations of [Kraus *et al.*, 1990], and a general theory of belief change [Alchourrón *et al.*, 1985]. A detailed description of this approach can be found in [Bochman, 2001]. The second, explanatory approach is older, and it includes default and modal nonmonotonic logics, as well as logic programming with negation as failure. In fact, all the papers in the famous 1980 issue of the Artificial Intelligence Journal on nonmonotonic reasoning could be seen as belonging to this latter camp (though McCarthy's circumscription is covered also by the preferential approach). The aim of this study consists of providing a systematic logical theory for this explanatory approach to nonmonotonic reasoning.

It might well be (and I certainly hope so) that both these approaches will some day become parts of a single future theory of nonmonotonic reasoning. Nevertheless, I also believe that a proper basis for a future unification should consist in a careful separation between these approaches. Such a separation, however, is not a trivial matter.

The difference between the two approaches can be found on a number

¹ They have been called, respectively, *classical* and *argumentative* nonmonotonic reasoning in [Bochman, 2001].

of levels. To begin with, there are two different senses in which a logical formalism, or a reasoning system, may be called nonmonotonic. First, it may be nonmonotonic in that its rules do not admit addition of new premises, that is, the system does not allow Strengthening the Antecedent. Second, it may be nonmonotonic in the sense that adding further rules to the system may possibly invalidate some of the conclusions obtained earlier. Now, it turns out that these two kinds of nonmonotonicity are largely independent. Thus, preferential inference relations (see [Kraus *et al.*, 1990]) are nonmonotonic in the first sense, since strengthening the antecedent does not hold for them (*Birds fly* does not imply *Penguins fly*). However, they are monotonic in the second sense, since addition of new preferential conditionals does not invalidate previous conclusions. On the other hand, default logic (see [Reiter, 1980]) exemplifies monotonicity of the first kind and nonmonotonicity of the second kind. Indeed, we will see that any default theory can be safely extended with default rules obtained from existing ones by strengthening their pre-requisites and justifications; such additional rules will not change the set of extensions. On the other hand, adding arbitrary new rules to the default theory may result in creating new extensions, so nonmonotonic conclusions made earlier will not, in general, be preserved.

Taken by itself, however, the above distinction is a purely formal difference between formalisms, and it still does not necessarily imply that the two approaches are essentially different. Actually, one of the main incentives behind the preferential approach, already explicitly expressed in [Shoham, 1988], was the hope that default logic and other explanatory nonmonotonic formalisms can be subsumed by some generalized version of the preferential approach in the sense that extensions could be viewed as preferred models under some generalized notion of preference. Unfortunately, subsequent studies have raised grave doubts about this hope. Thus, the nonmonotonic semantics of default logic has turned out to violate even the most basic postulates of cumulative inference (see [Makinson, 1989]). A similar situation has been found in logic programming (see [Dix, 1991]).

In a hindsight, this outcome should have been expected, since the selection of intended models in an explanatory approach is not preferential in a usual sense. Namely, the explanatory approach determines the intended models as models satisfying certain closure conditions with respect to the rules (see below). On a most abstract level, such models are usually expressible as fixed points of some operator which is not even monotonic. Accordingly, the supposed preference that singles out these models appears

to be a trivial, zero-one preference that basically differentiates only right models from bad ones. In this sense, the above formal difference can be viewed only as a symptom of deeper conceptual distinctions between the two approaches.

Both preferential and explanatory nonmonotonic reasoning can be seen as theories of a reasoned use of assumptions. Now, preferential reasoning treats such assumptions as *defaults*, namely as normality assumptions we can use whenever there is no evidence to the contrary. This understanding is combined with a general principle that a problem situation should be assumed as normal as it is consistently possible, given the known facts. This naturally creates a preferential setting, in which the normality of models is measured by the set of defaults they support (see [Bochman, 2001] where such a preferential order is formally described).

It turns out, however, that the explanatory nonmonotonic reasoning implicitly assigns a different role to assumptions. Using the name adopted by David Poole (see [Poole, 1989; Poole, 1990]) it makes such assumptions *conjectures*. Conjectures are assumptions that we make in order to explain observations. The supposition of normality (or abnormality, for that matter) is not essential for such conjectures. A certain combination of symptoms may lead to a conjecture about a rare and unusual disease, while in other cases some ‘ordinary’ illness will suffice for explaining the observations; it seems beside the point in this case to order diseases with respect to their ‘normality’. As was rightly noted by Poole, we make conjectures only if there is evidence that requires them for explanation, in contrast to defaults that can be freely assumed, unless they contradict the facts and other assumed defaults. It was also strongly argued by Poole that the distinction between normality defaults and conjectures is closely related to the distinction between prediction and explanation: while we use defaults in order to predict facts that are yet unknown, conjectures are invoked when we have to explain known facts.

Unfortunately, the above presumably clear distinction has been obscured in the short history of nonmonotonic reasoning. The reason was that, from the very beginning, the main formalisms of nonmonotonic reasoning, including default logic, modal nonmonotonic logics and circumscription, have claimed their rights and responsibility on representation of normality defaults. Thus, Ray Reiter has suggested in [Reiter, 1980] that we can identify such normality assertions with a special case of default rules