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Quantitative

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Business

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Analysis

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David Eugene Smith

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# Quantitative Business Analysis

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DAVID EUGENE SMITH

San Jose State University

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## **About the Author**

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David Eugene Smith is a professor of Business Administration at San Jose State University. He received his B.S. and M.S. degrees from San Francisco State University and his M.B.A. and Ph.D. from the University of Santa Clara. He joined the Marketing Department of Marketing and Quantitative Analysis at San Jose State University in the fall of 1969 where he teaches marketing, quantitative methods and future studies.



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## Preface

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In schools of business, the instructor of courses in quantitative analysis has a major problem. Students enrolled in these courses typically have both a wide diversity of educational needs and a wide diversity of quantitative sophistication. To satisfy these students, an effective text can be neither too quantitatively sophisticated nor too cursory in its presentation. Quantitative Business Analysis was written to solve this problem.

The text is intended to be highly readable and thorough in its coverage. It emphasizes the student's conceptual understanding. Concepts are presented with sound logic and causal reasoning in the context of applied illustration. The mathematical prerequisites have been kept to a minimum. The student needs only a reasonable background in basic algebra. A previous introduction to probability will be helpful, but it is not required. For students who lack this introduction or would like a review, Chapter 2, Quantifying Uncertainty, provides the required background.

Prepared primarily for use in schools of business, the text has been structured to provide flexibility in covering the topics. It is suitable for a survey course or for a full year sequence. To make it more adaptive, the material has been grouped into four major topic areas: decision theory, mathematical programming, inventory and queuing theory, and simulation. As illustrated in Figure p. 1, each area can be considered independently in the selection of topics. Appendix sections and specially designated chapters represent optional material. Frequently more technical, the optional material provides select extensions for a more thorough coverage. These sections can be excluded without loss of continuity.

A general guide for topic coverage is provided in Table p. 1. In a full year

sequence, the entire text can be covered comfortably. For a survey course, the topics listed in column 1, Table p. 1 represent a useful selection. This list will require modification depending on the level of the students' background, the amount of supplementary material, and the depth of coverage.

I would like to express my appreciation for the helpful suggestions provided by the reviewers and most particularly to Elwood S. Buffa, University of California, Los Angeles for his comments. In addition, a very special and well-deserved thanks to Janet Anaya, San Jose State University, for typing the entire manuscript, proofing the calculations in the illustrative examples, and classroom testing the materials.

David Eugene Smith

March 1977

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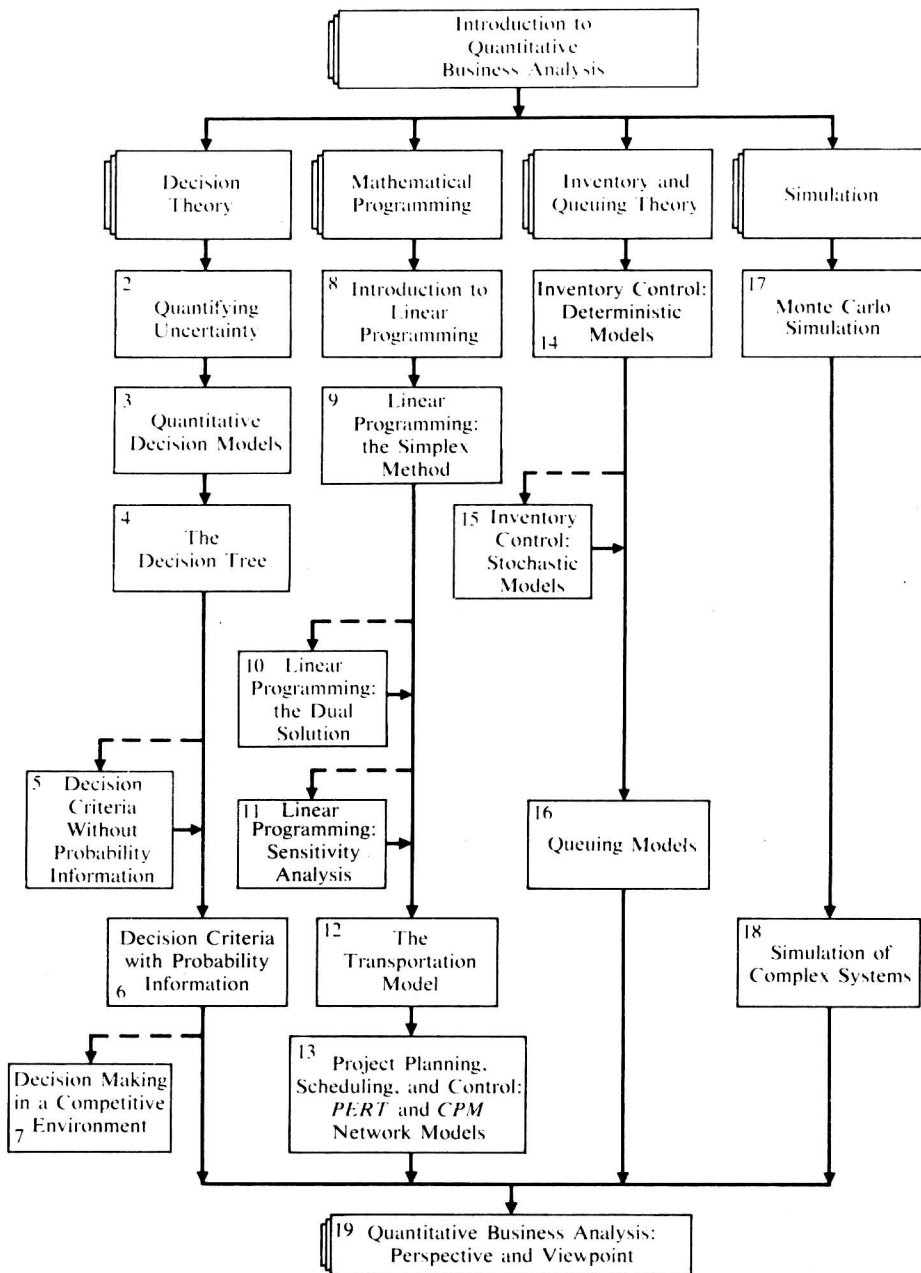


FIGURE P.1  
Organization of Topic Coverage

	Survey Course	Two Semesters	Two Quarters	Three Quarters
1 Introduction to Quantitative Business Analysis	✓	✓	✓	✓
2 Quantifying Uncertainty	✓	✓	✓	✓
3 Quantitative Decision Models	✓	✓	✓	✓
4 The Decision Tree	✓	✓	✓	✓
5 Decision Criteria Under Uncertainty Without Probability Information	—	✓	—	✓
6 Decision Criteria Under Uncertainty with Probability Information	✓	✓	✓	✓
7 Decision Making in a Competitive Environment: Game Theory	—	✓	—	✓
8 An Introduction to Linear Programming	✓	✓	✓	2nd Quarter ✓
9 Linear Programming: the Simplex Method	✓	✓	✓	✓
10 Linear Programming: the Dual Solution	—	✓	2nd Quarter —	✓
11 Linear Programming: Sensitivity Analysis	—	✓	—	✓
12 The Transportation Model	✓	2nd Semester ✓	✓	✓
13 Project Planning, Scheduling, and Control: <i>PERT</i> and <i>CPM</i> Network Models	✓	✓	✓	✓
14 Inventory Control: Deterministic Models	✓	✓	✓	3rd Quarter ✓
15 Inventory Control: Stochastic Models	—	✓	—	✓
16 Queuing Models	—	✓	✓	✓
17 Monte Carlo Simulation	✓	✓	✓	✓
18 Simulation of Complex Systems	—	✓	—	✓
19 Quantitative Business Analysis: Perspective and Viewpoint	✓	✓	✓	✓

TABLE P.1

A General Guide

# 1

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## **Introduction to Quantitative Business Analysis**

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Man is a decision maker. Throughout life, in every action, decisions are fundamental. They are unavoidable. The success or failure of an individual, of a society, or of mankind depends directly on the ability of individuals to render rational, intelligent, informed decisions.

Man lives in an environment of decisions, an environment of choice situations. His environment of choice is analogous to a decision to move on a chess board. The only major difference is the greater complexity: the board is not as finite, the pieces not as distinguishable or as few in number, the rules not as clearly defined. Nonetheless, each move opens future opportunities, fore-closes others. The player has no small stake in the outcome of the game.

An individual really has no rational choice but to choose, and to choose wisely. For not to choose is itself a choice, and not always the most desirable one. Some choices are more important than others, some more abstract and less clearly defined. Yet each choice will affect all other choices—the whole being a transient continuum weaving past to present, unfolding the future. If man is truly concerned with controlling his destiny, concerned with success rather than failure, he must masterfully learn the process of how to make better choices, how to make better decisions.

### **THE DECISION PROCESS**

While making countless decisions each day, an individual seldom if ever gives any conscious thought to the process by which decisions are made. All decisions,

however, have essential elements in common. The more clearly these elements can be understood, the more easily the decision maker can properly perceive, formulate, structure, and analyze a decision environment. With this understanding, the more easily the decision maker can make better choices, better decisions.

The essential elements of any decision are:

- (1) *the problem*: a conceptualization of a choice situation; the discovery of a situation providing an alternative selection;
- (2) *information*: the input into the system; a store of value; a store of symbolic models, images, and processes; a medium of transference;
- (3) *the value system*: a standard for measurement; the assignment of meaning to alternatives;
- (4) *decision models*: the analysis or synthesis of information; the process of using information in terms of the value system to evaluate and ultimately sort alternatives;
- (5) *the decision*: the selection of an alternative, the objective and output of the decision process.

The decision process, unfortunately, is not as easily conveyed as this simple list would suggest. The essential elements often vary in order and in importance. For example, in the discovery of a problem, the decision maker conducts a cursory, preliminary exploration of the decision environment. The decision maker must *decide* whether to become more fully involved in the decision process. This initial inquiry requires each of the essential elements—information, a value system, a method of analysis, and an alternative selection.

If this preliminary investigation is sufficiently promising, the decision maker begins a more formal involvement in the decision process. The initially perceived problem will typically contain subproblems with subproblems of their own. Each subproblem, again, may require each of the essential elements.

Information is indispensable at every stage. At times, desired information may not be available. It may be too costly or too time consuming to obtain. Especially for group decisions, an appropriate value system may be difficult to define. The means of analysis may vary substantially in formal sophistication at different stages throughout the decision process.

The decision process is complex—possibly as complex as man's perception of reality. Basic to any decision, however, are the essential elements in all of their varied forms.

### **The Problem: A Choice Situation**

The first essential element of the decision process is *the discovery of a problem*. A problem may be defined as the *conceptualization of a choice situation*. It is



the awareness of possible alternative courses of action that offer a different level of satisfaction. It is often the realization of the difference between what is and what could be.

Once a person realizes that there are better alternatives, he or she tends to be compelled to action. Not to select a better course of action is to forgo obtainable gains. To forgo an obtainable gain is to suffer a real *opportunity loss*. In turn, this perception of potential loss creates a state of psychological disequilibrium motivating the person to action, to further inquiry, to a more deliberate consideration of the other elements of the decision process. Irresistably, as a result of a personal striving for a better life, an individual's perception of potential transforms that person into a decision maker.

### **Information: The Medium of the Process**

*Information* plays an encompassing role in the decision process. It constitutes the only input into the system. It helps the decision maker to perceive the choice situation. Information clarifies the decision maker's perception of future environments and the definition of alternatives. It serves as a store of past experience. And as a store of past correlates between cause and effect, it contributes to the evolution and transformation of the value system. Information is the manipulative ingredient used in the analysis. It constitutes the output of the entire decision process.

The decision process can be conceptualized as an information-gathering, storing, processing, and producing system. Information serves as the catalyst and the medium of the system. *The decision process synthesizes information into choice—into action.*

### **The Value System: A Standard for Assigning Value**

In the process of selecting between alternatives, the decision maker assigns perceived worth or value to the alternatives. This is done by referring to a *value system* that serves as a master guide for scaling relative value. *The value system gives meaning and quantification to the alternatives.* It serves to guarantee a decision consistent with the decision maker's objectives.

The value system, therefore, is central to the process of defining and selecting alternatives. It clarifies the real meaning of each alternative and of the selection between alternatives. It assigns value to and sorts alternatives.

### **Decision Models: The Analysis and Synthesis of Information**

Once clearly defined and value weighted, the alternatives are analyzed. *The analysis consists of synthesizing the relevant information in terms of the value*