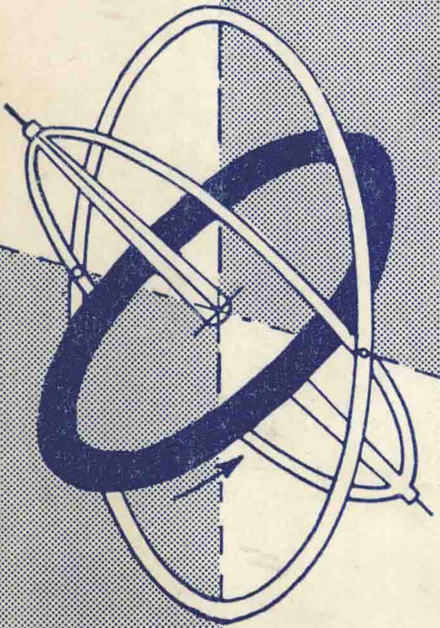
THE GYROSCOPE

THEORY AND APPLICATIONS



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For the first time in the English language, a sound, reliable account of the fundamental theory of the gyroscope and its more important applications.

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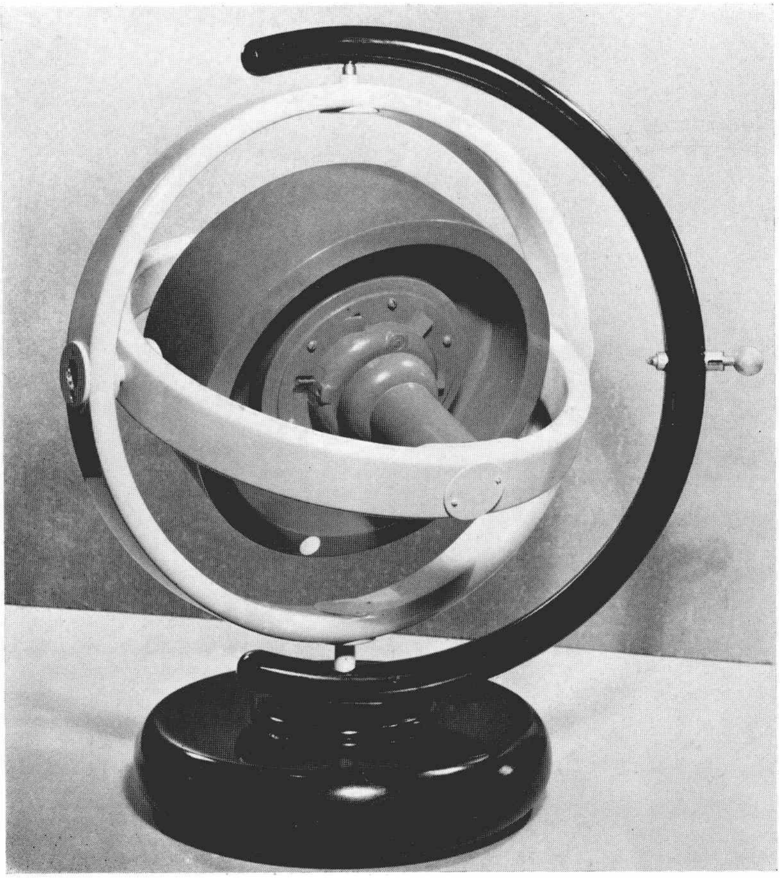

Simple Gyroscope
(Photograph courtesy Sperry Gyroscope Co.)

PREFACE

The aim of this book is to give a sound, systematic, unmistakably clear and reasonably complete treatment of the mathematical and mechanical aspects of the gyroscope and its more important applications. Since the book is concerned with fundamental principles, constructional details are not entered into except inso-far as they may aid in clarifying the principles. Friction has been left out of consideration in the motion of the gyroscope itself, because the construction, mounting and casing of modern gyro-scopes are such as to keep friction of all kinds as low as possible. Unavoidable friction is taken care of by the driving motor.

Because I think the mathematical theory of the gyroscope and its applications are best treated with the aid of vectors, I have used vector methods throughout the book. And since a knowledge of vector analysis is not assumed on the part of the reader, the first chapter is devoted to the exposition of the amount of vector analysis needed in the subsequent chapters.

The right-handed system of coordinate axes is used throughout the book, because of its advantages for this work.

A complete and exact mathematical treatment of gyroscopic motion becomes intractable almost at the beginning. Approxima-tions of minor importance must be made in order to obtain tractable and solvable equations. When making simplifying approximations, I have pointed out their nature and in some cases have shown by numerical examples that the errors thus introduced were of no consequence.

In the preparation of this book I have consulted the works of many previous writers, the most important of which are listed in the Bibliography at the end of the book.

It is a pleasure to record my thanks and obligations to the Sperry Gyroscope Company, Great Neck, New York, for their unstinted cooperation in furnishing information and photographs relating to various gyroscopic instruments and applications. I also wish to record my thanks to the following other manufacturers for

furnishing information and photographs: The Arma Corporation, Garden City, New York; The Minneapolis-Honeywell Regulator Company, Minneapolis, Minnesota; and The National Engineering Company, Chicago, Illinois. Finally, I wish to thank Mr. Donald Trumpy, of the yacht-building firm of John Trumpy and Sons, Annapolis, Maryland, for gyroscopic data concerning a 29,000-mile cruise which he took on a yacht equipped with a Sperry ship stabilizer.

J. B. SCARBOROUGH

August, 1957

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PART I

THEORY OF THE GYROSCOPE

CHAPTER I

Some Necessary Vector Analysis

The theory, behavior and applications of the gyroscope can be explained best by means of vectors. We therefore devote the present chapter to the exposition of the needed amount of vector analysis.

1. Scalar and vector quantities

The quantities which occur in physics are of two kinds: those which have magnitude only and those which have both magnitude and direction. The former are called *scalar* quantities and the latter are called *vector* quantities. Examples of scalar quantities are density, temperature and electric potential. Familiar examples of vector quantities are force, velocity and acceleration. Vectors may also be used to denote position, in which case they are called position vectors.

Vector quantities are represented geometrically by segments of straight lines, the line segments carrying arrow heads at one end to indicate the sense of the vector. The magnitude and direction of the directed quantity are indicated by the length and orientation of the vector.

In Fig. 1 is shown a vector $\overrightarrow{AB}$, which is also denoted by the single letter **r**. The projections of $\overrightarrow{AB}$ on the coordinate axes are the vectors **X**, **Y** and **Z** as shown.

A vector is designated in print by a single boldface letter, as **r** in Fig. 1; or by the letters designating its end-points, with an arrow written over the two letters, as $\overrightarrow{AB}$ in Fig. 1.

Vectors may be multiplied or divided at will by any numbers or scalars, the results always being vectors. Thus 3**r**, $\overrightarrow{AB}/l$, etc., are vectors having the same direction and sense as the original vectors. However, the multiplication or division of a vector by a negative number always reverses the sense of the vector.

Two vectors are *equal* when they have the same magnitude, direction and sense.

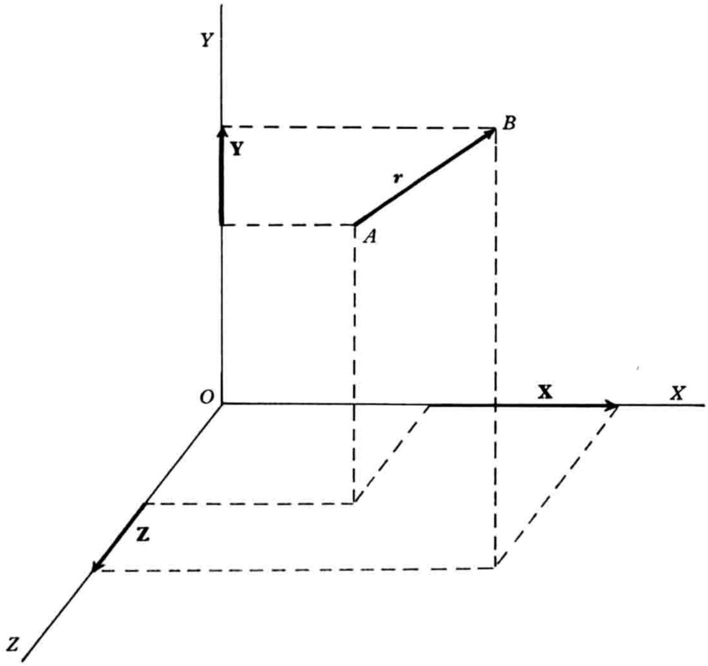

Fig. 1

2. Geometric addition and subtraction of vectors

To find the geometric sum of two vectors, place the initial point of the second vector at the terminal point of the first and then draw a line from the initial point of the first vector to the terminal point of the second. The vector thus drawn is the geometric sum of the given vectors. In Fig. 2, for example, $\overrightarrow{AC}$ is the geometric or vector sum of $\overrightarrow{AB}$ and $\overrightarrow{BC}$. Such addition is denoted by either of the equations

$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}, \tag{2·1}$$

$$\mathbf{P}+\mathbf{Q}=\mathbf{R}. \tag{2·2}$$

The geometric difference between two vectors is found by changing the sign of the subtrahend vector and then adding it to the other vector. Thus

$$\mathbf{P}-\mathbf{Q}=\mathbf{P}+(-\mathbf{Q}). \tag{2·3}$$

Reference to Fig. 2 will show that vectors are added and subtracted by the parallelogram law, the sum being given by one diagonal of the parallelogram and the difference by the other diagonal.

The geometric sum of any number of vectors is found by the same procedure as in the case of two vectors; that is, the initial point of the second vector is placed at the terminal point of the first, the

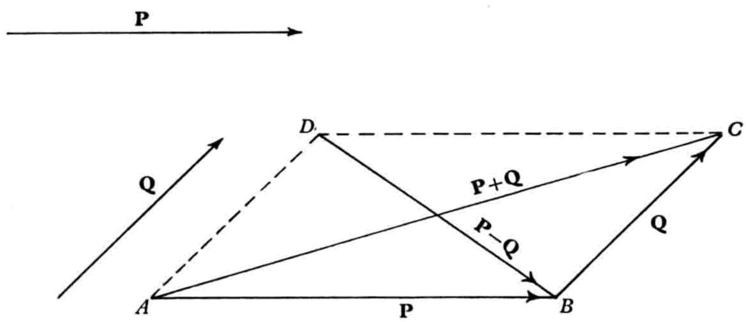

Fig. 2

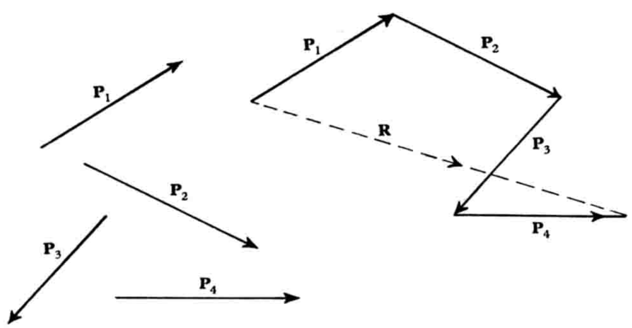

Fig. 3

initial point of the third is placed at the terminal point of the second, etc. The geometric sum of all the vectors is the vector drawn from the initial point of the first to the terminal point of the last, as indicated in Fig. 3.

3. Analytical addition of vectors

The sum or resultant of several vectors is found analytically by first resolving the vectors into rectangular components along coordinate axes, finding the algebraic sums of these components