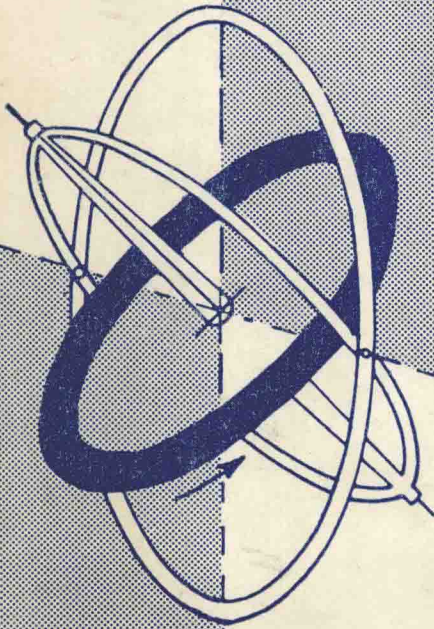


THE GYROSCOPE

THEORY AND APPLICATIONS



JAMES B. SCARBOROUGH

Professor Emeritus of Mathematics

U. S. Naval Academy, Annapolis, Maryland

For the first time in the English language, a sound, reliable account of the fundamental theory of the gyroscope and its more important applications.

INTERSCIENCE PUBLISHERS • NEW YORK • LONDON

52,13
S 285

THE GYROSCOPE

THEORY AND APPLICATIONS

JAMES B. SCARBOROUGH

Professor Emeritus of Mathematics
U.S. Naval Academy
Annapolis, Maryland

INTERSCIENCE PUBLISHERS, INC., NEW YORK

INTERSCIENCE PUBLISHERS LTD., LONDON

2215132

THE CYROSCOPE
THEORY AND APPLICATIONS
FIRST PRINTING 1958

ALL RIGHTS RESERVED

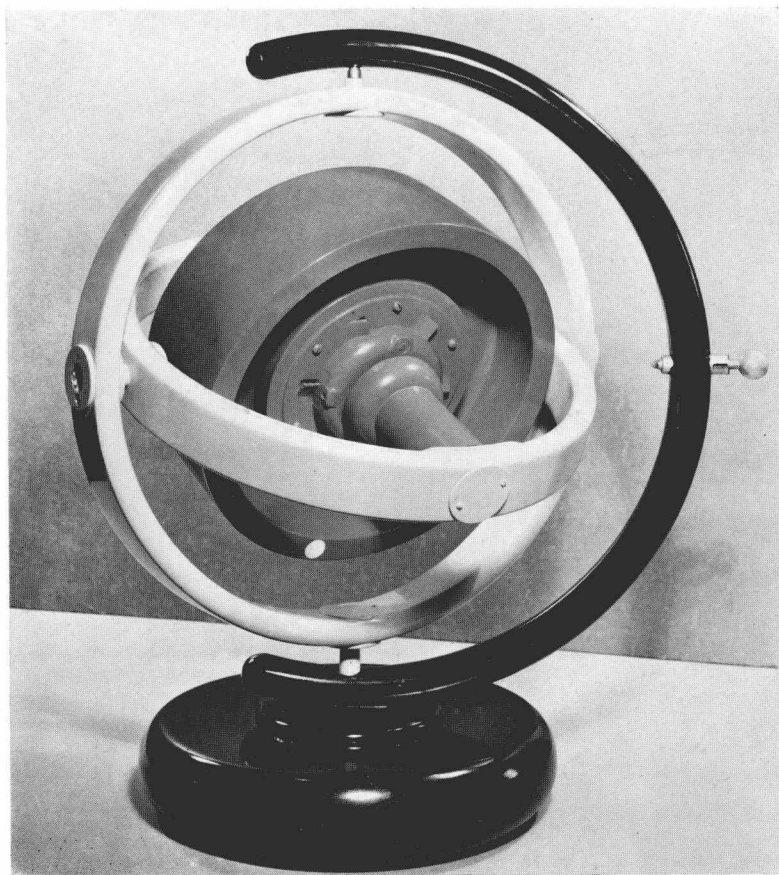
Library of Congress Catalog Card Number 57-13102

Interscience Publishers, Inc.
250 Fifth Avenue, New York 1, N.Y.

FOR GREAT BRITAIN AND NORTHERN IRELAND:

Interscience Publishers Ltd.
88/90 Chancery Lane, London, W.C. 2, England

INTERSCIENCE PUBLISHERS, INC. NEW YORK
PRINTED IN GREAT BRITAIN AT THE
UNIVERSITY PRESS, CAMBRIDGE
(BROOKE CRUTCHLEY, UNIVERSITY PRINTER)



Simple Gyroscope
(Photograph courtesy Sperry Gyroscope Co.)

PREFACE

The aim of this book is to give a sound, systematic, unmistakably clear and reasonably complete treatment of the mathematical and mechanical aspects of the gyroscope and its more important applications. Since the book is concerned with fundamental principles, constructional details are not entered into except insofar as they may aid in clarifying the principles. Friction has been left out of consideration in the motion of the gyroscope itself, because the construction, mounting and casing of modern gyroscopes are such as to keep friction of all kinds as low as possible. Unavoidable friction is taken care of by the driving motor.

Because I think the mathematical theory of the gyroscope and its applications are best treated with the aid of vectors, I have used vector methods throughout the book. And since a knowledge of vector analysis is not assumed on the part of the reader, the first chapter is devoted to the exposition of the amount of vector analysis needed in the subsequent chapters.

The right-handed system of coordinate axes is used throughout the book, because of its advantages for this work.

A complete and exact mathematical treatment of gyroscopic motion becomes intractable almost at the beginning. Approximations of minor importance must be made in order to obtain tractable and solvable equations. When making simplifying approximations, I have pointed out their nature and in some cases have shown by numerical examples that the errors thus introduced were of no consequence.

In the preparation of this book I have consulted the works of many previous writers, the most important of which are listed in the Bibliography at the end of the book.

It is a pleasure to record my thanks and obligations to the Sperry Gyroscope Company, Great Neck, New York, for their unstinted cooperation in furnishing information and photographs relating to various gyroscopic instruments and applications. I also wish to record my thanks to the following other manufacturers for

furnishing information and photographs: The Arma Corporation, Garden City, New York; The Minneapolis-Honeywell Regulator Company, Minneapolis, Minnesota; and The National Engineering Company, Chicago, Illinois. Finally, I wish to thank Mr. Donald Trumpy, of the yacht-building firm of John Trumpy and Sons, Annapolis, Maryland, for gyroscopic data concerning a 29,000-mile cruise which he took on a yacht equipped with a Sperry ship stabilizer.

J. B. SCARBOROUGH

August, 1957

CONTENTS

PART I

THEORY OF THE GYROSCOPE

CHAPTER I

SOME NECESSARY VECTOR ANALYSIS

1. Scalar and vector quantities	page 3
2. Geometric addition and subtraction of vectors	4
3. Analytical addition of vectors	5
4. The unit coordinate vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$	6
5. The scalar or dot product of two vectors	7
6. The vector or cross product of two vectors	9
7. The vector triple product $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$	15
8. Differentiation of vectors	16

CHAPTER II

SOME FUNDAMENTAL PRINCIPLES OF MECHANICS

9. Velocity and momentum	19
10. The fundamental equation of dynamics	19
11. D'Alembert's principle	20
12. Moment of a force about a point	20
13. Angular velocity	22
14. Moment of momentum of a rigid body about a fixed point	23
15. The equations of motion of a rigid body about a fixed point	25
16. The general differential equations of motion. Euler's equations	26
17. Kinetic energy of the body	31
18. Euler's angles	33
19. Components of the angular velocity in terms of Euler's angles	34
20. Rotation of a rigid body about a fixed axis	35

CHAPTER III

THEORY OF THE GYROSCOPE

21. Definition and behavior of a gyroscope	<i>page</i> 37
22. Steady or regular precession	38
23. Direction of precession	41
24. Free precession	42
25. Unsteady precession. General motion of a gyroscope	43
26. Steady precession of an unsymmetrical gyroscope	46
27. Gyroscopic resistance	48
28. Direction of the gyroscopic reaction moment	51
29. Motion of a free gyroscope	51
30. Stability of the motion of a free gyroscope	55
31. Motion of the spin axis in unsteady precession	61

CHAPTER IV

THEORY OF THE GYROSCOPE (*continued*)MOTION OF A GYROSCOPE UNDER THE ACTION
OF GRAVITY. THE TOP

32. The differential equations of motion	65
33. Forced steady precession	65
34. General motion of the top	68
35. The general precession	71
36. Nutation of the spin axis in the azimuthal plane	72
37. Path of the spin axis on a unit sphere	74
38. Periods of the cycloidal loops. Pseudo-regular precession	75
39. The sleeping top	83

PART II

APPLICATIONS OF THE GYROSCOPE

CHAPTER V

GYROSCOPIC ACTION IN VEHICLES AND
ROTATING BODIES

40. Gyroscopic effects in car wheels rounding a curve . . .	page 89
41. Derivation of the differential equations of motion of a gyroscope by application of the principle of gyroscopic reaction . . .	93
42. Gyroscopic effects of dynamos, motors, and turbines installed on ships	96
43. Rolling hoops and disks	100
44. Gyroscopic grinding mills	105
45. Gyroscopic action on oblong projectiles fired from rifled guns .	117
46. Gyroscopic effects on propeller-type airplanes	119

CHAPTER VI

THE GYROSCOPE AS A DIRECTION INDICATOR
AND STEERING DEVICE

A. THE GYROSCOPIC COMPASS

47. The principle of action	121
48. Differential equations of motion of a disturbed gyrocompass axle	124
49. Derivation of the differential equations by the G.R.M. method .	127
50. Solution of the equations	128
51. A numerical example	129
52. Simpler forms of the differential equations and their solution .	131
53. Comparison of the amplitudes of the horizontal and vertical oscillations. Locus of the end of the gyro axle	133
54. A third method of deriving the differential equations of motion	134
55. Motion of the gyroscopic compass with damping	135
56. The speed and course error	140
57. The Sperry mercury ballistic gyroscopic compass	141

B. GYROSCOPIC STEERING

58. Rate gyroscopes	page 143
59. Motion of a damped rate gyroscope	145
60. The rate gyroscope as a differentiator and integrator	146
61. Steering of torpedoes	147
62. Other steering devices	148

C. SOME RECENT TYPES OF GYROSCOPES

63. The Sperry Mark 22 gyrocompass	150
64. The Arma miniature gyrocompass	150
65. The Draper hermetic integrating gyro (HIG)	150
66. The Sperry gyrotron vibratory gyroscope	151

CHAPTER VII

THE GYROSCOPE AS A STABILIZER

A. THE GYROSCOPIC SPHERICAL PENDULUM

67. Definitions	161
68. Differential equations of motion	161
69. Solution of the equations	164
70. The spherical pendulum without gyroscope	169
71. Numerical comparison of the amplitudes of the gyroscopic and spherical pendulums	171
72. Gyroscopic pendulum with point of suspension subjected to a periodic force	172

B. OTHER TYPES OF GYROSCOPIC PENDULUMS

73. Gyroscopic pendulum suspended at center of gravity of gyro and with pendulous weight below	176
74. The inverted gyroscopic pendulum	177
75. Gyroscopic pendulum with gimbal axes rotating in azimuth	180

CHAPTER VIII

THE GYROSCOPE AS A STABILIZER (*continued*)

THE SHIP STABILIZER

A. SHIP ON A STRAIGHT COURSE

76. Schlick or brake-type stabilizer	page 187
77. Roll of the ship with gyro clamped	194
78. Numerical example	196
79. Sperry or active-type stabilizer	202

B. SHIP ON A CURVED PATH

80. Brake-type stabilizer	208
81. Active-type stabilizer	212

CHAPTER IX

THE GYROSCOPE AS A STABILIZER (*continued*)

MONORAIL CARS

A. MONORAIL CARS ON A STRAIGHT TRACK

82. Monorail car with gyro axis vertical (Scherl and Schilowsky type)	215
83. Monorail car with gyro axis horizontal (Brennan type)	220

B. MONORAIL CARS ON CURVES

84. Vertical axis type	224
85. Horizontal axis type	226

CHAPTER X

ASTRONOMICAL APPLICATIONS

86. Precession and nutation of the earth's axis	231
87. Other astronomical applications	243

BIBLIOGRAPHY	245
------------------------	-----

APPENDIX

SCHULER TUNING OF GYROSCOPIC COMPASSES AND
GYROSCOPIC PENDULUMS

1. Introduction	page 247
2. Schuler Tuning of a Gyroscopic Pendulum	249
3. Schuler Tuning of the Gyroscopic Compass	253
INDEX	255

PART I

THEORY OF THE GYROSCOPE

CHAPTER I

Some Necessary Vector Analysis

The theory, behavior and applications of the gyroscope can be explained best by means of vectors. We therefore devote the present chapter to the exposition of the needed amount of vector analysis.

1. Scalar and vector quantities

The quantities which occur in physics are of two kinds: those which have magnitude only and those which have both magnitude and direction. The former are called *scalar* quantities and the latter are called *vector* quantities. Examples of scalar quantities are density, temperature and electric potential. Familiar examples of vector quantities are force, velocity and acceleration. Vectors may also be used to denote position, in which case they are called position vectors.

Vector quantities are represented geometrically by segments of straight lines, the line segments carrying arrow heads at one end to indicate the sense of the vector. The magnitude and direction of the directed quantity are indicated by the length and orientation of the vector.

In Fig. 1 is shown a vector \vec{AB} , which is also denoted by the single letter \mathbf{r} . The projections of \vec{AB} on the coordinate axes are the vectors \mathbf{X} , \mathbf{Y} and \mathbf{Z} as shown.

A vector is designated in print by a single boldface letter, as \mathbf{r} in Fig. 1; or by the letters designating its end-points, with an arrow written over the two letters, as \vec{AB} in Fig. 1.

Vectors may be multiplied or divided at will by any numbers or scalars, the results always being vectors. Thus $3\mathbf{r}$, \vec{AB}/l , etc., are vectors having the same direction and sense as the original vectors. However, the multiplication or division of a vector by a negative number always reverses the sense of the vector.

Two vectors are *equal* when they have the same magnitude, direction and sense.

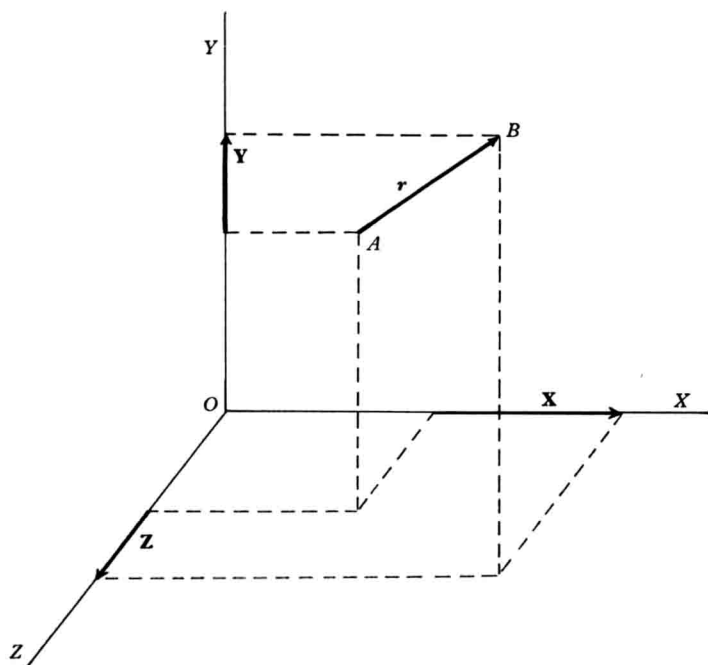


Fig. 1

2. Geometric addition and subtraction of vectors

To find the geometric sum of two vectors, place the initial point of the second vector at the terminal point of the first and then draw a line from the initial point of the first vector to the terminal point of the second. The vector thus drawn is the geometric sum of the given vectors. In Fig. 2, for example, \vec{AC} is the geometric or vector sum of \vec{AB} and \vec{BC} . Such addition is denoted by either of the equations

$$\vec{AB} + \vec{BC} = \vec{AC}, \quad (2.1)$$

$$\mathbf{P} + \mathbf{Q} = \mathbf{R}. \quad (2.2)$$

The geometric difference between two vectors is found by changing the sign of the subtrahend vector and then adding it to the other vector. Thus $\mathbf{P} - \mathbf{Q} = \mathbf{P} + (-\mathbf{Q})$.

$$(2.3)$$

Reference to Fig. 2 will show that vectors are added and subtracted by the parallelogram law, the sum being given by one

diagonal of the parallelogram and the difference by the other diagonal.

The geometric sum of any number of vectors is found by the same procedure as in the case of two vectors; that is, the initial point of the second vector is placed at the terminal point of the first, the

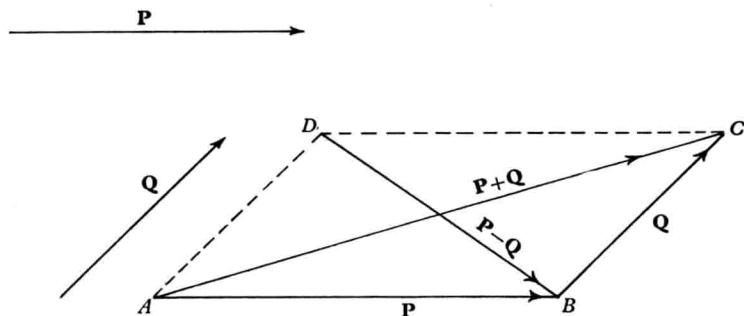


Fig. 2

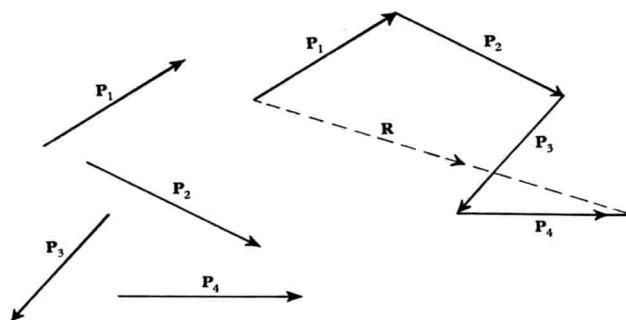


Fig. 3

initial point of the third is placed at the terminal point of the second, etc. The geometric sum of all the vectors is the vector drawn from the initial point of the first to the terminal point of the last, as indicated in Fig. 3.

3. Analytical addition of vectors

The sum or resultant of several vectors is found analytically by first resolving the vectors into rectangular components along coordinate axes, finding the algebraic sums of these components