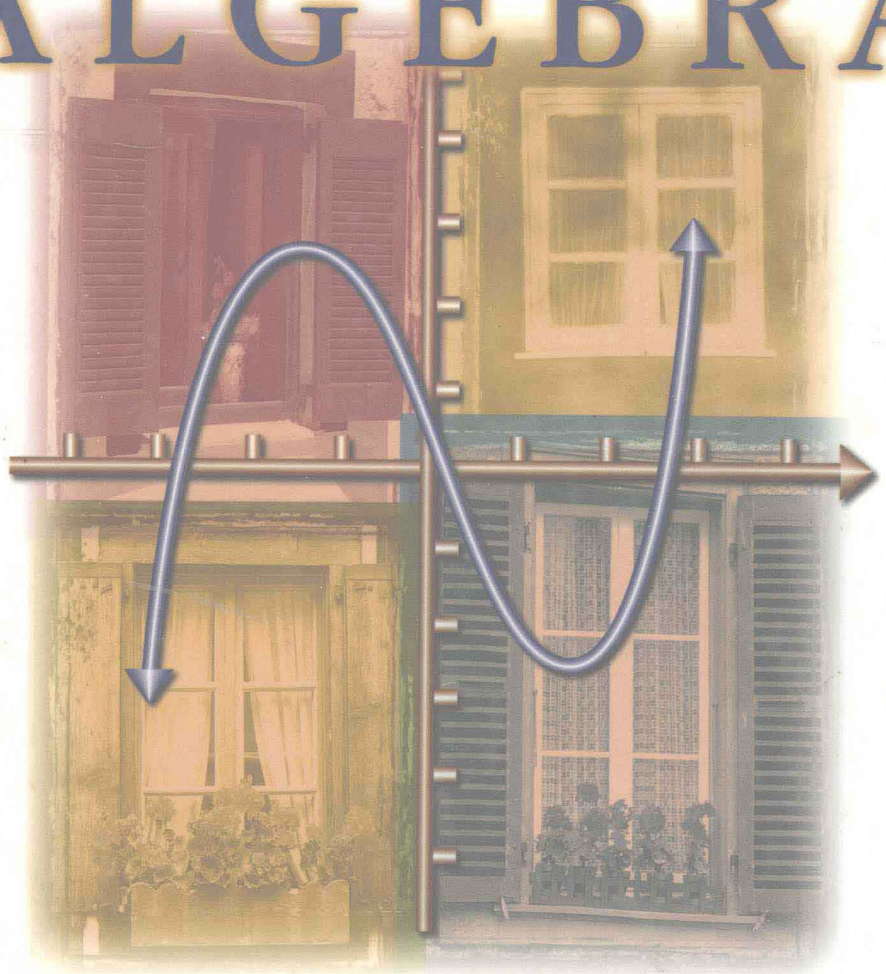
A

GRAPHICAL

APPROACH TO

COLLEGE ALGEBRA



SECOND EDITION

HORNSBY / LIAL

A

GRAPHICAL

APPROACH TO

COLLEGE ALGEBRA

SECOND EDITION

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To the memory of Jack Hornsby

Preface

In writing the previous edition of this text, we pursued the project with the firm commitment that it would not be merely an adaptation of our traditional text with only lip service paid to the use of technology. In so doing, we realized that a completely new approach would be necessary, based on the premise that all students would have graphing calculators on the first day of class and use them throughout the course.

The first edition was prepared and class-tested at the University of New Orleans from 1993 to 1995. It was published in 1996 after thousands of hours of work, not only by the authors but by reviewers, answer-checkers, editors, class-testers, and students.

This second edition reflects our combined years of experience as classroom teachers, emphasizes our enthusiasm for teaching with graphing calculators, and provides refinement of the first edition of the text through incorporation of the many helpful suggestions of both teachers and students.

PHILOSOPHY OF OUR APPROACH

Throughout the first five chapters, we present the various classes of functions studied in a standard college algebra text. Chapter 1 introduces functions and relations, using the linear function as the basis for the presentation. In this chapter we introduce the following approach used throughout the next four chapters.

After introducing a class of function:

- We first examine the nature of its graph.
- Next we discuss the analytic solution of equations based on that function.
- We then show how to provide graphical support for that solution using a graphing calculator.
- Having established these two methods of solving equations, we move on to the analytic methods of solving the associated inequalities.
- We then show how the analytic solutions of these inequalities can also be supported graphically. We use two approaches to graphical methods of solving equations and inequalities: the *x-intercept method* and the *intersection-of-graphs method*. We continually review and reinforce these methods throughout.
- Finally, once the student has a feel for the particular class of function under consideration, we use our analytic and graphical methods to solve interesting applications involving that function.

By consistently using this approach with all the different classes of functions, students become aware that we are always following the same general procedure and applying that procedure to a new kind of function.

THE APPROACH TO TECHNOLOGY

We wrote this text with the idea in mind that technology can be used to help us better understand the mathematical concepts. We continually emphasize that it is essential to understand the mathematical concepts and apply them hand-in-hand with the calculator. Our good friend, Peg Crider of Tomball College, says it best: "*Your brain is the most powerful tool in the whole process.*"

Because technology seems to be ever-changing, we feel strongly that this text should not attempt to teach the student how to use a particular model of calculator. However, we do use actual graphing calculator–generated screens in addition to the traditional art found in textbooks. All screens in the text can be duplicated by the TI-83 graphing calculator, manufactured by Texas Instruments. TI-Graph Link software was used to render the calculator screens. In addition, the *Graphing Calculator Manual* by Stuart Moskowitz that accompanies this text provides students with keystroke operations for many of the more popular graphing calculators.

OVERVIEW OF THE CONTENT

We have found that the function concept is frequently a difficult one for students to grasp. Rather than present (and possibly confuse) the student with a variety of functions at the outset, we begin with the linear function in Chapter 1, analyzing its graph, solving linear equations and inequalities, and then solving applications dealing exclusively with linear functions. In Chapter 1, we also immediately begin to explore the capabilities of graphing calculators to help students better understand algebraic concepts.

In Chapter 2 we examine the graphs of the basic algebraic functions and their associated symmetries, transformations, and operations. Here we use the absolute value function to extend the concepts presented in Chapter 1, again using the graph/equation/inequality/application approach that we feature throughout the text. In Chapter 3 we present polynomial functions, focusing first on quadratic functions and then expanding the discussion to higher degree functions. Chapter 4 covers the rational and root functions, using the same approach, and concludes with a section on inverse functions that leads naturally into Chapter 5 on exponential and logarithmic functions. In Chapter 6 we introduce the conic sections and parametric equations. Chapter 7 covers the various methods of solving linear and nonlinear systems and includes matrix methods for solving linear systems. The appropriate use of graphing calculators to help explain concepts and confirm solutions continues to be stressed in these two chapters. Chapter 8 covers various other topics in algebra, and Chapter R, a "reference" chapter for basic algebraic concepts, provides examples and exercises for review and reference.

NEW AND ENHANCED FEATURES

We have been very pleased with the response to the first edition of this text, and at the request of those who have used the book and our reviewers, we have included the following new or enhanced features.

Meaningful Applications of Mathematics With the assistance of Gary Rockswold of Mankato State University, we have provided more than 600 new applied

examples and exercises that focus on real-life applications of mathematics. To further supplement the material, we also open each chapter with an interesting application that can be solved using the methods introduced in that chapter. Additionally, all applications are titled, and an index of applications can be found on pages I-1–I-4.

Increased Emphasis on Modeling We have included a large number of applications that provide data, often in tabular form. These exercises provide opportunity for the students to construct and analyze mathematical models. Section 1.7 has been newly written to focus on linear models and regression. Here we first introduce the concept of modeling, and we then continue to feature data for other types of models throughout the subsequent chapters.

Increased Emphasis on Using Tables When we wrote the first edition of this text, tables were not found on some models of graphing calculators. This is no longer the case, and we have included table use in both examples and exercises in this edition.

Reference Chapter on Basic Algebraic Concepts The reference chapter has been updated and now includes exercises that test each of the concepts. Answers to the odd-numbered exercises in this chapter appear in the answer section.

New Quick Reference Guide This tear-out card, bound into the back of the text, serves as a handy guide to the Reference Chapter. For each section of the text, it suggests sections to review in the Reference Chapter before undertaking the study of the content in that section.

New Chapter Projects Each chapter concludes with a project that can be used as either an individual or collaborative learning activity. The project provides an opportunity for students to see how the material in the chapter they have just studied can be applied. The projects were written by Stuart Moskowitz of Humboldt State University, and we greatly appreciate the excellent contributions that he has made to both editions of this text.

New Chapter Summaries The chapter summaries are now provided in an easy-to-read grid format. They provide a section-by-section summary of important concepts that should assist students in reviewing and preparing for examinations.

New Chapter Tests We now offer a carefully written chapter test for each chapter. Students can use these to prepare for examinations, and instructors may wish to pattern their classroom tests after them.

New Sections At the request of users of the text and reviewers, we have included sections on Linear Models (Section 1.7) and Partial Fractions (Section 7.8) in this edition.

New Analytic and Graphical Solution Identification Many examples within the text highlight both ANALYTIC and GRAPHICAL solutions. This feature provides strong support for a multirepresentational approach to problem solving and shows students the value of solving analytically and supporting graphically.

New "What Went Wrong" Feature Using graphing technology to study mathematics opens up a whole new area of error analysis. In anticipation of typical student errors, we have included this feature that allows students and instructors to discuss such errors. This feature was suggested some time ago by a reviewer whose name we cannot remember, but we wish to thank that reviewer for this excellent suggestion.

New Web Site A new Web site has been established—designed to increase student success in the course by offering section-by-section tutorial help, enhancement of text chapter projects, downloadable programs for TI graphing calculators, and author tips. This icon  alerts students at times when this site would be helpful. The site will also be useful to instructors by providing dynamic resources for use in their classes. http://hepg.awl.com Keyword: Hornsby

CONTINUING FEATURES

The following features from the first edition have been retained.

Technology Notes Notes in the margin provide tips to students on how to use graphing calculators more effectively.

Cautions and Notes These warn of common errors and misconceptions.

For Group Discussion This feature appears within the exposition and offers material for instructors and students to discuss in a classroom setting.

Relating Concepts Exercises These groups of exercises tie together different topics and highlight the connections among various concepts and skills. By working the entire group in sequence, the student can appreciate the relationship among topics that earlier may have seemed unrelated.

Writing and Conceptual Exercises In addition to exercises that test concepts and skills or that present the mathematical concepts in a real-world applied setting, we have also included many writing (marked with a) and conceptual exercises. These are designed to help students reach a deeper level of understanding of the mathematical ideas being considered and to get them more actively involved in their own learning.

SUPPLEMENTS

FOR THE STUDENT

PRINTED SUPPLEMENTS

Student's Solutions Manual, ISBN 0-321-03945-9, *Norma James, New Mexico State University*

- Detailed solutions to odd-numbered Section Exercises, all Relating Concepts Exercises, odd-numbered Review Exercises and all Chapter Test Items.
- Ask your bookstore about ordering.

Graphing Calculator Manual, ISBN 0-321-03948-3, *Stuart Moskowitz, Humboldt State University*

- Graphing calculator usage instruction
- Keystroke operations for the following calculator models: TI-82®, TI-83®, TI-85®, TI-86®, Casio9850 Plus®, and HP38G®
- Worked-out examples taken directly from the text

MEDIA SUPPLEMENTS

Web Site

- Includes section-by-section tutorial help, enhanced Chapter Projects from the main text, study tips from the authors, downloadable TI-83® graphing calculator programs, and links to other sites
  http://hepg.awl.com Keyword: Hornsby

InterAct Math Tutorial Software, Windows ISBN 0-321-03547-X
Macintosh ISBN 0-321-03548-8
Windows/Macintosh CD-ROM ISBN 0-321-04317-0

Throughout the text, this icon indicates when this software would be helpful to students.

Interact Math Tutorial Software has been developed and designed by professional software engineers working closely with a team of experienced math educators. Interact Math Tutorial Software includes exercises that are linked with every objective in the

textbook and require the same computational and problem-solving skills as their companion exercises in the text. Each exercise has an example and an interactive guided solution that are designed to involve students in the solution process and to help them identify precisely where they are having trouble. The software recognizes common student errors and provides students with appropriate customized feedback. With its sophisticated answer recognition capabilities, Interact Math Tutorial Software recognizes appropriate forms of the same answer for any kind of input. It also tracks student activity and scores for each section which can then be printed out. The software is free to qualifying adopters or can be bundled with books for sale to students.

Videotape Series, ISBN 0-321-03951-3

Throughout the text, this icon  indicates when these videotapes would be helpful to students.

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- An engaging team of lecturers provide comprehensive coverage of each section.
- Selected odd-numbered exercises from the text presented
- Opportunity is given to solve a problem before the solution to the problem is given.
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FOR THE INSTRUCTOR

PRINTED SUPPLEMENTS

Instructor's Solutions Manual, ISBN 0-321-03943-2, *Norma James, New Mexico State University*

- Detailed solutions to all Section Exercises, Relating Concepts Exercises, Chapter Review Exercises, Chapter Tests, and Chapter Projects
- Contact your Addison-Wesley representative

Instructor's Testing Manual, ISBN 0-321-03949-1

- Contains four tests per chapter modeled on the chapter tests found in the text as well as the answers to all of the test questions included in the manual
- Contact your Addison-Wesley representative

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TestGen-EQ with QuizMaster-EQ, Windows ISBN 0-321-03541-0
Macintosh ISBN 0-321-03542-9

TestGen-EQ is a computerized test generator with algorithmically defined problems organized specifically for this textbook. Its user-friendly graphical interface enables instructors to select, view, edit and add test items, then print tests in a variety of fonts and forms. Seven question types are available, and search and sort features let the instructor quickly locate questions and arrange them in a preferred order. A built-in question editor gives the user the power to create graphs, import graphics, insert mathematical symbols and templates, and insert variable numbers or text. An "Export to HTML" feature lets instructors create practice tests that can be posted to a Web site. Tests created with TestGen-EQ can be used with QuizMaster-EQ, which enables stu-

dents to take exams on a computer network. QuizMaster-EQ automatically grades the exams, stores results on disk, and allows the instructor to view or print a variety of reports for individual students, classes, or courses. This program is available in Windows and Macintosh formats and is free to adopters of the text.

InterAct Math Plus Software Interact Math Plus combines course management and on-line testing with the features of the basic Interact Math Tutorial Software to create an invaluable teaching resource. Consult your Addison-Wesley representative for details.

ACKNOWLEDGMENTS

We wish to thank the many teachers and students who have given us valuable suggestions that have made this a better book. It has been deeply gratifying to have had some of you say "You've done it just the way it should be done." Such comments remain some of the most cherished memories of our professional careers. We wish to thank Anne Kelly, formerly of HarperCollins College Publishers, for signing the book and believing in it. Thanks also go out to those individuals who provided input into the first edition.

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A FINAL WORD

In the conclusion to the preface for the first edition, we wrote:

> We hope that this book begins to make a difference in the manner in which algebra is presented and learned as we move into the twenty-first century. We ask that both instructors and students pursue its contents with an open mind, ready to teach and learn in the manner that only now, after so many thousands of years, is possible. We, like Newton, can do so only because we "have stood on the shoulders of giants."

Judging from the many positive comments we have received since its publication, we feel that we are indeed on our way to making that difference. This is a most special project for us, and we are grateful to those who have contributed to its acceptance.

John Hornsby
Margaret L. Lial

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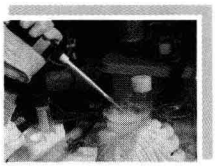

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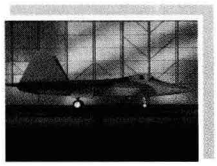

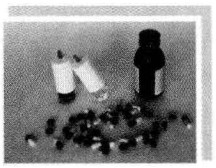

CHAPTER 1

Rectangular Coordinates, Functions, and Analysis of Linear Functions

CHAPTER OUTLINE

Approximating data with linear relations and functions is one of the most important and fundamental mathematical techniques we use today. Although most real-world applications are nonlinear, we can often use linear approximations to give accurate estimations. For example, the shape of Earth is round, not flat. Yet, when a building is constructed, the curvature of Earth's surface is seldom taken into account. Instead, it is assumed that the surface is level over the relatively small distance covered by the building. In this case, we use a linear approximation to accurately solve a nonlinear problem. However, when freeways were built across the United States, the curvature of Earth's surface had to be taken into account. If the distance or interval is small, linear approximations can lead to accurate estimations. Their advantage is that they are simple and easy to compute. On the other hand, if the distance or interval is large, then a linear approximation may lead to incorrect results.

Galaxy	Distance	Velocity
Virgo	15	1600
Ursa Minor	200	15,000
Corona Borealis	290	24,000
Bootes	520	40,000
Hydra	?	60,000

In the late 1920s the famous observational astronomer Edwin P. Hubble (1889–1953) determined by careful measurement both the distances to several galaxies and the velocities at which they were moving away from Earth. Four galaxies with their distances in megaparsecs and velocities in kilometers per second are listed in the table. (One megaparsec is approximately 1.9×10^{19} miles.)

Is there any relationship between the data that could be used to predict the distance from Earth to the galaxy Hydra? Could the age of the universe be estimated using these data? Edwin Hubble made one of the most important discoveries in astronomy when he determined that a linear relationship existed between the distance and velocity of a galaxy. His important finding resulted in Hubble's Law. Because of this significant contribution to the understanding of our expanding universe, the Hubble Space Telescope was named after him. For galaxies relatively close to Earth, Hubble's linear relationship has been

shown to be accurate. How far into deep space this linear relationship holds remains uncertain. Before we can calculate the distance to the galaxy Hydra or approximate the age of the universe, we must first understand relations and functions. Using relations and functions to approximate real data, we will be able to answer these and other important questions.*

1.1 REAL NUMBERS, LOGIC, AND COORDINATE SYSTEMS

Sets of Real Numbers ◆ Roots ◆ Logic ◆ Coordinate Systems ◆ Viewing Windows

SETS OF REAL NUMBERS

The idea of counting goes back to the beginning of our civilization. When people first counted they used only the **natural numbers,** written in set notation as

$$\{1, 2, 3, 4, 5, \ldots\}.$$

Much more recent is the idea of counting *no* object—that is, the idea of the number 0. Including 0 with the set of natural numbers gives the set of **whole numbers.**

$$\{0, 1, 2, 3, 4, 5, \ldots\}$$

(These and other sets of numbers are summarized later in this section.)

As the need for other kinds of numbers arose, additional sets of numbers were developed. Though the need for negative numbers may seem obvious to us today, they are a relatively recent development in the history of mathematics. The negatives of the natural numbers, included with the set of whole numbers, gives the very useful set of **integers,**

$$\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

Integers can be shown pictorially with a **number line.** (A number line is similar to a thermometer on its side.) As an example, the elements of the set $\{-3, -1, 0, 1, 3, 5\}$ are located on the number line in Figure 1.

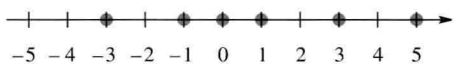

FIGURE 1

The result of dividing two integers, with a nonzero divisor, is called a *rational number*. By definition, the **rational numbers** are the elements of the set

$$\left\{ \frac{p}{q} \;\middle|\; p, q \text{ are integers and } q \neq 0 \right\}.$$

This definition, which is given in *set-builder notation*, is read "the set of all elements p/q such that p and q are integers and $q \neq 0$." Examples of rational numbers include $\frac{3}{4}$, $-\frac{5}{8}$, $\frac{7}{2}$, and $-\frac{14}{9}$. All integers are rational numbers, since any integer can be written as the quotient of itself and 1.

Rational numbers can be located on a number line by a process of subdivision. For example, $\frac{5}{8}$ can be located by dividing the interval from 0 to 1 into 8 equal parts, then

Sources: Acker, A., and C. Jaschek, *Astronomical Methods and Calculations*, John Wiley & Sons, 1986.

Sharov, A., and I. Novikov, *Edwin Hubble, The Discoverer of the Big Bang Universe*, Cambridge University Press, 1993.