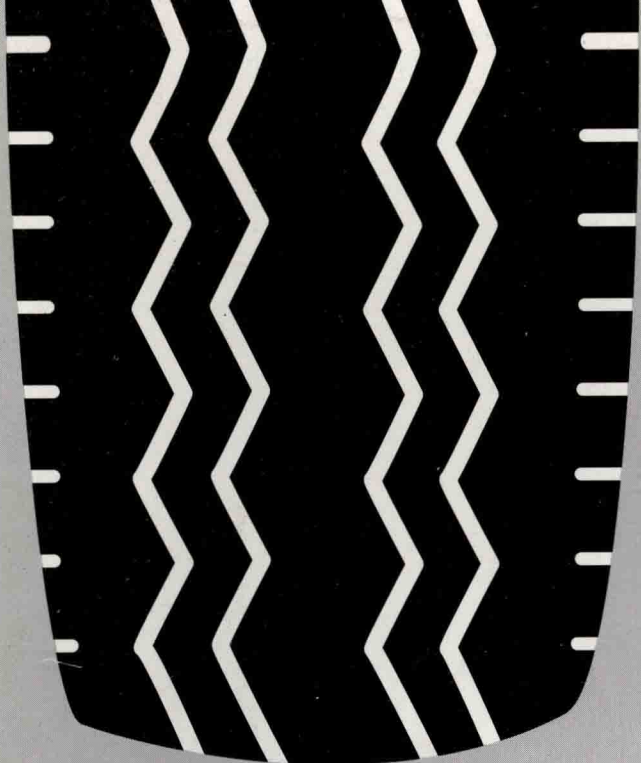




Vehicle, Tire, Pavement Interface

Henry/Wambold, editors



STP 1164

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Vehicle, Tire, Pavement Interface

J. J. Henry and James C. Wambold, editors

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Each paper published in this volume was evaluated by three peer reviewers. The authors addressed all of the reviewers' comments to the satisfaction of both the technical editor(s) and the ASTM Committee on Publications.

The quality of the papers in this publication reflects not only the obvious efforts of the authors and the technical editor(s), but also the work of these peer reviewers. The ASTM Committee on Publications acknowledges with appreciation their dedication and contribution to time and effort on behalf of ASTM.

Foreword

This book represents the proceedings of the conference on Vehicle, Tire, Pavement Interface sponsored by ASTM Committee E-17 on Pavement Management Technologies. The conference was held in Santa Barbara, CA on October 28, 1990. John J. Henry and James C. Wambold were chairmen of the conference and are coeditors of this publication.

Overview

The vehicle and tire meet the pavement at the vehicle-tire-pavement interface. Successful and safe travel by land transportation over pavement or rail and by air transportation as it relates to taxiways and runways is fundamental to a country's economy. In this age of specialization, many separate groups from the highway, rail, vehicle, structures, and other communities are studying elements of the vehicle-tire-pavement interface, but true system designs are not being developed. In recent years there has been a shift toward a multidisciplinary approach in research, with each member of the research team contributing his or her expertise to the solution of a given problem. Representatives of the vehicle, tire, and pavement fields agree that coordination of their research and technologies is essential for successful and efficient design and operation of the transportation system.

This Special Technical Publication (STP) has been published as a result of the October 1990 Engineering Foundation Conference on the Vehicle-Tire-Pavement Interface, held in Santa Barbara, California, in an effort to provide a means for members with expertise in the various fields to share the state of the art in their own fields and to witness the potential benefits of collaboration. The cooperation of such organizations as the American Society for Testing and Materials, the Permanent International Association of Road Congresses, the Transportation Research Board, and the Federal Highway Administration led to the organization of this conference.

This volume serves as a record of the conference and makes papers available to a broader public. The 11 papers in this volume are presented in the order in which they were presented at the conference: experimental studies of full-scale systems, ride quality, trade-off analysis between vehicle and way, pavement-induced vehicle wear, traction, and the effect of vehicle dynamics on the design of structures. In some of these subject areas, much attention was focused on the increasing amount of truck traffic and its effects on the nation's highways.

Analytic Methods for Vehicle-Structure Problems

The Use of Semi-Active Suspension to Reduce Pavement Damage—Hedrick

The importance of semi-active suspension was discussed relative to the question, "Should suspensions be developed to reduce dynamic loads or should smoother pavements be designed?" Based on the results of computer modeling in one study, semi-active suspensions have the potential to reduce pavement damage by significantly reducing dynamic loads. Reducing road roughness and improving tire design (for traction, low wear, and low noise) and suspension (good handling and low pavement loading) are two areas noted for future investigation in addition to laboratory and field testing.

Experimental Studies of Full-Scale Systems

Roughness Effects on Vehicle-Pavement Interactions—Hegmon

This paper offers a brief overview of problems caused by road roughness and the extensive work done over the years to understand and alleviate the effects of road roughness, with the

major portion dealing with current work in the FHWA laboratory, the equipment and methods used, and the expected results. The objective of this study was to decrease pavement damage and therefore increase pavement life. Current research emphasis is on ways for increasing pavement life through a reduction of pavement damage by heavy truck traffic, and cooperation among the trucking industry, manufacturers, and users is necessary to arrive at the least damaging combination of trucks and highways. Pavement surface characteristics can certainly affect the highway user. While roughness per se is undesirable, other micro- and macrot textures are needed for good skid resistance on wet pavements. Interactions among rough pavements and heavy trucks, field and laboratory measurements, and computer simulations to predict the interaction forces are discussed as well. This research is leading to a better understanding of the dynamic interactions between heavy trucks and pavements and the detrimental effects of the factors involved.

Ride Quality

An Examination of the Spectral Characteristics of Longitudinal Highway Profiles—Hayhoe

Pavement profiles typically contain pulse-like disturbances that are hypothesized to have a significant effect on the perceived ride quality of the pavement. In the study reported in this paper, pavement profiles were compared to subjective ratings of ride quality in an effort to explain anomalies in the relationship between ride quality and profile characteristics. The relationship between the two measures is obtained from a panel rating study in the form of a regression equation giving the best statistical fit between the two measures. The roughness measures that give the best fit do not have clear physical interpretation in terms of the usually accepted characteristics describing the response of human subjects to vibration.

Trade-Off Analysis Between Vehicle and Way

Pavement Damage as Related to Tires, Pressures, Axle Loads, and Configurations—Sebaaly

The performance of highway pavements is directly related to the magnitude and frequency of the applied load. The way in which traffic loads are transferred to the pavement surface and to the entire pavement structure is controlled by the tire-pavement interaction mechanism. Tire inflation pressure, tire type, axle load, and axle configuration all affect the response of asphalt concrete (flexible) pavements. But which of these factors has the greatest influence on pavement response? Prediction models in theoretical analyses are often used to try to identify the critical factor or factors. In situ instrumentation of pavements provides a means to monitor the response of a pavement when subjected to various combinations of these factors.

Pavement-Induced Vehicle Wear

Dynamic Force Measurement Vehicle (DFMV) and its Application to Measuring and Monitoring Road Roughness—Ashmore

Different road roughness measurement systems are in use worldwide, systems that measure the profile directly and systems that measure vehicle response to road roughness. The DFMV fits into the first category but is unique because it uses force-motion measurements made at each axle end to define the terrain elevation, while the majority of profilometers use motion measurements only. The DFMV is a multipurpose test vehicle that has been used for several terrain, tire, and suspension studies, and one of its functions is to measure terrain elevation profiles. The DFMV was designed for off-highway profiling, so that long sections of rough

durability test courses could be profiled within a day. A wave-number spectrum proved to be an efficient and accurate technique for presenting long sections of various terrain types, including paved roads and off-highway durability test courses.

The DFMV measures the dynamic vertical, longitudinal, and side forces at the tire/ground interface at four individual wheel ends. The life of a vehicle can be accelerated by up to 20 to 1 by selecting a test course environment that is slightly rougher than the cross-country terrain. Two measurements are needed to describe completely the operating environment of the vehicle system from a fatigue standpoint, the terrain and the material fatigue curves showing the number of cycles versus stress for each material type in the vehicle. DFMV methodology is an accurate and established profile technique that can be used to measure and monitor test course severity and represents a meaningful step toward defining the life-cycle requirements of a particular type of vehicle.

Vehicle Damage Induced by Road Surface Roughness—Poelman

An experiment was conducted to determine surface roughness effects, as measured by a response-type meter, on vehicle suspension. Results suggest that greatly accelerated vehicle suspension fatigue damage occurs on road surfaces with measured road roughness less than 1.5 on a pavement serviceability index (PSI) scale. The important implication of this result is that vehicle manufacturers should be aware of how their vehicles are used in the real world and they should design vehicles that can accept road load input from roads with a 1.5 PSI rating or less. Also, when determining maintenance and resurfacing schedules, road planners should be aware of the point at which the operator may experience significant vehicle damage. If road surfaces deteriorate to less than 1.5, then operators will begin to have accelerated vehicle durability problems.

Traction

Measurement and Modeling of Truck Tire Traction Characteristics—Kulakowski

Braking performance of trucks is a critical element of highway safety. One of the most important variables affecting the braking performance of trucks is the frictional force developed between the truck tire and the road surface when brakes are applied. The frictional force, tire traction, depends on variables representing both the vehicle and the pavement surface characteristics. Tire traction characteristics are determined by forces generated at the tire-pavement interface as a result of driving, steering, and braking actions applied to the vehicle. The objective of this research was to measure the frictional forces for different trucks on different pavements under various speed and tire vertical load conditions during braking as well as combined braking and cornering maneuvers. The results were then used in an attempt to derive regression models representing tire traction parameters as functions of vehicle and pavement variables. Emphasis was placed on the significance of field tests versus laboratory and computer simulation because the results of field tests reflect all elements of the actual system.

A Review of Tire Traction—Williams

Road safety under wet conditions remains a major area of concern. It is an accepted fact that the risk being involved in a wet-weather accident increases with decreasing tread depth. This paper includes a review of the practical knowledge of tire-to-road friction relative to the tire tread pattern, tread compound, and road surface condition. Also discussed is the influence

on tire wet roadhold performance of unevenly worn tires versus normally reported evenly worn tire conditions. Results for truck tires as well as car tires are discussed, and the author suggests that continued improvements in new tire wet roadhold performance will only increase the difference between the new tire and the tire worn to the legal minimum tread depth. It is recommended that the difference between new and worn tires be reduced by increasing the minimum legal tread depth or by designing a constant performance tire.

Structures

The effects of vehicle dynamics on structures are discussed from three perspectives: historical, experimental, and practical (observational).

Past and Future Analysis of Moving Dynamic Loads on Structures—Scheffey

This paper presents a history of the problem of dealing with the effects of moving loads on a bridge. In recent years, better dynamic instrumentation has produced a resurgence of full-scale studies of highway, rail, and rapid transit bridges. The importance of tire and suspension system characteristics for pavement and safety problems has made some accurate data on these elements available. The author hopes to stimulate a broad perspective on the dynamic problems of all types of bridges and transportation structures.

Impact Factors for Highway Bridges—Gangarao

The problem of bridge vibrations due to moving truck loads has been under way for some time. This paper presents a critical review of the literature on impact or dynamic allowance factors and suggests modifications to the impact factor in the current AASHTO specifications for highway bridges by properly accounting for dominant bridge parameters other than span length. Impact values were found to be less than AASHTO's maximum value.

Structural Implications of Poor Bridge Surface Performance—Yanev

Specific to metropolitan New York City bridges, the NYC DOT Bridge Bureau inspected bridges under their responsibility and found that 42% needed complete rehabilitation. The remaining 58% need considerable structural repairs in order to avoid the need for premature rebuilding of that portion as well. A further step is the identification of structural components requiring repair or replacement and the optimal frequency of such measures. A program of rehabilitation is expected to result from this study. The elimination of certain problem-prone structural components from future design can be considered as well. Ultimately, schedules for timely repairs can be generated in order to extend the useful life of a bridge. The following structural components emerged as the primary sources of problems on most of the bridges: joints, drainage, and wearing surface (deck). No other bridge component other than joints deteriorates faster. In all these cases, problems originate at the roadway surface and rapidly propagate throughout the structure. The progress of this path of rapid deterioration is traced for several bridge types frequently encountered in New York. Measures used with varying levels of success in the elimination of the surface and structural deficiencies are described.

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The Use of Semi-Active Suspensions to Reduce Pavement Damage

REFERENCE: Yi, K. and Hedrick, K., “The Use of Semi-Active Suspensions to Reduce Pavement Damage,” *Vehicle, Tire, Pavement Interface, ASTM STP 1164*, J. J. Henry and J. C. Wambold, Eds., American Society for Testing and Materials, Philadelphia, 1992, pp. 1–13.

ABSTRACT: Multiple axle groups of heavy trucks can generate peak dynamic tire loads that are greatly in excess of their static loads. This paper reviews previous simulation studies that compare the performance of alternative suspensions (for example, 4 spring, walking beam, and air suspension). In addition, the design of an electronically modulated shock absorber is presented for application on a tandem suspension. Simulation studies are then presented to compare the performance of this concept with the previously discussed suspension types. Recent laboratory data illustrating the performance of semi-active dampers to reduce dynamic tire loads will be presented.

KEYWORDS: semi-active suspension, heavy truck, modulable damper, pavement damage, vehicle dynamics

The vertical loading applied to the pavement by a vehicle has two components: the static load and the dynamic load. The static load depends on the weight and the geometry of the vehicle. Dynamic tire forces are caused by vibration of the vehicle due to the road excitation and are highly dependent on the suspension characteristics and roughness of the road surface. The “fourth power law” relating the static axle load to pavement damage was primarily due to the AASHO Road Test [1]. In addition to the static load, the dynamic tire forces accelerate pavement deterioration. Recent studies have shown that the dynamic vehicle loads have significant impact on the damage [2,3,4,5] and substantial improvements in pavement longevity may be possible via suspension optimization and active/semi-active suspensions [6,7].

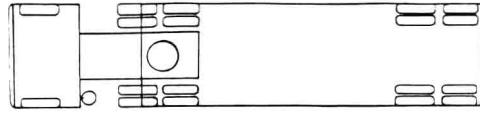
This paper describes the semi-active suspensions for heavy trucks to reduce the dynamic tire forces and presents the performance of an electronically modulated semi-active damper. The truck simulation package, VESYM, and the flexible pavement simulation program, VESYS, were used to investigate the influence of the dynamic loads on the pavement damage. Experimental study was performed using a laboratory half-car model.

Vehicle Dynamics-Pavement Damage Study

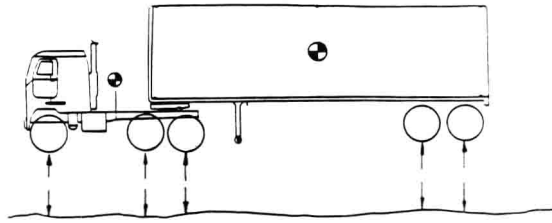
The truck model used in the simulation study is a five-axle tractor/semi-trailer (see Fig. 1). The tire forces fluctuate due to vehicle/pavement interaction as a vehicle runs on a road. A simulated time history of the driving axle dynamic tire force is shown in Fig. 2 for the case of a leaf spring tandem suspension with 88 km/h (55 mph) vehicle speed and road slope variance

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Five-Axle 18 wheel Tractor/Semi-Trailer Model



Vehicle/Pavement Interaction

FIG. 1—Five-axle 18 wheel tractor/semi-trailer model and vehicle/pavement interaction.

of 10×10^{-6} . The question that arises is how much the pavement damage is increased due to the dynamic component of the total tire force in addition to that due to the static tire force. Assuming a normal distribution of the dynamic forces, we can estimate the road damage by [2]:

$$\Phi = KP_{stat}^4[1 + 6s^2 + 3s^4] \quad (1)$$

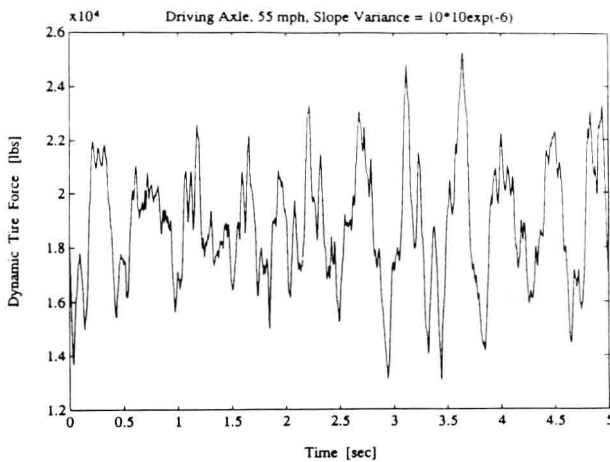


FIG. 2—Simulated time history of driving axle tire force of a five-axle tractor/semi-trailer vehicle on a typical flexible road at 55 mph (1 lbs = 4.448 N).

where

- P_{stat} = the mean axle load,
- s = the coefficient of variation of the dynamic tire force, and
- K = a constant.

Strictly, s is not equal to the Dynamic Load Coefficient (DLC). However, for the purpose of comparing between suspension types, it is appropriate to use DLC in place of s in Eq 1 [2]. The dynamic load stress factor is defined by:

$$\nu = \frac{\Phi}{KP_{stat}^4} = 1 + 6 \text{ DLC}^2 + 3 \text{ DLC}^4 \quad (2)$$

and the DLC is defined by:

$$\text{DLC} = \frac{\text{RMS dynamic tire force}}{\text{static tire force}} \quad (3)$$

It is worth noting that the dynamic road stress factor given in eq. 2 may be considered a lower estimate because it assumes that the distribution of loading at a specific point of the pavement is normal. Because the dynamic component of the tire force is due to the interaction between vehicle and pavement, the higher loads will tend to recur at a specific point in the pavement [2,4,5]. If it is assumed that the dynamic tire force distributions are normal, the 95th percentile impact factor may be estimated as follows:

$$IF_{95th} = 1 + 1.645 \text{ DLC} \quad (4)$$

and the road stress factor associated with this force is given by the fourth power law

$$\nu_{95th} = (IF_{95th})^4 \quad (5)$$

The severity of the dynamic loading for the most common types of heavy truck suspensions is shown in Table 1. The DLCs for each suspension type were estimated using the heavy vehicle simulation package, VESYM, under typical highway conditions (vehicle speed = 88 Km/h and road slope variance = 10×10^{-6}). The 95th percentile road stress factors shown in Table 1 imply that the pavement damage at specific points in the pavement may be increased by 82–159% due to the dynamic tire force.

Sweatman [2] performed an experimental investigation of the dynamic wheel forces in a number of axle group suspension systems for the speed range between 40 Km/h and 80 Km/h and with road roughness ranging from new-construction to maximum tolerable roughness.

TABLE 1—Severity of dynamic loading.

Suspension Type	DLC	Dynamic Road Stress Factor	95th Percentile Road Stress Factor
Walking Beam	0.163	1.160	2.586
Four-leaf Tandem	0.133	1.107	2.207
Air-bag Tandem	0.098	1.058	1.818

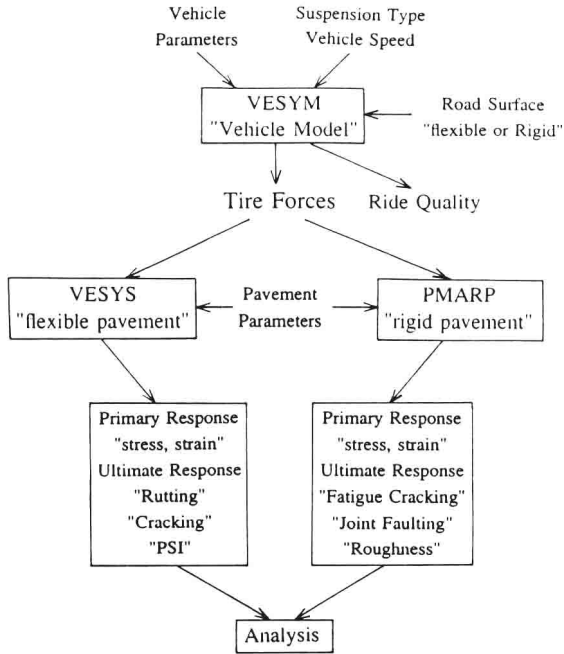


FIG. 3—Flow chart for vehicle/pavement studies.

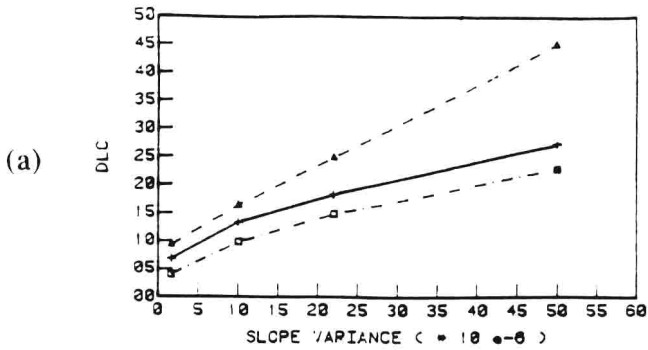
He concluded that DLC was strongly affected by speed and road roughness, and that the order of increasing dynamic loads is:

Air spring, Torsion bar < Four leaf < Walking beam

Cebon [4,5] performed a heavy vehicle simulation study and experimental validation. He investigated road response, road damage criteria and the effect of the dynamic tire forces in roadway damage. He concluded that the dynamic wheel loads increase theoretical road damage by a factor of 1.2 to 4 for typical vehicles and operating conditions.

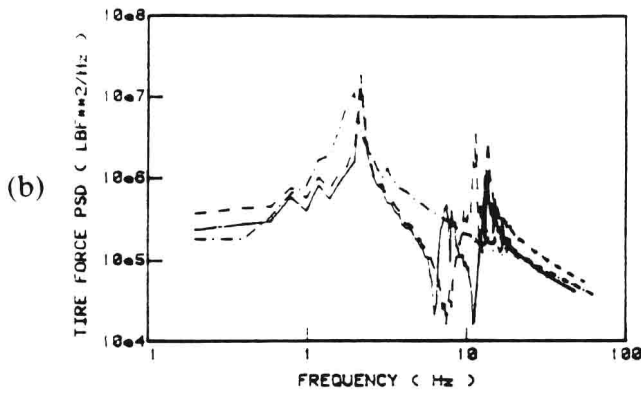
Hedrick et al. developed and modified simulation tools (VESYM, VESYS and PMARP) that allow the analysis of vehicle/pavement interaction [8]. They investigated the influence of heavy vehicle dynamics on pavement response for both flexible and rigid pavement. A large parametric study of vehicle and pavement variables was performed using the simulation packages. They concluded that the dynamic loads have significant impact on pavement damage; and that air spring suspensions were found to be the least damaging, walking beam suspensions the most damaging, and four leaf spring suspensions in between. They also concluded that pavement damage may be reduced significantly by careful suspension design.

Our vehicle dynamics/pavement damage study consists of two parts: vehicle analysis and the evaluation of the dynamic effects on pavement damage. Figure 3 shows our procedure for combined vehicle/pavement studies. VESYS is a viscoelastic flexible pavement simulation program developed at a number of institutions and PMARP is a rigid pavement simulation package. Coupled to a data file describing a particular pavement structure and seasonal testing cycle, VESYS(PMARP) calculates the stresses and strains that the tire load impresses on the pavement, then predicts the pavement damage that results from years of repeated load application. The effect of suspension type on vehicle response and road damage, shown in Fig. 4



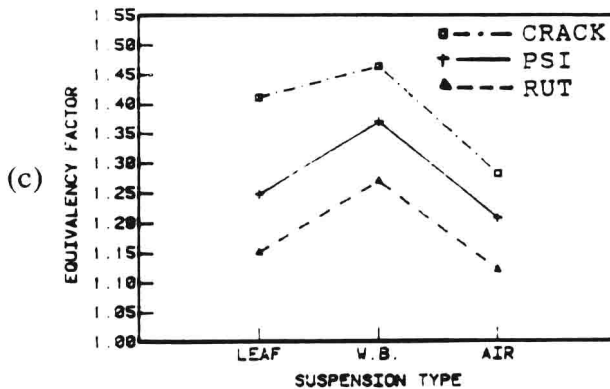
(a) Leading drive axle DLC versus road slope variance:

+ — + Leaf spring/short rocker tandem
 Δ - - Δ Walking beam tandem
 $\square$ - · - $\square$ Air spring tandem



(b) Leading drive axle tire force PSD at $SV = 22 \times 10^{-6}$:

— Leaf spring/short rocker tandem
 - - - Walking beam tandem
 - · - Air spring tandem



(c) The effects of suspension type on pavement damage.

FIG. 4—The effect of suspension type on vehicle response and road damage.

[6], indicates that the air spring tandem suspension is the least damaging. The slope variance is defined as the variance of the road slope. The equivalency factors are defined for cracking, rutting and present serviceability (PSI) and are given by:

$$EF = \frac{NF_{st}}{NF_{dy}}$$

where NF_{st} is the number of cycles to failure for a particular damage mode, with no vehicle dynamics, that is when the DLCs of the axle are held to zero. This case is referred as the static case and NF_{dy} is the number of cycles to failure when the vehicle dynamics are considered, that is, DLC's are not zero. This case is referred to as the dynamic case.

Semi-Active Suspensions

It was shown that substantial improvement in pavement longevity may be possible via suspension optimization and/or active/semi-active suspension technology [6,7]. Although an active suspension provides good performance, it requires a controller, high power actuators with good frequency response at high frequency, a relatively large power supply and sensors such as accelerometers, deflection sensors and velocity transducers. However, a semi-active suspension provides good performance compared to passive suspensions and does not require high power actuators or a large power supply and therefore is less expensive.

Figure 5 illustrates a block diagram of a semi-active suspension control system and shows a schematic diagram of a semi-active suspension implemented on a four-leaf spring tandem suspension. The objective of a semi-active suspension control for heavy vehicles is to minimize the dynamic tire force under random road input. The control input is the damping rate of modulable shock absorbers.

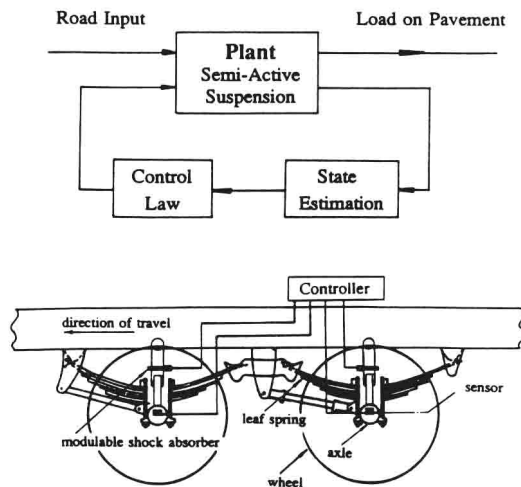


FIG. 5—Block diagram of a semi-active suspension system and schematic diagram of a semi-active suspension implemented on a leaf spring tandem suspension.

Control Law for a Semi-Active Suspension

In this section, we outline the derivation of an active control law using a state space model and an optimal control technique, then derive a semi-active control law by the minimizing instantaneous squared control input error between active and semi-active inputs. It was shown that the above procedure provides an excellent semi-active control law [9]. An active suspension system is described by the following state equation:

$$\dot{x} = Ax + Bu + Lh \quad (6)$$

where x , u , h are the state, control force and road input, respectively. For a quarter-car model case, the states are defined as follows:

$$x = \begin{bmatrix} \text{suspension deflection} \\ \text{sprung mass velocity} \\ \text{tire deflection} \\ \text{axle velocity} \end{bmatrix}$$

and matrices A , B , L are given as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{k_t}{m_u} & -\frac{b_s}{m_u} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix}, L = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

where m_s , m_u , k_s , k_t , b_s are sprung mass, axle mass, suspension stiffness, tire stiffness and shock absorber damping rate, respectively. Let the cost for control force u applied to the system be given by:

$$J = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [x^T Q x + 2x^T M u + u^T R u] dt \quad (7)$$

where $Q - MR^{-1}M^T \geq 0$ and $R > 0$. Q , M and R are the design parameters. LQ (Linear Quadratic) Optimal control theory provides the following state feedback control law:

$$\begin{aligned} u^{opt} &= F_{active} \\ &= -G_x \\ &= -R^{-1}(B^T H + M^T)x \end{aligned} \quad (8)$$

where G is the state feedback gain and H is the solution of the algebraic Riccati equation:

$$0 = -(A - BR^{-1}M^T)^T H - (A - BR^{-1}M^T) - Q(-MR^T) - HBR^T H \quad (9)$$

A semi-active suspension model is represented by the following bilinear state equation:

$$\dot{x} = Ax + NxV + Lh$$