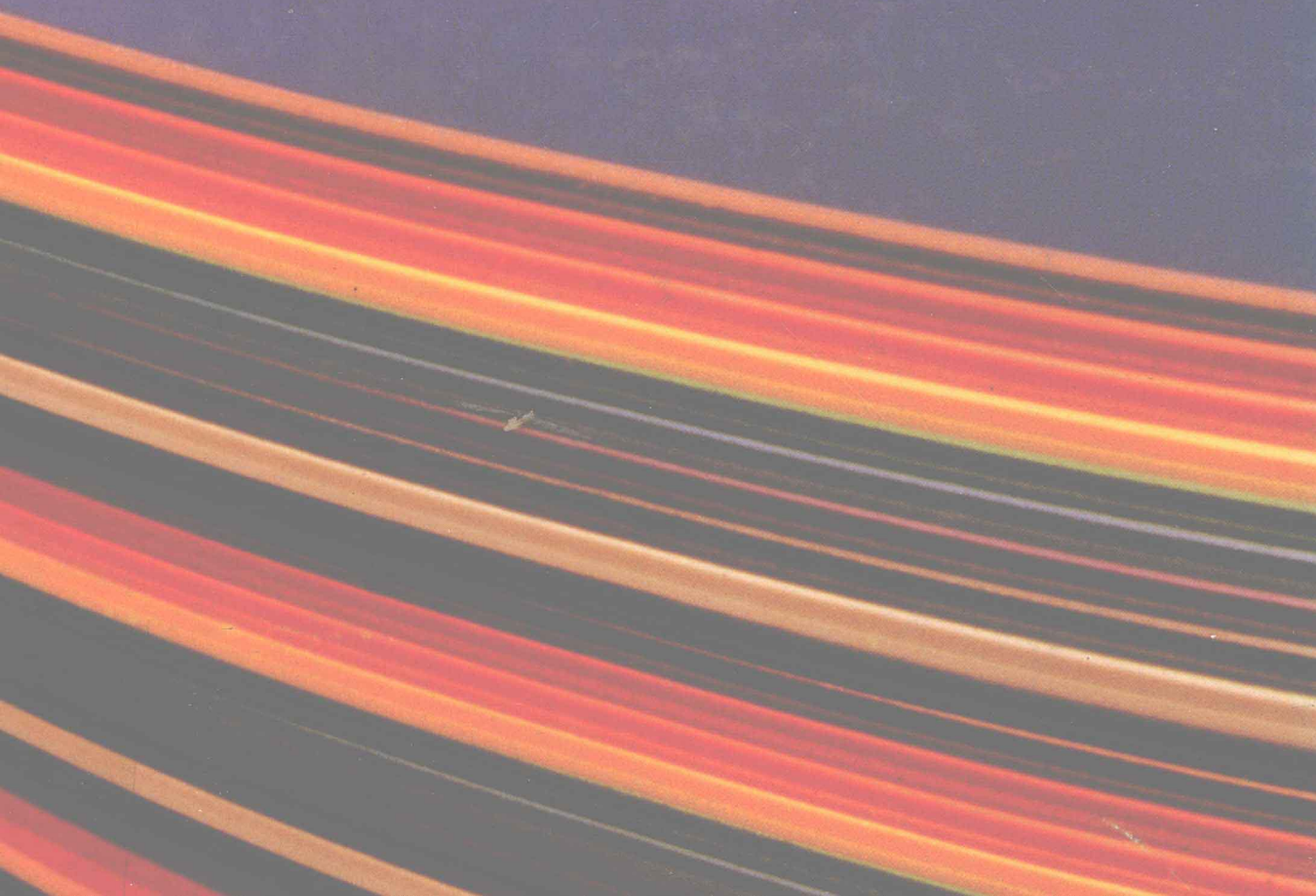


Intermediate Algebra

Fourth Edition

Alfonse Gobran



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Los Angeles Harbor College



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Intermediate Algebra

To My Parents



Preface

Intermediate Algebra, Fourth Edition, provides students with a sound algebraic background for further study in college mathematics. This text can be equally effective for use in a terminal course in intermediate algebra.

I believe that mathematics is best understood by applying concepts to specific examples. Consequently, the explanatory material is concise, and numerous examples and exercises are provided throughout the text.

The solving of word problems is given particular emphasis. Word problems are introduced gradually and covered repeatedly with numerous exercises. The translation of verbal expressions into algebraic equations is introduced in Chapter 3, followed by an introduction to solving equations and word problems. A wide variety of word problems are also included in Chapters 6, 9, 10, and 13.

These changes are based on responses from current users of the text as well as others teaching intermediate algebra. The revisions include the following:

- Many new examples ease the transition from elementary to more complicated concepts and methods.
- Over 1700 new exercises incorporated into the exercise sets.
- Cumulative Review sections added after Chapters 4, 8, 11, 14, and 17.
- More than 9500 exercises included in this edition.

I want to thank all those people who are using my text for their comments to the PWS–KENT sales staff. I also thank the following people, whose written evaluations have contributed to this revision: Cecilia M. Cooper, William Rainey Harper College; Ronald D. Ferguson, San Antonio College; Seymour Gottlieb, LA Pierce College; Paul N. Huchens, St. Louis Community College at Florissant Valley; Nancy Hyde, Broward Community College; Norma A. Innes, Miami Dade College; Marvin Johnson, College of Lake County; Pamela E. Matthews, Chabot College; Ellen Smith, Santa Fe Community College; Howard Sorkin, Broward Community College; and Richard Watkins, Tidewater Community College.

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Alfonse Gobran

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chapter 1

Sets and Real Numbers

- 1.1 Introduction
- 1.2 The number line
- 1.3 Sets
- 1.4 The set of real numbers
- 1.5 The set of whole numbers
- 1.6 The set of integers
- 1.7 The set of rational numbers
- 1.8 Irrational numbers and real numbers

1.1 Introduction

The **Hindu-Arabic system** of numeration started with nine symbols to represent the numbers 1 through 9, inclusive. The concept of zero came much later and was used to express a lack of objects. Today we use an extension and modernization of the Hindu-Arabic system. We use ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 to represent numbers. These symbols are combined in a **place-value system** to represent any number we need to express.

The numbers 1, 2, 3, etc. are called **counting numbers** or **natural numbers**. The numbers 0, 1, 2, etc. are called **whole numbers**.

1.2 The Number Line

Numbers developed from the need to count. Sometimes it is convenient to use geometry to illustrate or prove some important results in algebra. Thus we would like to have a geometrical representation of the whole numbers. To accomplish this, we draw a straight line and choose a point on the line to represent the number 0. We call this point the **origin**. Take another point on the line at some distance from the origin to the right of it. Associate that point with the number 1. The segment of the line from the origin to the point representing the number 1 is our unit measurement; it is the **scale** we use on the line. Now, at a distance of 1 unit to the right of the point representing the number 1, put another point and let it represent the number 2. We continue this procedure as far as we like, thus establishing an association between the whole numbers and points on the line.

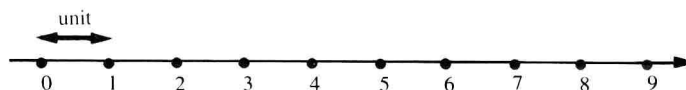


FIGURE 1.1

Figure 1.1 shows the **number line**. The arrow at the end of the line indicates that we continue in this manner and also in the direction of increasing numbers. The segment of the line representing a unit distance, that is, the scale on the line, is taken to suit our purpose (see Figure 1.2).

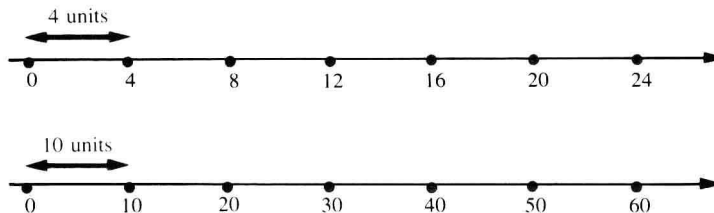


FIGURE 1.2