

STATISTICAL METHODS

Snedecor and Cochran

SIXTH EDITION

Sixth Edition

METHODS

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Preface

In preparing the sixth edition we have kept in mind the two purposes this book has served during the past thirty years. Prior editions have been used extensively both as texts for introductory courses in statistics and as reference sources of statistical techniques helpful to research workers in the interpretation of their data.

As a text, the book contains ample material for a course extending throughout the academic year. For a one-term course, a suggested list of topics is given on the page preceding the Table of Contents. As in past editions, the mathematical level required involves little more than elementary algebra. Dependence on mathematical symbols has been kept to a minimum. We realize, however, that it is hard for the reader to use a formula with full confidence until he has been given proof of the formula or its derivation. Consequently, we have tried to help the reader's understanding of important formulas either by giving an algebraic proof where this is feasible or by explaining on common-sense grounds the roles played by different parts of the formula.

This edition retains also one of the characteristic features of the book—the extensive use of experimental sampling to familiarize the reader with the basic sampling distributions that underlie modern statistical practice. Indeed, with the advent of electronic computers, experimental sampling in its own right has become much more widely recognized as a research weapon for solving problems beyond the current skills of the mathematician.

Some changes have been made in the structure of the chapters, mainly at the suggestion of teachers who have used the book as a text. The former chapter 8 (Large Sample Methods) has disappeared, the retained material being placed in earlier chapters. The new chapter 8 opens with an introduction to probability, followed by the binomial and Poisson distributions (formerly in chapter 16). The discussion of multiple regression (chapter 13) now precedes that of covariance and multiple covariance (chapter 14).

Chapter 16 contains two related topics, the analysis of two-way classifications with unequal numbers of observations in the sub-classes and the analysis of proportions in two-way classifications. The first of these topics was formerly at the end of a long chapter on factorial arrangements; the second topic is new in this edition. This change seemed advisable for two reasons. During the past twenty years there has been a marked increase in observational studies in the social sciences, in medicine and public health, and in operations research. In their analyses, these studies often involve the handling of multiple classifications which present complexities appropriate to the later sections of the book.

Finally, in response to almost unanimous requests, the statistical tables in the book have been placed in an Appendix.

A number of topics appear for the first time in this edition. As in past editions, the selection of topics was based on our judgment as to those likely to be most useful. In addition to the new material on the analysis of proportions in chapter 16, other new topics are as follows:

- The analysis of data recorded in scales having only a small number of distinct values (section 5.8);
- In linear regression, the prediction of the independent variable X from the dependent variable Y , sometimes called linear calibration (section 6.14);
- Linear regression when X is subject to error (section 6.17);
- The comparison of two correlated estimates of variance (section 7.12);
- An introduction to probability (section 8.2);
- The analysis of proportions in ordered classifications (section 9.10);
- Testing a linear trend in proportions (section 9.11);
- The analysis of a set of 2×2 contingency tables (section 9.14);
- More extensive discussion of the effects of failures in the assumptions of the analysis of variance and of remedial measures (sections 11.10–11.13);
- Recent work on the selection of variates for prediction in multiple regression (section 13.13);
- The discriminant function (sections 13.14, 13.15);
- The general method of fitting non-linear regression equations and its application to asymptotic regression (sections 15.7–15.8).

Where considerations of space permitted only a brief introduction to the topic, references were given to more complete accounts.

Most of the numerical illustrations continue to be from biological investigations. In adding new material, both in the text and in the examples to be worked by the student, we have made efforts to broaden the

range of fields represented by data. One of the most exhilarating features of statistical techniques is the extent to which they are found to apply in widely different fields of investigation.

High-speed electronic computers are rapidly becoming available as a routine resource in centers in which a substantial amount of data are analyzed. Flexible standard programs remove the drudgery of computation. They give the investigator vastly increased power to fit a variety of mathematical models to his data; to look at the data from different points of view; and to obtain many subsidiary results that aid the interpretation. In several universities their use in the teaching of introductory courses in statistics is being tried, and this use is sure to increase.

We believe, however, that in the future it will be just as necessary that the investigator learn the standard techniques of analysis and understand their meaning as it was in the desk machine age. In one respect, computers may change the relation of the investigator to his data in an unfortunate way. When calculations are handed to a programmer who translates them into the language understood by the computer, the investigator, on seeing the printed results, may lack the self-assurance to query or detect errors that arose because the programmer did not fully understand what was wanted or because the program had not been correctly debugged. When data are being programmed it is often wise to include a similar example from this or another standard book as a check that the desired calculations are being done correctly.

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George W. Snedecor
William G. Cochran

A SHORT COURSE IN THE ELEMENTS OF STATISTICAL METHOD

CHAPTER	PAGES
1 Attributes.....	3- 31
2 Measurements.....	32- 61
3 Sampling distributions.....	{ 66- 74
	{ 77- 79
4 Comparison of two samples.....	91-104
5 Non-Parametric Methods.....	120-128
6 Regression.....	{ 135-145
	{ 149-157
7 Correlation.....	172-177
8 Binomial distribution.....	199-219
9 One-way classifications—Attributes.....	{ 228-231
	{ 236-238
10 One-way classifications—Measurements.....	258-271
11 Two-way classifications.....	299-310

Table of contents

Chapter 1. Sampling of Attributes

1.1	Introduction.....	3
1.2	Purpose of this chapter.....	4
1.3	The twin problems of sampling.....	4
1.4	A sample of farm facts. Point and interval estimates.....	5
1.5	Random sampling.....	10
1.6	Tables of random digits.....	12
1.7	Confidence interval: verification of theory.....	14
1.8	The sampled population.....	15
1.9	The frequency distribution and its graphical representation.....	16
1.10	Hypotheses about populations.....	20
1.11	Chi-square, an index of dispersion.....	20
1.12	The formula for chi-square.....	21
1.13	An experiment in sampling chi-square; the sampling distribution.....	22
1.14	Comparison with the theoretical distribution.....	25
1.15	The test of a null hypothesis or test of significance.....	26
1.16	Tests of significance in practice.....	28
1.17	Summary of technical terms.....	29

Chapter 2. Sampling From a Normally Distributed Population

2.1	Normally distributed population.....	32
2.2	Reasons for the use of the normal distribution.....	35
2.3	Tables of the normal distribution.....	35
2.4	Estimators of μ and σ	39
2.5	The array and its graphical representation.....	40
2.6	Algebraic notation.....	41
2.7	Deviations from sample mean.....	42
2.8	Another estimator of σ ; the sample standard deviation.....	44
2.9	Comparison of the two estimators of σ	46
2.10	Hints on the computation of s	47
2.11	The standard deviation of sample means.....	49
2.12	The frequency distribution of sample means.....	51
2.13	Confidence intervals for μ when σ is known.....	56
2.14	Size of sample.....	58
2.15	"Student's" t -distribution.....	59
2.16	Confidence limits for μ based on the t -distribution.....	61
2.17	Relative variation. Coefficient of variation.....	62

Chapter 3. Experimental Sampling From a Normal Population

3.1	Introduction	66
3.2	A finite population simulating the normal	66
3.3	Random samples from a normal distribution	69
3.4	The distribution of sample means	70
3.5	Sampling distribution of s^2 and s	72
3.6	Interval estimates of σ^2	74
3.7	Test of a null hypothesis value of σ^2	76
3.8	The distribution of t	77
3.9	The interval estimate of μ ; the confidence interval	78
3.10	Use of frequency distributions for computing \bar{X} and s	80
3.11	Computation of \bar{X} and s in large samples: example	81
3.12	Tests of normality	84
3.13	A test of skewness	86
3.14	Tests for kurtosis	86
3.15	Effects of skewness and kurtosis	88

Chapter 4. The Comparison of Two Samples

4.1	Estimates and tests of differences	91
4.2	A simulated paired experiment	92
4.3	Example of a paired experiment	94
4.4	Conditions for pairing	97
4.5	Tests of other null hypotheses about μ	97
4.6	Comparison of the means of two independent samples	100
4.7	The variance of a difference	100
4.8	A pooled estimate of variance	101
4.9	An experiment comparing two groups of equal size	102
4.10	Groups of unequal sizes	104
4.11	Paired versus independent groups	106
4.12	Precautions against bias-randomization	109
4.13	Sample size in comparative experiments	111
4.14	Analysis of independent samples when $\sigma_1 \neq \sigma_2$	114
4.15	A test of the equality of two variances	116

Chapter 5. Shortcut and Non-parametric Methods

5.1	Introduction	120
5.2	The t -test based on range	120
5.3	Median, percentiles, and order statistics	123
5.4	The sign test	125
5.5	Non-parametric methods: ranking of differences between measurements	128
5.6	Non-parametric methods: ranking for unpaired measurements	130
5.7	Comparison of rank and normal tests	132
5.8	Scales with limited values	132

Chapter 6. Regression

6.1	Introduction	135
6.2	The regression of blood pressure on age	135
6.3	Shortcut methods of computation in regression	139
6.4	The mathematical model in linear regression	141
6.5	\hat{Y} as an estimator of $\mu = \alpha + \beta x$	144
6.6	The estimator of $\sigma_{y \cdot x}^2$	145

6.7	The method of least squares	147
6.8	The value of b in some simple cases	147
6.9	The situation when X varies from sample to sample	149
6.10	Interval estimates of β and tests of null hypotheses	153
6.11	Prediction of the population regression line	153
6.12	Prediction of an individual Y	155
6.13	Testing a deviation that looks suspiciously large	157
6.14	Prediction of X from Y . Linear calibration	159
6.15	Partitioning the sum of squares of the dependent variate	160
6.16	Galton's use of the term "regression"	164
6.17	Regression when X is subject to error	164
6.18	Fitting a straight line through the origin	166
6.19	The estimation of ratios	170
6.20	Summary	170

Chapter 7. Correlation

7.1	Introduction	172
7.2	The sample correlation coefficient r	173
7.3	Relation between the sample coefficients of correlation and regression	175
7.4	The bivariate normal distribution	177
7.5	Sampling variation of the correlation coefficient. Common elements	181
7.6	Testing the null hypothesis $\rho = 0$	184
7.7	Confidence limits and tests of hypotheses about ρ	185
7.8	Practical utility of correlation and regression	188
7.9	Variances of sums and differences of correlated variables	190
7.10	The calculation of r in a large sample	191
7.11	Non-parametric methods. Rank correlation	193
7.12	The comparison of two correlated variances	195

Chapter 8. Sampling From the Binomial Distribution

8.1	Introduction	199
8.2	Some simple rules of probability	199
8.3	The binomial distribution	202
8.4	Sampling the binomial distribution	205
8.5	Mean and standard deviation of the binomial distribution	207
8.6	The normal approximation and the correction for continuity	209
8.7	Confidence limits for a proportion	210
8.8	Test of significance of a binomial proportion	211
8.9	The comparison of proportions in paired samples	213
8.10	Comparison of proportions in two independent samples: the 2×2 table	215
8.11	Test of the independence of two attributes	219
8.12	A test by means of the normal deviate z	220
8.13	Sample size for comparing two proportions	221
8.14	The Poisson distribution	223

Chapter 9. Attribute Data With More Than One Degree of Freedom

9.1	Introduction	228
9.2	Single classifications with more than two classes	228
9.3	Single classifications with equal expectations	231
9.4	Additional tests	233
9.5	The χ^2 test when the expectations are small	235
9.6	Single classifications with estimated expectations	236
9.7	Two-way classifications. The $2 \times C$ contingency table	238

9.8	The variance test for homogeneity of the binomial distribution.....	240
9.9	Further examination of the data.....	242
9.10	Ordered classifications.....	243
9.11	Test for a linear trend in proportions.....	246
9.12	Heterogeneity χ^2 in testing Mendelian ratios.....	248
9.13	The $R \times C$ table.....	250
9.14	Sets of 2×2 tables.....	253

Chapter 10. One-Way Classifications. Analysis of Variance

10.1	Extension from two samples to many.....	258
10.2	An experiment with four samples.....	258
10.3	The analysis of variance.....	260
10.4	Effect of differences between the population means.....	264
10.5	The variance ratio, F	265
10.6	Analysis of variance with only two classes.....	267
10.7	Comparisons among class means.....	268
10.8	Inspection of all differences between pairs of means.....	271
10.9	Shortcut computation using ranges.....	275
10.10	Model I. Fixed treatment effects.....	275
10.11	Effects of errors in the assumptions.....	276
10.12	Samples of unequal sizes.....	277
10.13	Model II. Random effects.....	279
10.14	Structure of model II illustrated by sampling.....	282
10.15	Confidence limits for σ_A^2	284
10.16	Samples within samples. Nested classifications.....	285
10.17	Samples within samples. Mixed model.....	288
10.18	Samples of unequal sizes. Random effects.....	289
10.19	Samples within samples. Unequal sizes.....	291
10.20	Intraclass correlation.....	294
10.21	Tests of homogeneity of variance.....	296

Chapter 11. Two-Way Classifications

11.1	Introduction.....	299
11.2	An experiment with two criteria of classification.....	299
11.3	Comparisons among means.....	301
11.4	Algebraic notation.....	302
11.5	Mathematical model for a two-way classification.....	303
11.6	Partitioning the treatments sum of squares.....	308
11.7	Efficiency of blocking.....	311
11.8	Latin squares.....	312
11.9	Missing data.....	317
11.10	Non-conformity to model.....	321
11.11	Gross errors: rejection of extreme observations.....	321
11.12	Lack of independence in the errors.....	323
11.13	Unequal error variances due to treatments.....	324
11.14	Non-normality. Variance-stabilizing transformations.....	325
11.15	Square-root transformation for counts.....	325
11.16	Arcsin transformation for proportions.....	327
11.17	The logarithmic transformation.....	329
11.18	Non-additivity.....	330
11.19	Tukey's test of additivity.....	331
11.20	Non-additivity in a Latin square.....	334

Chapter 12. Factorial Experiments

12.1	Introduction	339
12.2	The single factor versus the factorial approach	339
12.3	Analysis of the 2^2 factorial experiment	342
12.4	The 2^2 factorial when interaction is present	344
12.5	The general two-factor experiment	346
12.6	Response curves	349
12.7	Response curves in two-factor experiments	352
12.8	Example of a response surface	354
12.9	Three-factor experiments; the 2^3	359
12.10	Three-factor experiments; a $2 \times 3 \times 4$	361
12.11	Expected values of mean squares	364
12.12	The split-plot or nested design	369
12.13	Series of experiments	375
12.14	Experiments with perennial crops	377

Chapter 13. Multiple Regression

13.1	Introduction	381
13.2	Two independent variables	381
13.3	The deviations mean square and the F -test	385
13.4	Alternative method of calculation. The inverse matrix	389
13.5	Standard errors of estimates in multiple regression	391
13.6	The interpretation of regression coefficients	393
13.7	Relative importance of different X -variables	398
13.8	Partial and multiple correlation	400
13.9	Three or more independent variables. Computations	403
13.10	Numerical example. Computing the b 's	405
13.11	Numerical example. Computing the inverse matrix	409
13.12	Deletion of an independent variable	412
13.13	Selection of variates for prediction	412
13.14	The discriminant function	414
13.15	Numerical example of the discriminant function	416

Chapter 14. Analysis of Covariance

14.1	Introduction	419
14.2	Covariance in a completely randomized experiment	420
14.3	The F -test of the adjusted means	424
14.4	Covariance in a 2-way classification	425
14.5	Interpretation of adjusted means in covariance	429
14.6	Comparison of regression lines	432
14.7	Comparison of the "Between Classes" and the "Within Classes" regressions	436
14.8	Multiple covariance	438
14.9	Multiple covariance in a 2-way table	443

Chapter 15. Curvilinear Regression

15.1	Introduction	447
15.2	The exponential growth curve	449
15.3	The second degree polynomial	453
15.4	Data having several Y 's at each X value	456
15.5	Test of departure from linear regression in covariance analysis	460

xiv Contents

15.6	Orthogonal polynomials	460
15.7	A general method of fitting non-linear regressions.	465
15.8	Fitting an asymptotic regression.	467

Chapter 16. Two-Way Classifications With Unequal Numbers and Proportions

16.1	Introduction	472
16.2	Unweighted analysis of cell means.	475
16.3	Equal numbers within rows.	477
16.4	Proportional sub-class numbers.	478
16.5	Disproportionate numbers. The 2×2 table.	483
16.6	Disproportionate numbers. The $R \times 2$ table.	484
16.7	The $R \times C$ table. Least squares analysis.	488
16.8	The analysis of proportions in 2-way tables.	493
16.9	Analysis in the p scale: a 2×2 table.	495
16.10	Analysis in the p scale: a 3×2 table.	496
16.11	Analysis of logits in an $R \times C$ table.	497
16.12	Numerical example.	498

Chapter 17. Design and Analysis of Sampling

17.1	Populations.	504
17.2	A simple example.	505
17.3	Probability sampling.	508
17.4	Listing the population.	509
17.5	Simple random sampling.	511
17.6	Size of sample.	516
17.7	Systematic sampling.	519
17.8	Stratified sampling.	520
17.9	Choice of sample sizes in the individual strata.	523
17.10	Stratified sampling for attributes.	526
17.11	Sampling in two stages.	528
17.12	The allocation of resources in two-stage sampling.	531
17.13	Selection with probability proportional to size.	534
17.14	Ratio and regression estimates.	536
17.15	Further reading.	538

Appendix

List of Appendix Tables and Notes.	541
Appendix Tables.	543
Author Index.	577
Index to Numerical Examples.	581
Subject Index.	585

STATISTICAL METHODS



Sampling of attributes

1.1—Introduction. The subject matter of the field of statistics has been described in various ways. According to one definition, statistics deals with techniques for collecting, analyzing, and drawing conclusions from data. This description helps to explain why an introduction to statistical methods is useful to students who are preparing themselves for a career in one of the sciences and to persons working in any branch of knowledge in which much quantitative research is carried out. Such research is largely concerned with gathering and summarizing observations or measurements made by planned experiments, by questionnaire surveys, by the records of a sample of cases of a particular kind, or by combing past published work on some problem. From these summaries, the investigator draws conclusions that he hopes will have broad validity.

The same intellectual activity is involved in much other work of importance. Samples are extensively used in keeping a continuous watch on the output of production lines in industry, in obtaining national and regional estimates of crop yields and of business and employment conditions, in the auditing of financial statements, in checking for the possible adulteration of foods, in gauging public opinion and voter preferences, in learning how well the public is informed on current issues, and so on.

Acquaintance with the main ideas in statistical methodology is also an appropriate part of a general education. In newspapers, books, television, radio, and speeches we are all continuously exposed to statements that draw general conclusions: for instance, that the cost of living rose by 0.3% in the last month, that the smoking of cigarettes is injurious to health, that users of "Blank's" toothpaste have 23% fewer cavities, that a television program had 18.6 million viewers. When an inference of this kind is of interest to us, it is helpful to be able to form our own judgment about the truth of the statement. Statistics has no magic formula for doing this in all situations, for much remains to be learned about the problem of making sound inferences. But the basic ideas in statistics assist us in thinking clearly about the problem, provide some guidance about the conditions that must be satisfied if sound inferences are to be made, and enable us to detect many inferences that have no good logical foundation.

1.2—Purpose of this chapter. Since statistics deals with the collection, analysis, and interpretation of data, a book on the subject might be expected to open with a discussion of methods for collecting data. Instead, we shall begin with a simple and common type of data already collected, the replies to a question given by a sample of the farmers in a county, and discuss the problem of making a statement from this sample that will apply to all farmers in the county. We begin with this problem of making inferences beyond the data because the type of inference that we are trying to make governs the way in which the data must be collected. In earlier days, and to some extent today also, many workers did not appreciate this fact. It was a common experience for statisticians to be approached with: Here are my results. What do they show? Too often the data were incapable of showing anything that would have been of interest to an investigator, because the method of collecting the data failed to meet the conditions needed for making reliable inferences beyond the data.

In this chapter, some of the principal tools used in statistics for making inferences will be presented by means of simple illustrations. The mathematical basis of these tools, which lies in the theory of probability, will not be discussed until later. Consequently, do not expect to obtain a full understanding of the techniques at this stage, and do not worry if the ideas seem at first unfamiliar. Later chapters will give you further study of the properties of these techniques and enhance your skill in applying them to a broad range of problems.

1.3—The twin problems of sampling. A *sample* consists of a small collection from some larger aggregate about which we wish information. The sample is examined and the facts about it learned. Based on these facts, the problem is to make correct inferences about the *aggregate* or *population*. It is the sample that we observe, but it is the population which we seek to know.

This would be no problem were it not for ever-present variation. If all individuals were alike, a sample consisting of a single one would give complete information about the population. Fortunately, there is endless variety among individuals as well as their environments. A consequence is that successive samples are usually different. Clearly, the facts observed in a sample cannot be taken as facts about the population. Our job then is to reach appropriate conclusions about the population despite sampling variation.

But not every sample contains information about the population sampled. Suppose the objective of an experimental sampling is to determine the growth rate in a population of young mice fed a new diet. Ten of the animals are put in a cage for the experiment. But the cage gets located in a cold draught or in a dark corner. Or an unnoticed infection spreads among the mice in the cage. If such things happen, the growth rate in the sample may give no worthwhile information about that in the population of normal mice. Again, suppose an interviewer in an opinion