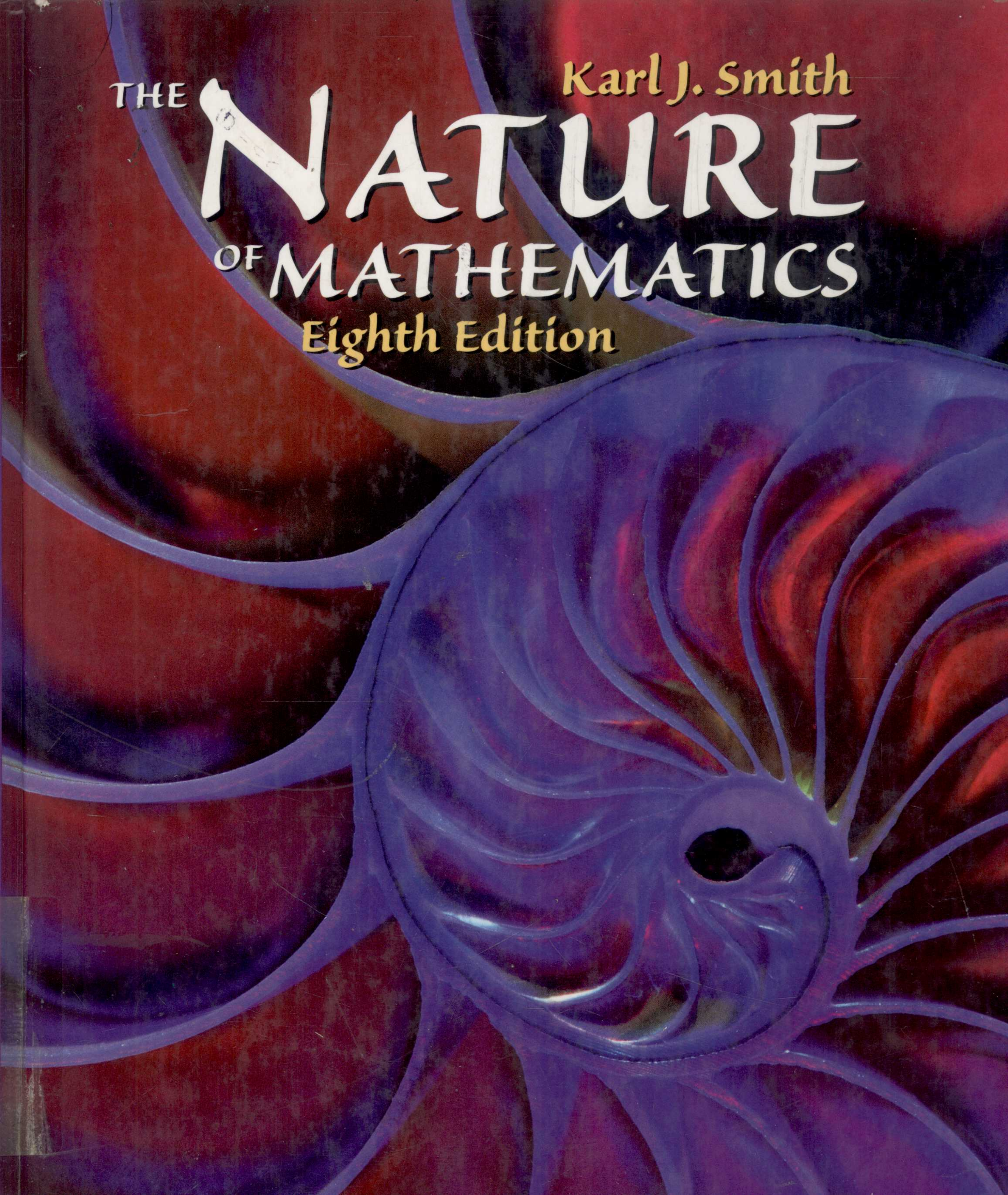
Karl J. Smith

THE NATURE OF MATHEMATICS

Eighth Edition



The Nature of Mathematics

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For more information, contact:

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THIS BOOK IS PRINTED ON ACID-FREE RECYCLED PAPER

I dedicate this book with love to
Melissa and Benjamin.

Preface

This book was written for those students who need a mathematics course to satisfy the general university competency requirement in mathematics. Because of the university requirement, many students enrolling in a course that uses my book have postponed taking this course as long as possible, are dreading the experience, and are coming with a great deal of anxiety. I wrote this book with one overriding goal: to create a positive attitude toward mathematics. Rather than simply presenting the technical details needed to proceed to the next course, I have attempted to give insight into what mathematics is, what it accomplishes, and how it is pursued as a human enterprise. However, at the same time, I have included in this eighth edition a great deal of material to help students estimate, calculate, and solve problems *outside* the classroom or textbook setting.

I frequently encounter people who tell me about their unpleasant experiences with mathematics. I have a true sympathy for those people, and I recall one of my elementary school teachers who assigned additional arithmetic problems as punishment. This can only create negative attitudes toward mathematics, which is indeed unfortunate. If elementary school teachers and parents have positive attitudes toward mathematics, their children cannot help but see some of the beauty of the subject. I want students to come away from this course with the feeling that mathematics can be pleasant, useful, and practical—and enjoyed for its own sake.

The prerequisites for this course vary considerably, as do the backgrounds of students. Some schools have no prerequisites, but other schools have an intermediate algebra prerequisite. The students, as well, have heterogeneous backgrounds. Some have little or no mathematics skills; others have had a great deal of mathematics. Even though the usual prerequisite for using this book is intermediate algebra, a careful selection of topics and chapters would allow a class with a beginning algebra prerequisite to study effectively from this book.

This book was written to meet the needs of all of these students and schools. How did I accomplish that goal? First, the chapters are almost independent of one another, and can be covered in any order appropriate to a particular audience. Second, the problems are designed to be the core of the course. There are problems that every student will find easy and will provide the opportunity for success; there are also problems that are very challenging. Much interesting material appears in the problems, and students should get into the habit of reading (not necessarily working) all the problems whether they are assigned or not.

A Problems: mechanical or drill problems

B Problems: require understanding of the concepts

Problem Solving Problems: require problem-solving skills or original thinking

Individual Research: requires research or library work

Group Research: requires not only research or library work, but also group participation (See the index for a list of group projects.)

The major themes of this book are problem solving and estimation in the context of presenting the great ideas in the history of mathematics. I believe that *learning to solve problems is the principal reason for studying mathematics.* Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in most textbooks is one form of problem solving, but students also should be faced with non–text-type problems. In the first section of this edition I introduce students to Polya's problem-solving techniques, and these techniques are used throughout the book to solve non–text-type problems. These problem-solving examples are found throughout the book (marked as **Polya's Method** examples). Also new to this edition are problems in *each* section that require Polya's method for problem solving.

Students should learn the language and notation of mathematics. Most students who have trouble with mathematics do not realize that mathematics *does require hard work.* The usual pattern for most mathematics students is to open the book to the assigned page of problems, and begin working. Only after getting "stuck" is an attempt made to "find it in the book." The final resort is reading the text. In this book the students are asked not only to "do math problems," but also to "experience mathematics." This means it is necessary to become involved with the **concepts** being presented, not "just get answers." In fact, the slogan "Mathematics Is Not a Spectator Sport" is not just an advertising slogan, but an invitation which suggests that the only way to succeed in mathematics is to become involved with it. Students will learn to receive mathematical ideas through listening, reading, and visualizing. They are expected to present mathematical ideas by speaking, writing, drawing pictures and graphs, and demonstrating with concrete models. The problems in each section that are designated **In Your Own Words** provide practice in communication skills.

A Personal Note

Writing a mathematics textbook is both enjoyable and challenging. To make mathematics come alive, I have included many items not usually found in a textbook. For example, I've included cartoons and quotations, and have used the margins for news clippings and historical notes. The historical notes are not strictly biographical reports, but instead focus on the people to convey some of the humanness of mathematics. Nearly every major mathematician (and many minor ones) has some part of his or her life to tell on the pages of this book. At the end of each chapter there is an interview of a living mathematician. I sat down and made a list of those persons who are the most famous, or those whom I greatly respect. I did not know how my request for an interview would be received, but to my surprise each of these persons was most gracious in providing me not only with biographical information, but with personal details of their lives so that I could share some of their humanness with users of this book. I treasure the correspondence I had with these people.

A Note for Instructors

Feel free to arrange the material in a different order from that presented in the text. I have written the chapters to be as independent of one another as possible. There is much more material in this book than could be covered in a single course. This book can be used in classes designed for liberal arts, teacher training, finite mathematics, college algebra, or a combination of these.

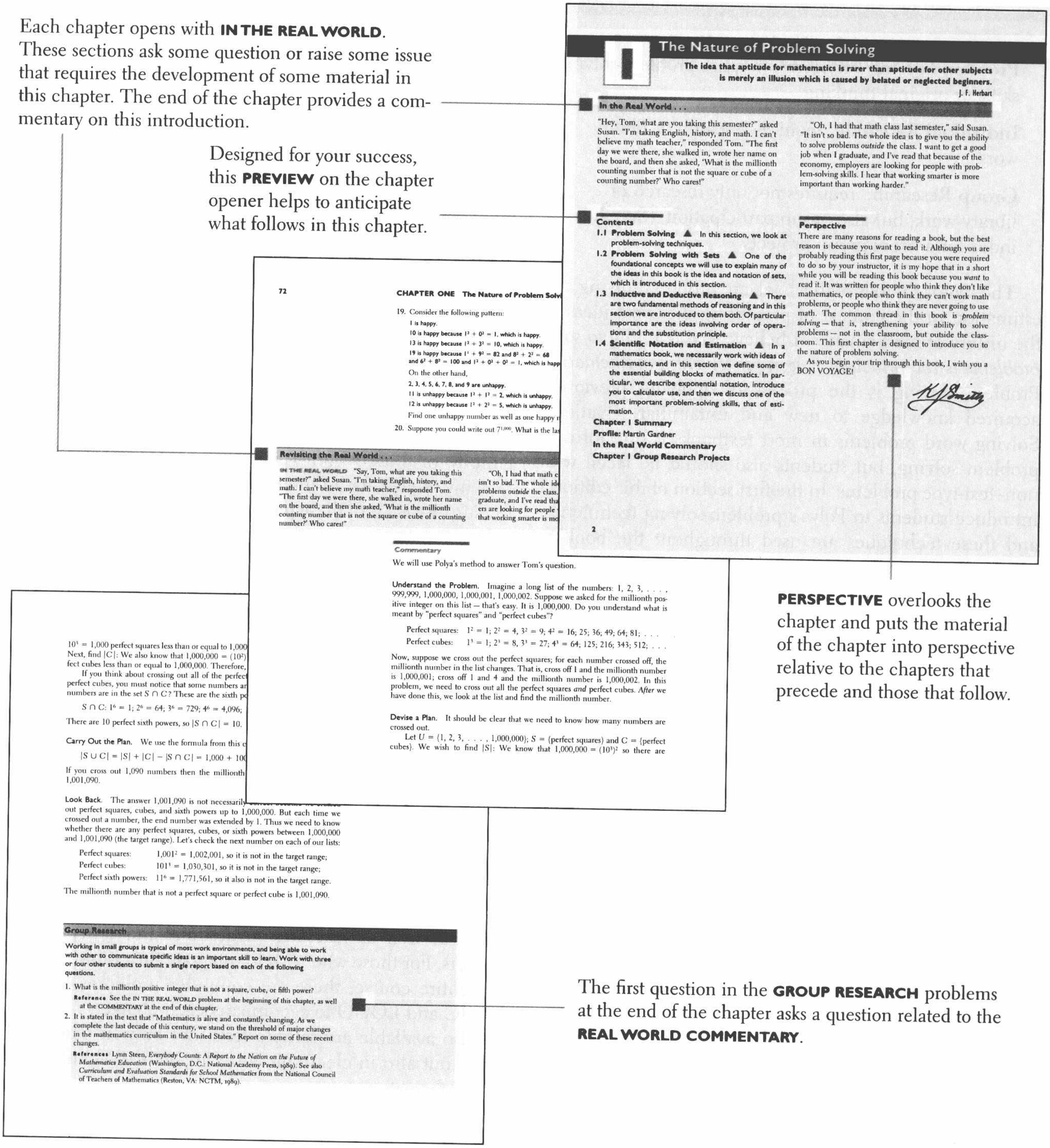
I have written an extensive *Instructor's Manual* to accompany this book. It includes the complete solutions to all the problems (including the "Problem Solving" problems) as well as teaching suggestions and transparency masters. For those who wish to integrate the computer into the entire course, there are computer problems in both BASIC and LOGO to accompany each chapter.

Also available are sample tests, not only in hard copy form, but also in electronic form for both IBM and Macintosh formats.

Some of the significant features of the book are shown on the following pages.

Each chapter opens with **IN THE REAL WORLD**. These sections ask some question or raise some issue that requires the development of some material in this chapter. The end of the chapter provides a commentary on this introduction.

Designed for your success, this **PREVIEW** on the chapter opener helps to anticipate what follows in this chapter.



1 The Nature of Problem Solving

The idea that aptitude for mathematics is rarer than aptitude for other subjects is merely an illusion which is caused by belated or neglected beginners.

J. F. Herbart

In the Real World . . .

"Hey, Tom, what are you taking this semester?" asked Susan. "I'm taking English, history, and math. I can't believe my math teacher," responded Tom. "The first day we were there, she walked in, wrote her name on the board, and then she asked, 'What is the millionth counting number that is not the square or cube of a counting number?' Who cares!"

"Oh, I had that math class last semester," said Susan. "It isn't so bad. The whole idea is to give you the ability to solve problems *outside* the class. I want to get a good job when I graduate, and I've read that because of the economy, employers are looking for people with problem-solving skills. I hear that working smarter is more important than working harder."

Contents

1.1 Problem Solving ▲ In this section, we look at problem-solving techniques.

1.2 Problem Solving with Sets ▲ One of the foundational concepts we will use to explain many of the ideas in this book is the idea and notation of sets, which is introduced in this section.

1.3 Inductive and Deductive Reasoning ▲ There are two fundamental methods of reasoning and in this section we are introduced to them both. Of particular importance are the ideas involving order of operations and the substitution principle.

1.4 Scientific Notation and Estimation ▲ In a mathematics book, we necessarily work with ideas of mathematics, and in this section we define some of the essential building blocks of mathematics. In particular, we describe exponential notation, introduce you to calculator use, and then we discuss one of the most important problem-solving skills, that of estimation.

Chapter 1 Summary

Profile: Martin Gardner

In the Real World Commentary

Chapter 1 Group Research Projects

Perspective

There are many reasons for reading a book, but the best reason is because you want to read it. Although you are probably reading this first page because you were required to do so by your instructor, it is my hope that in a short while you will be reading this book because you *want* to read it. It was written for people who think they don't like mathematics, or people who think they can't work math problems, or people who think they are never going to use math. The common thread in this book is *problem solving* — that is, strengthening your ability to solve problems — not in the classroom, but outside the classroom. This first chapter is designed to introduce you to the nature of problem solving.

As you begin your trip through this book, I wish you a BON VOYAGE!

K J Smith

2

72 **CHAPTER ONE The Nature of Problem Solv**

19. Consider the following pattern:

1 is happy.

10 is happy because $1^2 + 0^2 = 1$, which is happy.

13 is happy because $1^2 + 3^2 = 10$, which is happy.

19 is happy because $1^1 + 9^2 = 82$ and $8^2 + 2^2 = 68$ and $6^2 + 8^2 = 100$ and $1^2 + 0^2 + 0^2 = 1$, which is happ

On the other hand,

2, 3, 4, 5, 6, 7, 8, and 9 are unhappy.

11 is unhappy because $1^2 + 1^2 = 2$, which is unhappy.

12 is unhappy because $1^2 + 2^2 = 5$, which is unhappy.

Find one unhappy number as well as one happy n

20. Suppose you could write out $7^{1,000}$. What is the las

Revisiting the Real World . . .

IN THE REAL WORLD "Say, Tom, what are you taking this semester?" asked Susan. "I'm taking English, history, and math. I can't believe my math teacher," responded Tom. "The first day we were there, she walked in, wrote her name on the board, and then she asked, 'What is the millionth counting number that is not the square or cube of a counting number?' Who cares!"

"Oh, I had that math c isn't so bad. The whole id problems *outside* the class. graduate, and I've read tha ers are looking for people that working smarter is mo

Commentary

We will use Polya's method to answer Tom's question.

Understand the Problem. Imagine a long list of the numbers: 1, 2, 3, . . . , 999,999, 1,000,000, 1,000,001, 1,000,002. Suppose we asked for the millionth positive integer on this list — that's easy. It is 1,000,000. Do you understand what is meant by "perfect squares" and "perfect cubes"?

Perfect squares: $1^2 = 1$; $2^2 = 4$, $3^2 = 9$; $4^2 = 16$; 25; 36; 49; 64; 81; . . .

Perfect cubes: $1^3 = 1$; $2^3 = 8$, $3^3 = 27$; $4^3 = 64$; 125; 216; 343; 512; . . .

Now, suppose we cross out the perfect squares; for each number crossed off, the millionth number in the list changes. That is, cross off 1 and the millionth number is 1,000,001; cross off 1 and 4 and the millionth number is 1,000,002. In this problem, we need to cross out all the perfect squares *and* perfect cubes. *After* we have done this, we look at the list and find the millionth number.

Devise a Plan. It should be clear that we need to know how many numbers are crossed out.

Let $U = \{1, 2, 3, \ldots, 1{,}000{,}000\}$; S = {perfect squares} and C = {perfect cubes}. We wish to find $|S|$: We know that $1{,}000{,}000 = (10^3)^2$ so there are

$10^3 = 1{,}000$ perfect squares less than or equal to 1,000 Next, find $|C|$: We also know that $1{,}000{,}000 = (10^2)$ fect cubes less than or equal to 1,000,000. Therefore,

If you think about crossing out all of the perfec perfect cubes, you must notice that some numbers ar numbers are in the set $S \cap C$? These are the sixth po

$S \cap C$: $1^6 = 1$; $2^6 = 64$; $3^6 = 729$; $4^6 = 4{,}096$;

There are 10 perfect sixth powers, so $|S \cap C| = 10$.

Carry Out the Plan. We use the formula from this c

$|S \cup C| = |S| + |C| - |S \cap C| = 1{,}000 + 10$

If you cross out 1,090 numbers then the millionth 1,001,090.

Look Back. The answer 1,001,090 is not necessarily out perfect squares, cubes, and sixth powers up to 1,000,000. But each time we crossed out a number, the end number was extended by 1. Thus we need to know whether there are any perfect squares, cubes, or sixth powers between 1,000,000 and 1,001,090 (the target range). Let's check the next number on each of our lists:

Perfect squares: $1{,}001^2 = 1{,}002{,}001$, so it is not in the target range;

Perfect cubes: $101^3 = 1{,}030{,}301$, so it is not in the target range;

Perfect sixth powers: $11^6 = 1{,}771{,}561$, so it also is not in the target range.

The millionth number that is not a perfect square or perfect cube is 1,001,090.

Group Research

Working in small groups is typical of most work environments, and being able to work with other to communicate specific ideas is an important skill to learn. Work with three or four other students to submit a single report based on each of the following questions.

1. What is the millionth positive integer that is not a square, cube, or fifth power?

   Reference See the IN THE REAL WORLD problem at the beginning of this chapter, as well at the COMMENTARY at the end of this chapter.

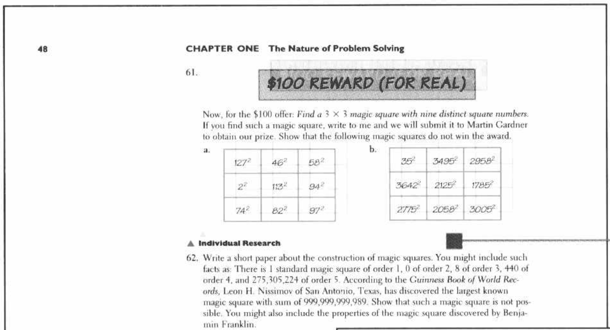
2. It is stated in the text that "Mathematics is alive and constantly changing. As we complete the last decade of this century, we stand on the threshold of major changes in the mathematics curriculum in the United States." Report on some of these recent changes.

   References Lynn Steen, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (Washington, D.C.: National Academy Press, 1989). See also *Curriculum and Evaluation Standards for School Mathematics* from the National Council of Teachers of Mathematics (Reston, VA: NCTM, 1989).

PERSPECTIVE overlooks the chapter and puts the material of the chapter into perspective relative to the chapters that precede and those that follow.

The first question in the **GROUP RESEARCH** problems at the end of the chapter asks a question related to the **REAL WORLD COMMENTARY**.

It is important to be able to communicate your ideas. This book gives you many such opportunities.



61. $100 REWARD (FOR REAL)

Now, for the $100 offer: *Find a 3 × 3 magic square with nine distinct square numbers.* If you find such a magic square, write to me and we will submit it to Martin Gardner to obtain our prize. Show that the following magic squares do not win the award.

a.

127^2	46^2	58^2
2^2	113^2	94^2
74^2	82^2	97^2

b.

35^2	3495^2	2958^2
3642^2	2125^2	1785^2
2775^2	2058^2	3005^2

▲ **Individual Research**

62. Write a short paper about the construction of magic squares. You might include such facts as: There is 1 standard magic square of order 1, 0 of order 2, 8 of order 3, 440 of order 4, and 275,305,224 of order 5. According to the *Guinness Book of World Records,* Leon H. Nissimov of San Antonio, Texas, has discovered the largest known magic square with sum of 999,999,999,989. Show that such a magic square is not possible. You might also include the properties of the magic square discovered by Benjamin Franklin.

References
William H. Benson and Oswald Jaco
Dover Publications, 1976).
John Fults, *Magic Squares* (La Salle,
Martin Gardner, "The Magic of 3 ×
Square of Squares?" *Quantum,* Jan
Martin Gardner, "Mathematical Gan
pp. 118–122.

63. A process for producing an artistic
cle, "An Art-Ful Application Using
ematics Teacher, January 1982, pp.
square art pieces.

64. An **alphamagic square,** invented b
the numbers spelled out in words f
the words also form a magic square

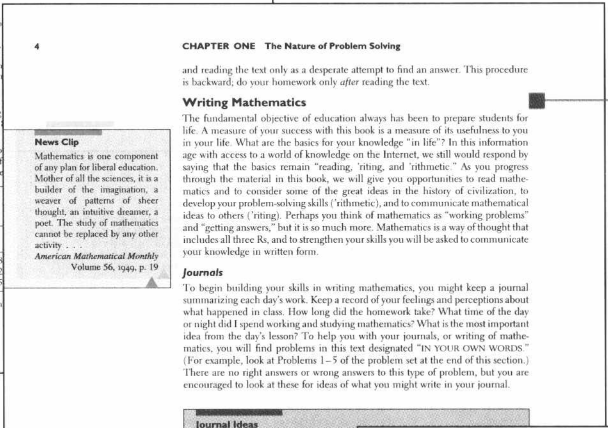
five
twenty-eight
twelve

gives rise to two magic squares:

5	22	18
28	15	2
12	8	25

The first magic square comes from

INDIVIDUAL RESEARCH problems, included in almost every section, provide original problems for your consideration.



News Clip

Mathematics is one component of any plan for liberal education. Mother of all the sciences, it is a builder of the imagination, a weaver of patterns of sheer thought, an intuitive dreamer, a poet. The study of mathematics cannot be replaced by any other activity . . .

American Mathematical Monthly
Volume 56, 1949, p. 19

and reading the text only as a desperate attempt to find an answer. This procedure is backward; do your homework only *after* reading the text.

Writing Mathematics

The fundamental objective of education always has been to prepare students for life. A measure of your success with this book is a measure of its usefulness to you in your life. What are the basics for your knowledge "in life"? In this information age with access to a world of knowledge on the Internet, we still would respond by saying that the basics remain "reading, 'riting, and 'rithmetic." As you progress through the material in this book, we will give you opportunities to read mathematics and to consider some of the great ideas in the history of civilization, to develop your problem-solving skills ('rithmetic), and to communicate mathematical ideas to others ('riting). Perhaps you think of mathematics as "working problems" and "getting answers," but it is so much more. Mathematics is a way of thought that includes all three Rs, and to strengthen your skills you will be asked to communicate your knowledge in written form.

Journals

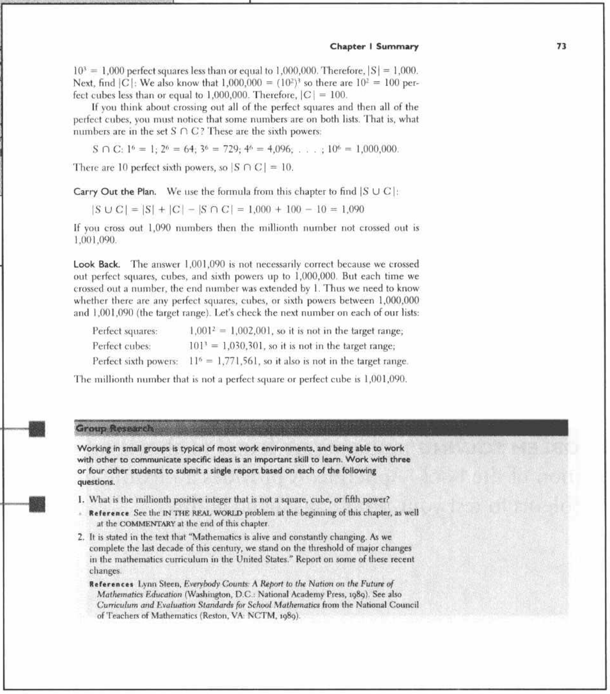
To begin building your skills in writing mathematics, you might keep a journal summarizing each day's work. Keep a record of your feelings and perceptions about what happened in class. How long did the homework take? What time of the day or night did I spend working and studying mathematics? What is the most important idea from the day's lesson? To help you with your journals, or writing of mathematics, you will find problems in this text designated "IN YOUR OWN WORDS." (For example, look at Problems 1–5 of the problem set at the end of this section.) There are no right answers or wrong answers to this type of problem, but you are encouraged to look at these for ideas of what you might write in your journal.

Journal Ideas

Write in your journal every day.
Include important ideas.
Include new words, ideas, formul
Include questions that you want t
If possible, carry your journal with
get an idea.

Reasons for Keeping a Journal

It will record ideas you might oth
It will keep a record of your progr
If you have trouble later, it may
improvement.
It will build your writing skills.



$10^3 = 1{,}000$ perfect squares less than or equal to 1,000,000. Therefore, $|S| = 1{,}000$. Next, find $|C|$: We also know that $1{,}000{,}000 = (10^2)^3$ so there are $10^2 = 100$ perfect cubes less than or equal to 1,000,000. Therefore, $|C| = 100$.

If you think about crossing out all of the perfect squares and then all of the perfect cubes, you must notice that some numbers are on both lists. That is, what numbers are in the set $S \cap C$? These are the sixth powers:

$$S \cap C\text{: } 1^6 = 1;\ 2^6 = 64;\ 3^6 = 729;\ 4^6 = 4{,}096;\ \ldots;\ 10^6 = 1{,}000{,}000.$$

There are 10 perfect sixth powers, so $|S \cap C| = 10$.

Carry Out the Plan. We use the formula from this chapter to find $|S \cup C|$:

$$|S \cup C| = |S| + |C| - |S \cap C| = 1{,}000 + 100 - 10 = 1{,}090$$

If you cross out 1,090 numbers then the millionth number not crossed out is 1,001,090.

Look Back. The answer 1,001,090 is not necessarily correct because we crossed out perfect squares, cubes, and sixth powers up to 1,000,000. But each time we crossed out a number, the end number was extended by 1. Thus we need to know whether there are any perfect squares, cubes, or sixth powers between 1,000,000 and 1,001,090 (the target range). Let's check the next number on each of our lists:

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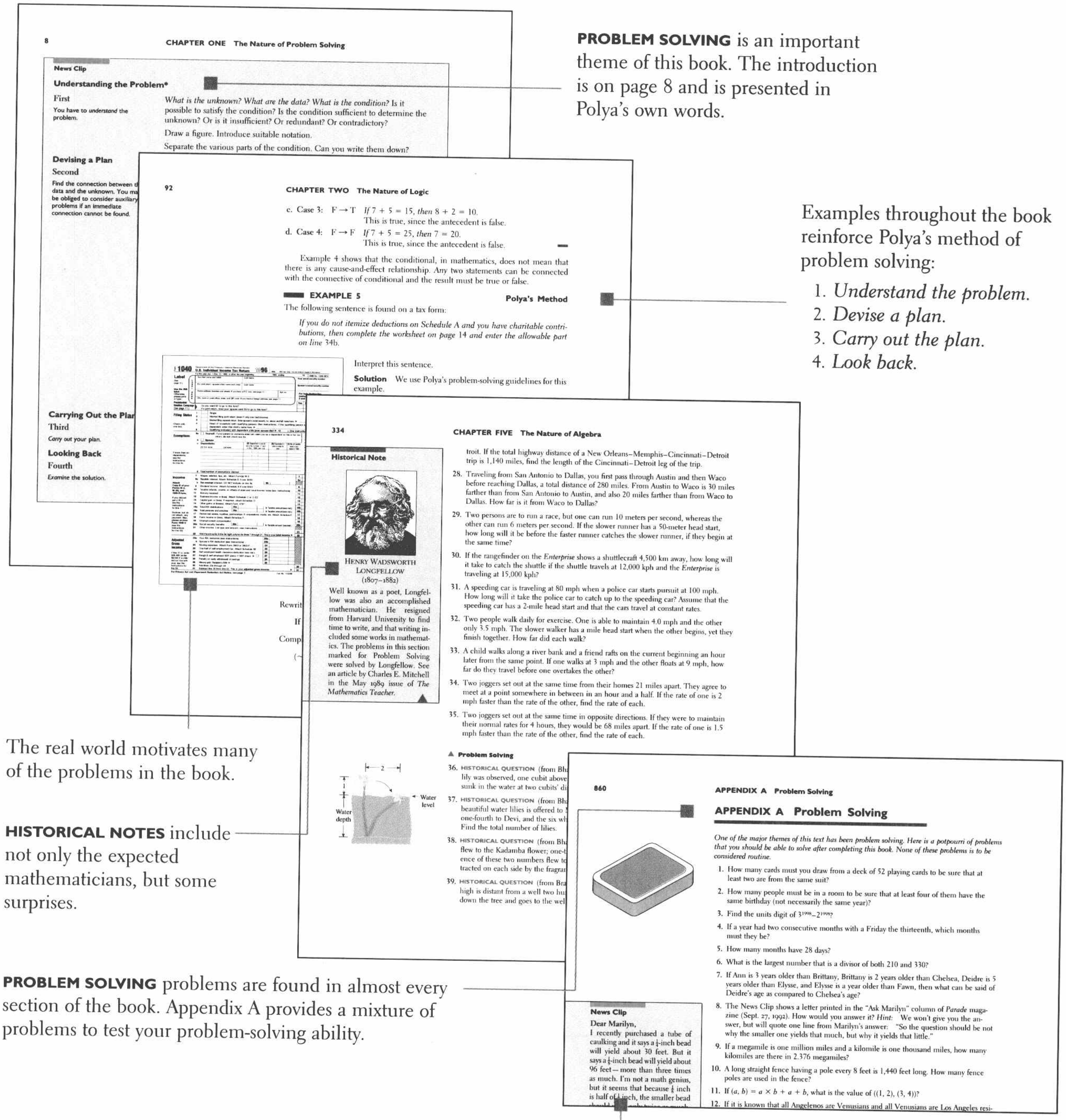
   References Lynn Steen, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (Washington, D.C.: National Academy Press, 1989). See also *Curriculum and Evaluation Standards for School Mathematics* from the National Council of Teachers of Mathematics (Reston, VA: NCTM, 1989).

It is often necessary to collaborate with others and agree on a solution or reach a consensus. These **GROUP RESEARCH** problems are designed to facilitate group work.

The first question relates to the introductory remarks in the chapter called **IN THE REAL WORLD.**



8 CHAPTER ONE The Nature of Problem Solving

News Clip

Understanding the Problem*

First

You have to *understand* the problem.

What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

Devising a Plan

Second

Find the connection between t data and the unknown. You ma be obliged to consider auxiliary problems if an immediate connection cannot be found.

Carrying Out the Plan

Third

Carry out your plan.

Looking Back

Fourth

Examine the solution.

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c. Case 3: F → T *If* $7 + 5 = 15$, *then* $8 + 2 = 10$.
This is true, since the antecedent is false.

d. Case 4: F → F *If* $7 + 5 = 25$, *then* $7 = 20$.
This is true, since the antecedent is false.

Example 4 shows that the conditional, in mathematics, does not mean that there is any cause-and-effect relationship. *Any* two statements can be connected with the connective of conditional and the result must be true or false.

EXAMPLE 5 **Polya's Method**

The following sentence is found on a tax form:

If you do not itemize deductions on Schedule A and you have charitable contributions, then complete the worksheet on page 14 and enter the allowable part on line 34b.

Interpret this sentence.

Solution We use Polya's problem-solving guidelines for this example.

Rewrit

If

Comp

(~

334 CHAPTER FIVE The Nature of Algebra

Historical Note

HENRY WADSWORTH LONGFELLOW
(1807–1882)

Well known as a poet, Longfellow was also an accomplished mathematician. He resigned from Harvard University to find time to write, and that writing included some works in mathematics. The problems in this section marked for Problem Solving were solved by Longfellow. See an article by Charles E. Mitchell in the May 1989 issue of *The Mathematics Teacher*.

troit. If the total highway distance of a New Orleans–Memphis–Cincinnati–Detroit trip is 1,140 miles, find the length of the Cincinnati–Detroit leg of the trip.

28. Traveling from San Antonio to Dallas, you first pass through Austin and then Waco before reaching Dallas, a total distance of 280 miles. From Austin to Waco is 30 miles farther than from San Antonio to Austin, and also 20 miles farther than from Waco to Dallas. How far is it from Waco to Dallas?
29. Two persons are to run a race, but one can run 10 meters per second, whereas the other can run 6 meters per second. If the slower runner has a 50-meter head start, how long will it be before the faster runner catches the slower runner, if they begin at the same time?
30. If the rangefinder on the *Enterprise* shows a shuttlecraft 4,500 km away, how long will it take to catch the shuttle if the shuttle travels at 12,000 kph and the *Enterprise* is traveling at 15,000 kph?
31. A speeding car is traveling at 80 mph when a police car starts pursuit at 100 mph. How long will it take the police car to catch up to the speeding car? Assume that the speeding car has a 2-mile head start and that the cars travel at constant rates.
32. Two people walk daily for exercise. One is able to maintain 4.0 mph and the other only 3.5 mph. The slower walker has a mile head start when the other begins, yet they finish together. How far did each walk?
33. A child walks along a river bank and a friend rafts on the current beginning an hour later from the same point. If one walks at 3 mph and the other floats at 9 mph, how far do they travel before one overtakes the other?
34. Two joggers set out at the same time from their homes 21 miles apart. They agree to meet at a point somewhere in between in an hour and a half. If the rate of one is 2 mph faster than the rate of the other, find the rate of each.
35. Two joggers set out at the same time in opposite directions. If they were to maintain their normal rates for 4 hours, they would be 68 miles apart. If the rate of one is 1.5 mph faster than the rate of the other, find the rate of each.

▲ **Problem Solving**

36. HISTORICAL QUESTION (from Bh lily was observed, one cubit above sunk in the water at two cubits' di
37. HISTORICAL QUESTION (from Bh beautiful water lilies is offered to one-fourth to Devi, and the six wh Find the total number of lilies.
38. HISTORICAL QUESTION (from Bh flew to the Kadamba flower; one-t ence of these two numbers flew to tracted on each side by the fragran
39. HISTORICAL QUESTION (from Bra high is distant from a well two hu down the tree and goes to the wel

860 APPENDIX A Problem Solving

APPENDIX A Problem Solving

One of the major themes of this text has been problem solving. Here is a potpourri of problems that you should be able to solve after completing this book. None of these problems is to be considered routine.

1. How many cards must you draw from a deck of 52 playing cards to be sure that at least two are from the same suit?
2. How many people must be in a room to be sure that at least four of them have the same birthday (not necessarily the same year)?
3. Find the units digit of $3^{1998}-2^{1998}$?
4. If a year had two consecutive months with a Friday the thirteenth, which months must they be?
5. How many months have 28 days?
6. What is the largest number that is a divisor of both 210 and 330?
7. If Ann is 3 years older than Brittany, Brittany is 2 years older than Chelsea, Deidre is 5 years older than Elysse, and Elysse is a year older than Fawn, then what can be said of Deidre's age as compared to Chelsea's age?
8. The News Clip shows a letter printed in the "Ask Marilyn" column of *Parade* magazine (Sept. 27, 1992). How would you answer it? *Hint:* We won't give you the answer, but will quote one line from Marilyn's answer: "So the question should be not why the smaller one yields that much, but why it yields that little."
9. If a megamile is one million miles and a kilomile is one thousand miles, how many kilomiles are there in 2.376 megamiles?
10. A long straight fence having a pole every 8 feet is 1,440 feet long. How many fence poles are used in the fence?
11. If $(a, b) = a \times b + a + b$, what is the value of $((1, 2), (3, 4))$?
12. If it is known that all Angelenos are Venusians and all Venusians are Los Angeles resi-

News Clip

Dear Marilyn,
I recently purchased a tube of caulking and it says a $\frac{1}{4}$-inch bead will yield about 30 feet. But it says a $\frac{1}{8}$-inch bead will yield about 96 feet—more than three times as much. I'm not a math genius, but it seems that because $\frac{1}{8}$ inch is half of $\frac{1}{4}$ inch, the smaller bead

PROBLEM SOLVING is an important theme of this book. The introduction is on page 8 and is presented in Polya's own words.

Examples throughout the book reinforce Polya's method of problem solving:

1. *Understand the problem.*
2. *Devise a plan.*
3. *Carry out the plan.*
4. *Look back.*

The real world motivates many of the problems in the book.

HISTORICAL NOTES include not only the expected mathematicians, but some surprises.

PROBLEM SOLVING problems are found in almost every section of the book. Appendix A provides a mixture of problems to test your problem-solving ability.

NEWS CLIPS motivate many of the problems. Here is a "Dear Marilyn" problem.

Problems, Problems, Problems, Problems....

HISTORICAL QUESTIONS are taken from antiquity.

Common-sense and estimation problems are common.

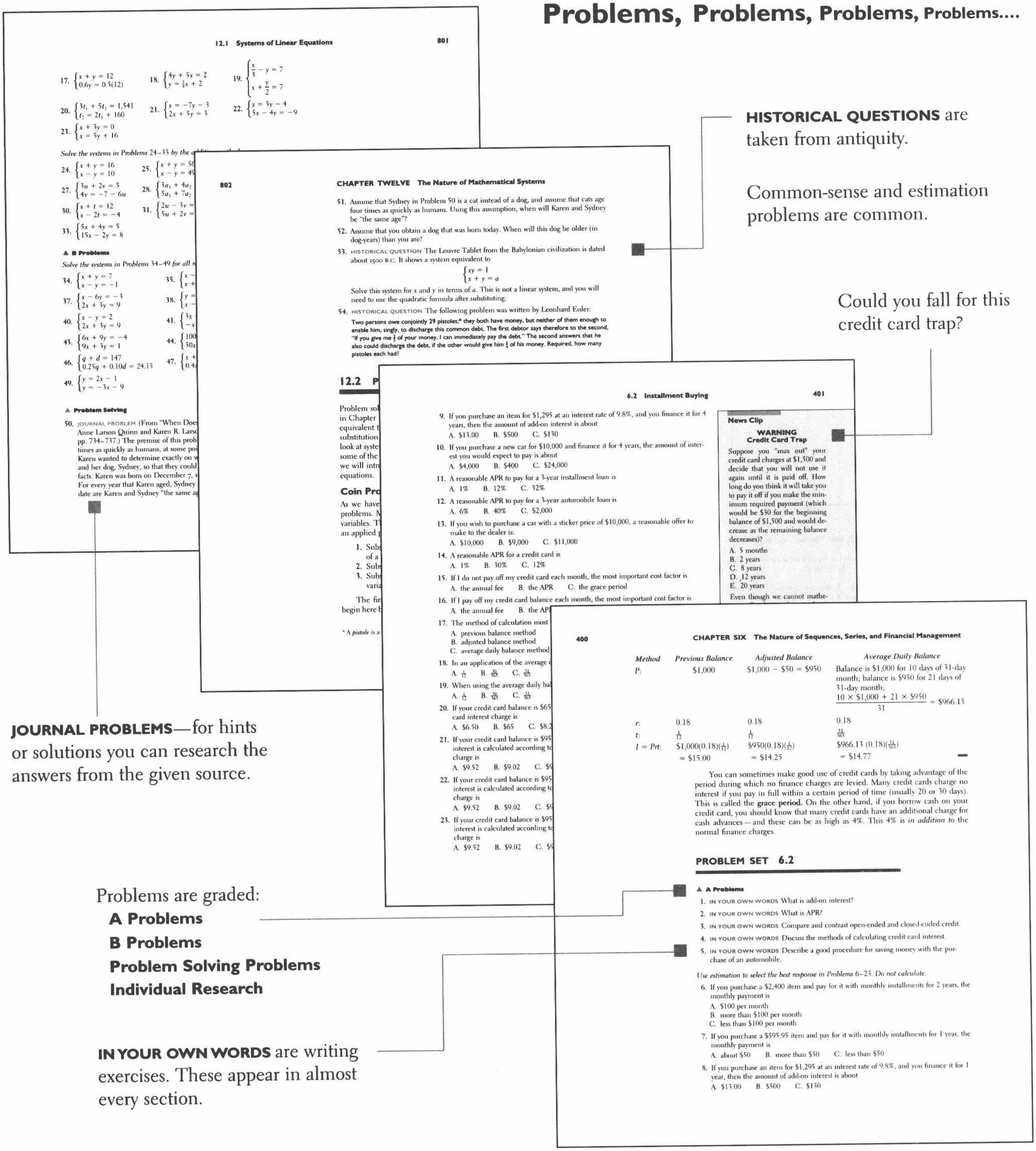
Could you fall for this credit card trap?

JOURNAL PROBLEMS—for hints or solutions you can research the answers from the given source.

Problems are graded:

A Problems
B Problems
Problem Solving Problems
Individual Research

IN YOUR OWN WORDS are writing exercises. These appear in almost every section.



12.1 Systems of Linear Equations 801

17. $\begin{cases} x + y = 12 \\ 0.6y = 0.5(12) \end{cases}$ 18. $\begin{cases} 4y + 5x = 2 \\ y = \frac{5}{4}x + 2 \end{cases}$ 19. $\begin{cases} \frac{x}{3} - y = 7 \\ x + \frac{y}{2} = 7 \end{cases}$

20. $\begin{cases} 3t_1 + 5t_2 = 1{,}541 \\ t_2 = 2t_1 + 160 \end{cases}$ 21. $\begin{cases} x = -7y - 3 \\ 2x + 5y = 3 \end{cases}$ 22. $\begin{cases} x = 3y - 4 \\ 5x - 4y = -9 \end{cases}$

23. $\begin{cases} x + 3y = 0 \\ x = 5y + 16 \end{cases}$

Solve the systems in Problems 24–33 by the

24. $\begin{cases} x + y = 16 \\ x - y = 10 \end{cases}$ 25. $\begin{cases} x + y = 5 \\ x - y = 4 \end{cases}$

27. $\begin{cases} 3u + 2v = 5 \\ 4v = -7 - 6u \end{cases}$ 28. $\begin{cases} 3a_1 + 4a_2 \\ 5a_1 + 7a_2 \end{cases}$

30. $\begin{cases} s + t = 12 \\ s - 2t = -4 \end{cases}$ 31. $\begin{cases} 2u - 3v = \\ 5u + 2v = \end{cases}$

33. $\begin{cases} 5x + 4y = 5 \\ 15x - 2y = 8 \end{cases}$

▲ **B Problems**

Solve the systems in Problems 34–49 for all r

34. $\begin{cases} x + y = 7 \\ x - y = -1 \end{cases}$ 35. $\begin{cases} x - \\ x + \end{cases}$

37. $\begin{cases} x - 6y = -3 \\ 2x + 3y = 9 \end{cases}$ 38. $\begin{cases} y = \\ x - \end{cases}$

40. $\begin{cases} x - y = 2 \\ 2x + 3y = 9 \end{cases}$ 41. $\begin{cases} 3x \\ -x \end{cases}$

43. $\begin{cases} 6x + 9y = -4 \\ 9x + 3y = 1 \end{cases}$ 44. $\begin{cases} 100 \\ 50x \end{cases}$

46. $\begin{cases} q + d = 147 \\ 0.25q + 0.10d = 24.15 \end{cases}$ 47. $\begin{cases} x + \\ 0.4 \end{cases}$

49. $\begin{cases} y = 2x - 1 \\ y = -3x - 9 \end{cases}$

▲ **Problem Solving**

50. JOURNAL PROBLEM (From "When Doe
Anne Larson Quinn and Karen R. Lars
pp. 734–737.) The premise of this prob
times as quickly as humans, at some poi
Karen wanted to determine exactly on w
and her dog, Sydney, so that they could
facts. Karen was born on December 7,
For every year that Karen aged, Sydney
date are Karen and Sydney "the same a

802 CHAPTER TWELVE The Nature of Mathematical Systems

51. Assume that Sydney in Problem 50 is a cat instead of a dog, and assume that cats age four times as quickly as humans. Using this assumption, when will Karen and Sydney be "the same age"?

52. Assume that you obtain a dog that was born today. When will this dog be older (in dog-years) than you are?

53. HISTORICAL QUESTION The Louvre Tablet from the Babylonian civilization is dated about 1500 B.C. It shows a system equivalent to

$$\begin{cases} xy = 1 \\ x + y = a \end{cases}$$

Solve this system for x and y in terms of a. This is not a linear system, and you will need to use the quadratic formula after substituting.

54. HISTORICAL QUESTION The following problem was written by Leonhard Euler:

Two persons owe conjointly 29 pistoles;* they both have money, but neither of them enough to enable him, singly, to discharge this common debt. The first debtor says therefore to the second, "If you give me $\frac{2}{3}$ of your money, I can immediately pay the debt." The second answers that he also could discharge the debt, if the other would give him $\frac{3}{4}$ of his money. Required, how many pistoles each had?

12.2 P

Problem sol
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* A *pistole* is a

6.2 Installment Buying 401

9. If you purchase an item for $1,295 at an interest rate of 9.8%, and you finance it for 4 years, then the amount of add-on interest is about
A. $13.00 B. $500 C. $130

10. If you purchase a new car for $10,000 and finance it for 4 years, the amount of interest you would expect to pay is about
A. $4,000 B. $400 C. $24,000

11. A reasonable APR to pay for a 3-year installment loan is
A. 1% B. 12% C. 32%

12. A reasonable APR to pay for a 3-year automobile loan is
A. 6% B. 40% C. $2,000

13. If you wish to purchase a car with a sticker price of $10,000, a reasonable offer to make to the dealer is:
A. $10,000 B. $9,000 C. $11,000

14. A reasonable APR for a credit card is
A. 1% B. 30% C. 12%

15. If I do not pay off my credit card each month, the most important cost factor is
A. the annual fee B. the APR C. the grace period

16. If I pay off my credit card balance each month, the most important cost factor is
A. the annual fee B. the AP

17. The method of calculation most
A. previous balance method
B. adjusted balance method
C. average daily balance method

18. In an application of the average
A. $\frac{1}{12}$ B. $\frac{30}{365}$ C. $\frac{31}{365}$

19. When using the average daily ba
A. $\frac{1}{12}$ B. $\frac{30}{365}$ C. $\frac{31}{365}$

20. If your credit card balance is $65
card interest charge is
A. $6.50 B. $65 C. $8.2

21. If your credit card balance is $95
interest is calculated according t
charge is
A. $9.52 B. $9.02 C. $9

22. If your credit card balance is $95
interest is calculated according t
charge is
A. $9.52 B. $9.02 C. $9

23. If your credit card balance is $95
interest is calculated according t
charge is
A. $9.52 B. $9.02 C. $9

News Clip

WARNING
Credit Card Trap

Suppose you "max out" your credit card charges at $1,500 and decide that you will not use it again until it is paid off. How long do you think it will take you to pay it off if you make the minimum required payment (which would be $30 for the beginning balance of $1,500 and would decrease as the remaining balance decreases)?

A. 5 months
B. 2 years
C. 8 years
D. 12 years
E. 20 years

Even though we cannot mathe-

400 CHAPTER SIX The Nature of Sequences, Series, and Financial Management

Method	*Previous Balance*	*Adjusted Balance*	*Average Daily Balance*
P:	$1,000	$1,000 − $50 = $950	Balance is $1,000 for 10 days of 31-day month; balance is $950 for 21 days of 31-day month; $\frac{10 \times \$1{,}000 + 21 \times \$950}{31} = \$966.13$
r:	0.18	0.18	0.18
t:	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{31}{365}$
$I = Prt$:	$\$1{,}000(0.18)(\frac{1}{12})$ $= \$15.00$	$\$950(0.18)(\frac{1}{12})$ $= \$14.25$	$\$966.13\ (0.18)(\frac{31}{365})$ $= \$14.77$

You can sometimes make good use of credit cards by taking advantage of the period during which no finance charges are levied. Many credit cards charge no interest if you pay in full within a certain period of time (usually 20 or 30 days). This is called the **grace period.** On the other hand, if you borrow cash on your credit card, you should know that many credit cards have an additional charge for cash advances — and these can be as high as 4%. This 4% is *in addition to* the normal finance charges.

PROBLEM SET 6.2

▲ **A Problems**

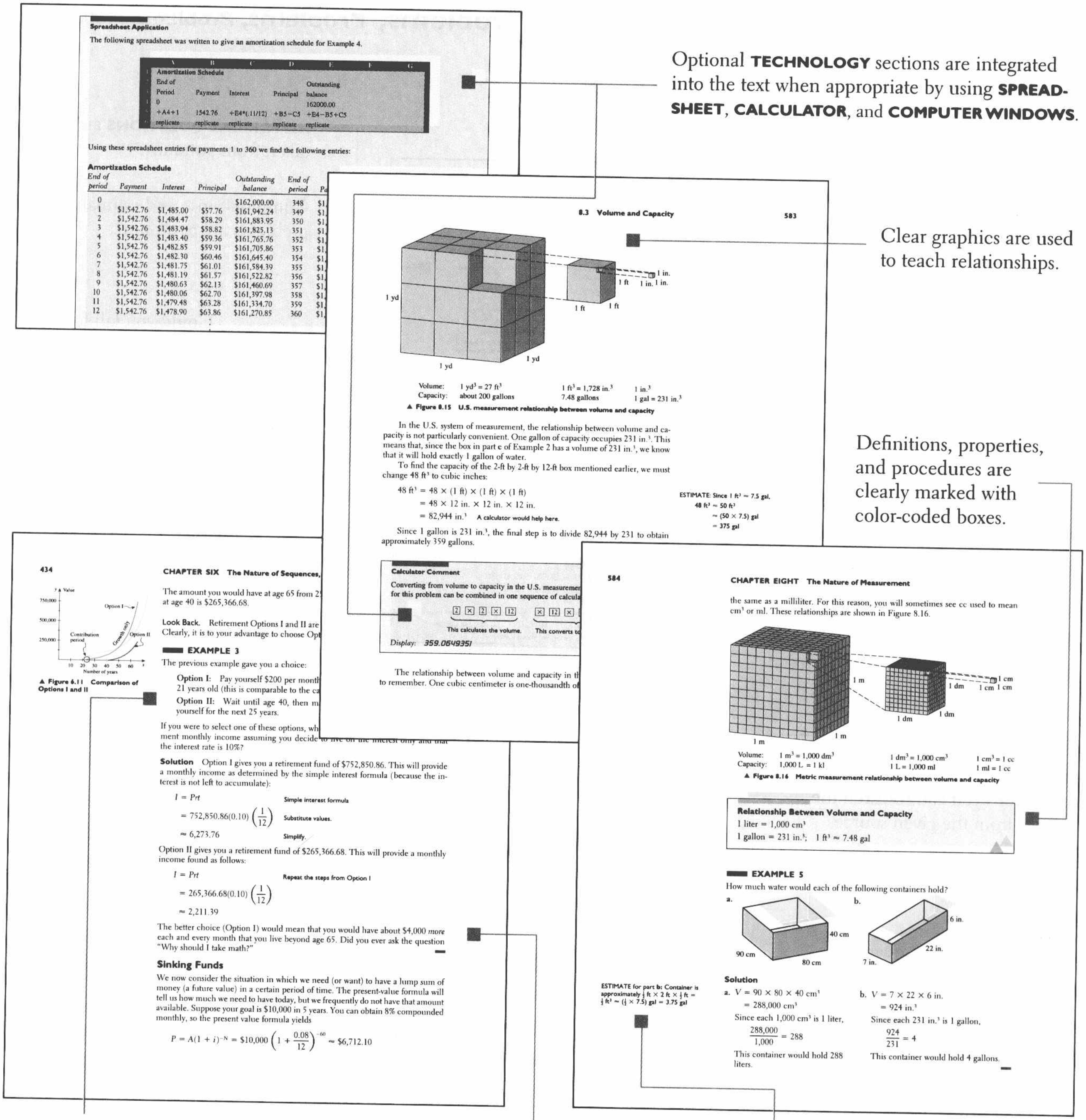
1. IN YOUR OWN WORDS What is add-on interest?
2. IN YOUR OWN WORDS What is APR?
3. IN YOUR OWN WORDS Compare and contrast open-ended and closed-ended credit.
4. IN YOUR OWN WORDS Discuss the methods of calculating credit card interest.
5. IN YOUR OWN WORDS Describe a good procedure for saving money with the purchase of an automobile.

Use estimation to select the best response in Problems 6–23. Do not calculate.

6. If you purchase a $2,400 item and pay for it with monthly installments for 2 years, the monthly payment is
A. $100 per month
B. more than $100 per month
C. less than $100 per month

7. If you purchase a $595.95 item and pay for it with monthly installments for 1 year, the monthly payment is
A. about $50 B. more than $50 C. less than $50

8. If you purchase an item for $1,295 at an interest rate of 9.8%, and you finance it for 1 year, then the amount of add-on interest is about
A. $13.00 B. $500 C. $130



Spreadsheet Application

The following spreadsheet was written to give an amortization schedule for Example 4.

	A	B	C	D	E	F	G
1	Amortization Schedule						
2	End of				Outstanding		
3	Period	Payment	Interest	Principal	balance		
4	0				162000.00		
5	+A4+1	1542.76	+B4*(.11/12)	+B5−C5	+E4−B5+C5		
6	replicate	replicate	replicate	replicate	replicate		

Using these spreadsheet entries for payments 1 to 360 we find the following entries:

Amortization Schedule

End of period	*Payment*	*Interest*	*Principal*	*Outstanding balance*	*End of period*	*Pa*
0				$162,000.00	348	$1,
1	$1,542.76	$1,485.00	$57.76	$161,942.24	349	$1,
2	$1,542.76	$1,484.47	$58.29	$161,883.95	350	$1,
3	$1,542.76	$1,483.94	$58.82	$161,825.13	351	$1,
4	$1,542.76	$1,483.40	$59.36	$161,765.76	352	$1,
5	$1,542.76	$1,482.85	$59.91	$161,705.86	353	$1,
6	$1,542.76	$1,482.30	$60.46	$161,645.40	354	$1,
7	$1,542.76	$1,481.75	$61.01	$161,584.39	355	$1,
8	$1,542.76	$1,481.19	$61.57	$161,522.82	356	$1,
9	$1,542.76	$1,480.63	$62.13	$161,460.69	357	$1,
10	$1,542.76	$1,480.06	$62.70	$161,397.98	358	$1,
11	$1,542.76	$1,479.48	$63.28	$161,334.70	359	$1,
12	$1,542.76	$1,478.90	$63.86	$161,270.85	360	$1,

8.3 Volume and Capacity 583

Volume:	1 yd^3 = 27 ft^3	1 ft^3 = 1,728 in.^{3}	1 in.^{3}
Capacity:	about 200 gallons	7.48 gallons	1 gal = 231 in.^{3}

▲ **Figure 8.15 U.S. measurement relationship between volume and capacity**

In the U.S. system of measurement, the relationship between volume and capacity is not particularly convenient. One gallon of capacity occupies 231 in.^{3}. This means that, since the box in part c of Example 2 has a volume of 231 in.^{3}, we know that it will hold exactly 1 gallon of water.

To find the capacity of the 2-ft by 2-ft by 12-ft box mentioned earlier, we must change 48 ft^3 to cubic inches:

$$\begin{aligned} 48 \text{ ft}^3 &= 48 \times (1 \text{ ft}) \times (1 \text{ ft}) \times (1 \text{ ft}) \\ &= 48 \times 12 \text{ in.} \times 12 \text{ in.} \times 12 \text{ in.} \\ &= 82{,}944 \text{ in.}^3 \quad \textbf{A calculator would help here.} \end{aligned}$$

ESTIMATE: Since 1 ft^3 ≈ 7.5 gal,
48 ft^3 ≈ 50 ft^3
≈ (50 × 7.5) gal
= 375 gal

Since 1 gallon is 231 in.^{3}, the final step is to divide 82,944 by 231 to obtain approximately 359 gallons.

Calculator Comment

Converting from volume to capacity in the U.S. measuremer
for this problem can be combined in one sequence of calcula

$\underbrace{\boxed{2}\ \boxed{\times}\ \boxed{2}\ \boxed{\times}\ \boxed{12}}_{\text{This calculates the volume.}}$ $\underbrace{\boxed{\times}\ \boxed{12}\ \boxed{\times}}_{\text{This converts to}}$

Display: 359.0649351

The relationship between volume and capacity in th
to remember. One cubic centimeter is one-thousandth of

434 **CHAPTER SIX The Nature of Sequences,**

▲ **Figure 6.11 Comparison of Options I and II**

The amount you would have at age 65 from 2
at age 40 is $265,366.68.

Look Back. Retirement Options I and II are
Clearly, it is to your advantage to choose Opt

EXAMPLE 3

The previous example gave you a choice:

Option I: Pay yourself $200 per month
21 years old (this is comparable to the ca

Option II: Wait until age 40, then m
yourself for the next 25 years.

If you were to select one of these options, wh
ment monthly income assuming you decide
the interest rate is 10%?

Solution Option I gives you a retirement fund of $752,850.86. This will provide a monthly income as determined by the simple interest formula (because the interest is not left to accumulate):

$$\begin{aligned} I &= Prt && \textbf{Simple interest formula} \\ &= 752{,}850.86(0.10)\left(\frac{1}{12}\right) && \textbf{Substitute values.} \\ &\approx 6{,}273.76 && \textbf{Simplify.} \end{aligned}$$

Option II gives you a retirement fund of $265,366.68. This will provide a monthly income found as follows:

$$\begin{aligned} I &= Prt && \textbf{Repeat the steps from Option I} \\ &= 265{,}366.68(0.10)\left(\frac{1}{12}\right) \\ &\approx 2{,}211.39 \end{aligned}$$

The better choice (Option I) would mean that you would have about $4,000 *more* each and every month that you live beyond age 65. Did you ever ask the question "Why should I take math?"

Sinking Funds

We now consider the situation in which we need (or want) to have a lump sum of money (a future value) in a certain period of time. The present-value formula will tell us how much we need to have today, but we frequently do not have that amount available. Suppose your goal is $10,000 in 5 years. You can obtain 8% compounded monthly, so the present value formula yields

$$P = A(1+i)^{-N} = \$10{,}000\left(1+\frac{0.08}{12}\right)^{-60} \approx \$6{,}712.10$$

584 **CHAPTER EIGHT The Nature of Measurement**

the same as a milliliter. For this reason, you will sometimes see cc used to mean cm^3 or ml. These relationships are shown in Figure 8.16.

Volume:	1 m^3 = 1,000 dm^3	1 dm^3 = 1,000 cm^3	1 cm^3 = 1 cc
Capacity:	1,000 L = 1 kl	1 L = 1,000 ml	1 ml = 1 cc

▲ **Figure 8.16 Metric measurement relationship between volume and capacity**

Relationship Between Volume and Capacity
1 liter = 1,000 cm^3
1 gallon = 231 in.^{3}; 1 ft^3 ≈ 7.48 gal

EXAMPLE 5

How much water would each of the following containers hold?

a.

b.

Solution

a. $V = 90 \times 80 \times 40 \text{ cm}^3$
$= 288{,}000 \text{ cm}^3$
Since each 1,000 cm^3 is 1 liter,
$\frac{288{,}000}{1{,}000} = 288$
This container would hold 288 liters.

b. $V = 7 \times 22 \times 6$ in.
$= 924 \text{ in.}^3$
Since each 231 in.^{3} is 1 gallon,
$\frac{924}{231} = 4$
This container would hold 4 gallons.

ESTIMATE for part b: Container is approximately $\frac{1}{2}$ ft × 2 ft × $\frac{1}{2}$ ft = $\frac{1}{2}$ ft^3 ≈ ($\frac{1}{2}$ × 7.5) gal = 3.75 gal

Optional **TECHNOLOGY** sections are integrated into the text when appropriate by using **SPREADSHEET**, **CALCULATOR**, and **COMPUTER WINDOWS**.

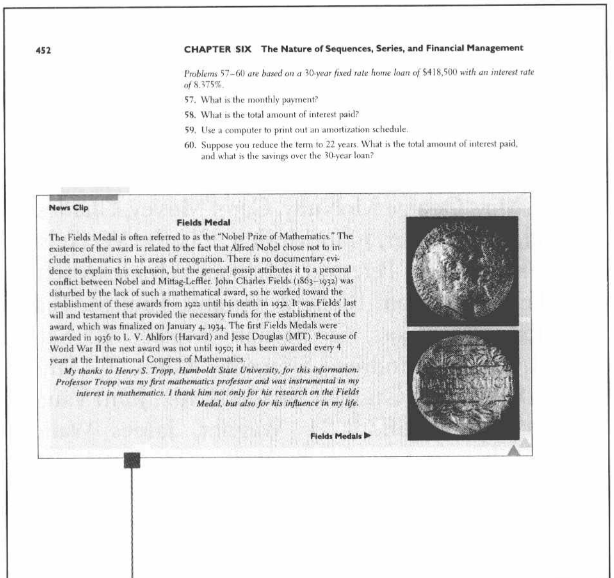
Clear graphics are used to teach relationships.

Definitions, properties, and procedures are clearly marked with color-coded boxes.

Examples are clearly marked; author's notes are provided to explain steps.

"Why should I take math?" or, in terms of this question, "Would I rather have $6,273.76/mo or $2,211.39/mo?" Here is how!

Author's notes are frequent.



452 **CHAPTER SIX The Nature of Sequences, Series, and Financial Management**

Problems 57–60 are based on a 30-year fixed rate home loan of $418,500 *with an interest rate of* 8.375%.

57. What is the monthly payment?
58. What is the total amount of interest paid?
59. Use a computer to print out an amortization schedule.
60. Suppose you reduce the term to 22 years. What is the total amount of interest paid, and what is the savings over the 30-year loan?

News Clip

Fields Medal

The Fields Medal is often referred to as the "Nobel Prize of Mathematics." The existence of the award is related to the fact that Alfred Nobel chose not to include mathematics in his areas of recognition. There is no documentary evidence to explain this exclusion, but the general gossip attributes it to a personal conflict between Nobel and Mittag-Leffler. John Charles Fields (1863–1932) was disturbed by the lack of such a mathematical award, so he worked toward the establishment of these awards from 1922 until his death in 1932. It was Fields' last will and testament that provided the necessary funds for the establishment of the award, which was finalized on January 4, 1934. The first Fields Medals were awarded in 1936 to L. V. Ahlfors (Harvard) and Jesse Douglas (MIT). Because of World War II the next award was not until 1950; it has been awarded every 4 years at the International Congress of Mathematics.

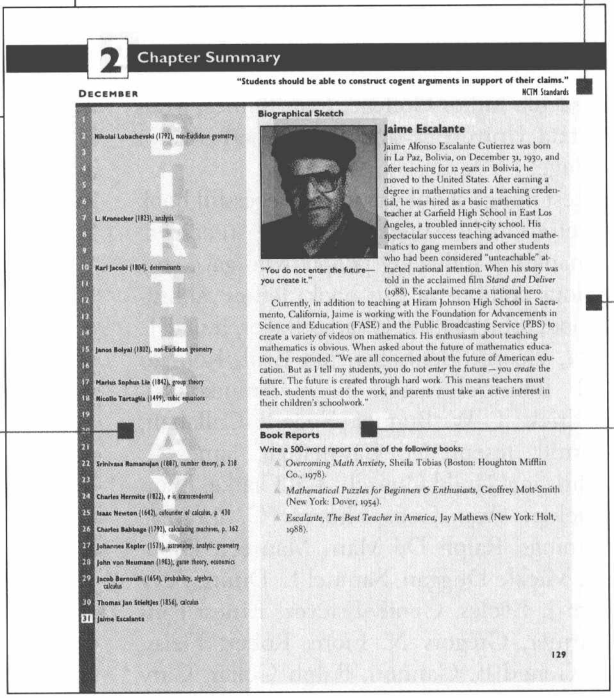
My thanks to Henry S. Tropp, Humboldt State University, for this information. Professor Tropp was my first mathematics professor and was instrumental in my interest in mathematics. I thank him not only for his research on the Fields Medal, but also for his influence in my life.

Fields Medals ▶

You have heard of the Nobel Prize, but have you ever heard of the Fields Medal? The text uses a variety of techniques to make mathematics alive and contemporary.

Mathematics involves real people. This interview with Jaime Escalante, subject of the movie *Stand and Deliver* (whose birthday, by the way, is December 31), speaks directly to the readers of this book.

This book conforms to the NCTM *Standards*.



2 Chapter Summary

"Students should be able to construct cogent arguments in support of their claims."
NCTM Standards

DECEMBER

BIRTHDAYS

1	
2	**Nikolai Lobachevski** (1792), non-Euclidean geometry
3	
4	
5	
6	
7	**L. Kronecker** (1823), analysis
8	
9	
10	**Karl Jacobi** (1804), determinants
11	
12	
13	
14	
15	**Janos Bolyai** (1802), non-Euclidean geometry
16	
17	**Marius Sophus Lie** (1842), group theory
18	**Nicollo Tartaglia** (1499), cubic equations
19	
20	
21	
22	**Srinivasa Ramanujan** (1887), number theory, p. 218
23	
24	**Charles Hermite** (1822), e is transcendental
25	**Isaac Newton** (1642), cofounder of calculus, p. 430
26	**Charles Babbage** (1792), calculating machines, p. 162
27	**Johannes Kepler** (1571), astronomy, analytic geometry
28	**John von Neumann** (1903), game theory, economics
29	**Jacob Bernoulli** (1654), probability, algebra, calculus
30	**Thomas Jan Stieltjes** (1856), calculus
31	**Jaime Escalante**

Biographical Sketch

Jaime Escalante

"You do not enter the future—you create it."

Jaime Alfonso Escalante Gutierrez was born in La Paz, Bolivia, on December 31, 1930, and after teaching for 12 years in Bolivia, he moved to the United States. After earning a degree in mathematics and a teaching credential, he was hired as a basic mathematics teacher at Garfield High School in East Los Angeles, a troubled inner-city school. His spectacular success teaching advanced mathematics to gang members and other students who had been considered "unteachable" attracted national attention. When his story was told in the acclaimed film *Stand and Deliver* (1988), Escalante became a national hero.

Currently, in addition to teaching at Hiram Johnson High School in Sacramento, California, Jaime is working with the Foundation for Advancements in Science and Education (FASE) and the Public Broadcasting Service (PBS) to create a variety of videos on mathematics. His enthusiasm about teaching mathematics is obvious. When asked about the future of mathematics education, he responded. "We are all concerned about the future of American education. But as I tell my students, you do not *enter* the future—you *create* the future. The future is created through hard work. This means teachers must teach, students must do the work, and parents must take an active interest in their children's schoolwork."

Book Reports

Write a 500-word report on one of the following books:

- *Overcoming Math Anxiety*, Sheila Tobias (Boston: Houghton Mifflin Co., 1978).
- *Mathematical Puzzles for Beginners & Enthusiasts*, Geoffrey Mott-Smith (New York: Dover, 1954).
- *Escalante, The Best Teacher in America*, Jay Mathews (New York: Holt, 1988).

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Birthdates of famous mathematicians are shown, month by month.

Reading about mathematics is encouraged with suggested book reports at the end of each chapter.

Changes from the Previous Edition

You will find this edition substantially unchanged from the seventh edition. I have added group research projects at the end of each chapter, included *z*-scores in statistics, and you will find many new and interesting problems, as well as a glossary of terms. I have retained all of the familiar features that were in the previous edition.

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Karl J. Smith
Sebastopol, CA
email: smithkjs@wco.com

To the Student

A Fable

Once upon a time, two young ladies, Shelley and Cindy, came to a town called Mathematics. People had warned them that this is a particularly confusing town. Many people who arrived in Mathematics were very enthusiastic, but could not find their way around, became frustrated, gave up, and left town.

Shelley was strongly determined to succeed. She was going to learn her way through the town. For example, to learn how to go from her dorm to class, she concentrated on memorizing this clearly essential information: she had to walk 325 steps south, then 253 steps west, then 129 steps in a diagonal (southwest), and finally 86 steps north. It was not easy to remember all of that, but fortunately she had a very good instructor who helped her to walk this same path 50 times. To stick to the strictly necessary information, she ignored much of the beauty along the route, such as the color of the adjacent buildings or the existence of trees, bushes, and nearby flowers. She always walked blindfolded. After repeated exercising, she succeeded in learning her way to class and also to the cafeteria. But she could not learn the way to the grocery store, the bus station, or a nice restaurant; there were just too many routes to memorize. It was so overwhelming! Finally she gave up and left town; Mathematics was too complicated for her.

Cindy, on the other hand, was of a much less serious nature. To the dismay of her instructor, she did not even intend to memorize the number of steps of her walks. Neither did she use the standard blindfold that students need for learning. She was always curious, looking at the different buildings, trees, bushes, and nearby flowers or anything else not necessarily related to her walk. Sometimes she walked down dead-end alleys to find out where they were leading, even if this was obviously superfluous. Curiously, Cindy succeeded in learning how to walk from one place to another. She even found it easy and enjoyed the scenery. She eventually built a building on a vacant lot in the city of Mathematics.*

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